Solutions for Tutorial 5
Dynamic Behavior of Typical Dynamic Systems

5.1 **First order System:** A model for a first order system is given in the following equation.

$$\frac{dY}{dt} = X_{in} - X_{out} \qquad (5.1.1)$$

What conditions have to be satisfied for the system to be self-regulating?

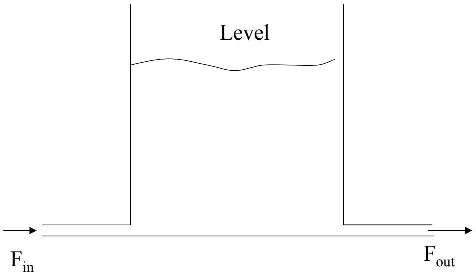
A stable self-regulating system has an output variable that tends to a steady state after the input variable has reached an altered steady-state value. The system described in equation (5.1.1) will not necessarily be self-regulating.

If both X_{in} and X_{out} are independent of Y, the derivative of the output variable is independent of the input variable. For example, the following condition could occur.

$$\frac{dY}{dt} = X_{in} - X_{out} = 5 - 2 = 3$$

Since the derivative is a constant, the output variable would increase without limit. Therefore, the system is non-self-regulating and is unstable.

Let's look at a physical system that is stable and self-regulatory. The level in the tank is affected by the flow in to and out of the tank. The overall material balance has the form of equation (5.1.1) and gives the following for a tank with straight sides.



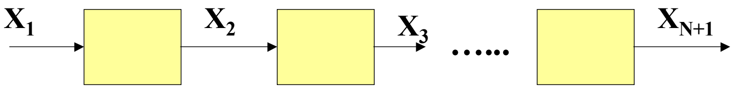
$$\frac{dL}{dt} = F_{in} - F_{out}$$

As the level increases, the flow in decreases, which is a stabilizing effect. Also, as the level increases, the flow out increases, which is a stabilizing effect. This is a self-regulating, first-order system.

Note that a self-regulating system is not guaranteed to behave well. For the level example, a large increase in the flow in (due to an increase in the source pressure) will cause the level to increase. The flow out will also increase, but not necessarily enough to reach a constant level before the level overflows. We see that the magnitude of a disturbance will influence whether the variables in a self-regulating system remain within acceptable limits.

We can draw two conclusions from Question 5.1.
1. We seek to design processes without non-self-regulating variables. (This is not always possible.)
2. We must control non-self-regulating variables. (This is possible; see Chapter 18 for details.)

5.2 Second and higher order systems can be over or under damped. Which is more likely to occur in chemical processes?



Most chemical processes are interconnections of first-order systems, resulting from material and energy balances. These interconnections involve interacting and non-interacting first order systems, which are overdamped. Therefore, the vast majority of chemical process - without feedback control - are overdamped.

However, we will see that the application of feedback control to these processes can, and often does, result in underdamped systems. So, even though models developed in Chapters 3-5 are overdamped, engineers must deal with underdamped behavior.

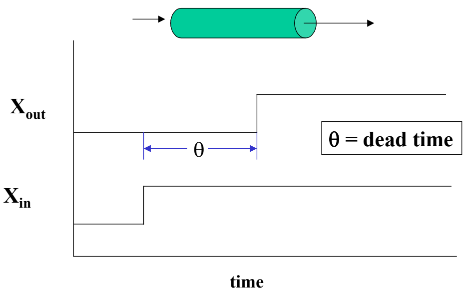
Most chemical process without control are overdamped.

5.3 You are working in a plant and need to estimate the delay for flow through a pipe. How can you evaluate the dead time?

There are two obvious ways.

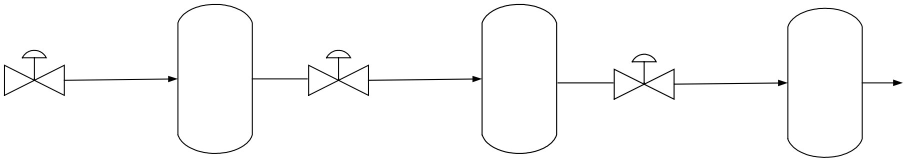
1. Measure the length of the pipe (L). Then, determine the velocity of the fluid in the pipe (v). For turbulent flow (with a flat velocity profile), the dead time would be $\theta = L/v$.

2. Perhaps, the pipe is underground, and we do not know the path taken. We can perform an experiment to evaluate dead time. We can introduce a step change of a tracer component with a small flow rate, so that the tracer does not modify the process behavior. The dead time is the time between the introduction of the tracer at the inlet to the



pipe and the first time that the tracer appeared at the pipe outlet.

5.4 Are the pressures in the vessels in Figure 5.4 self-regulating or non-self-regulating? The fluid is a gas, and the feed and exhaust pressures are constant. In answering this question, think about the response of the system to a change in the percent opening of the first valve.



Qualitative analysis: We begin by recognizing that the flow rate through a pipe-valve combination depends on the pressure difference (P_{in} - P_{out}), assuming that the flow rate is sub-sonic. When the first valve opening is increased, the flow into the first vessel increases. The increase in vessel pressure will offer greater resistance to the flow in and a greater driving force for the flow out. Therefore, the vessel pressure is self-regulating.

Modelling: The mass balance for the gas in a vessel is given by the following.

$$\frac{d(mass)}{dt} = \rho_{in} F_{in} - \rho_{out} F_{out} = \rho_{in} C_v (v_{i-1}) \sqrt{P_{i-1} - P_i} - \rho_{out} C_v (v_i) \sqrt{P_i - P_{i+1}}$$

Also, the mass in the vessel can be related to the pressure by the ideal gas law. If the temperature is assumed constant, the derivative of mass is simply a constant times the derivative of pressure.

$$mass = \frac{PV(MW)}{RT}$$

$$\frac{dP}{dt} = \frac{RT}{V(MW)} \frac{d(mass)}{dt}$$

Substituting, yields the expression that demonstrates the reliance of the pressure derivative on the pressure

$$\frac{V(MW)}{RT} \frac{dP_i}{dt} = \rho_{in} C_v (v_{i-1}) \sqrt{P_{i-1} - P_i} - \rho_{out} C_v (v_i) \sqrt{P_i - P_{i+1}}$$

Therefore, the pressure in each vessel is self-regulating.

Note that the process is an interacting series of first-order systems.

5.5 You have obtained the graph in Figure 5.5 by making a step to a valve opening and observing the dynamic response of the temperature. From the results of this experiment, describe the physical process (order, dead time, etc.)

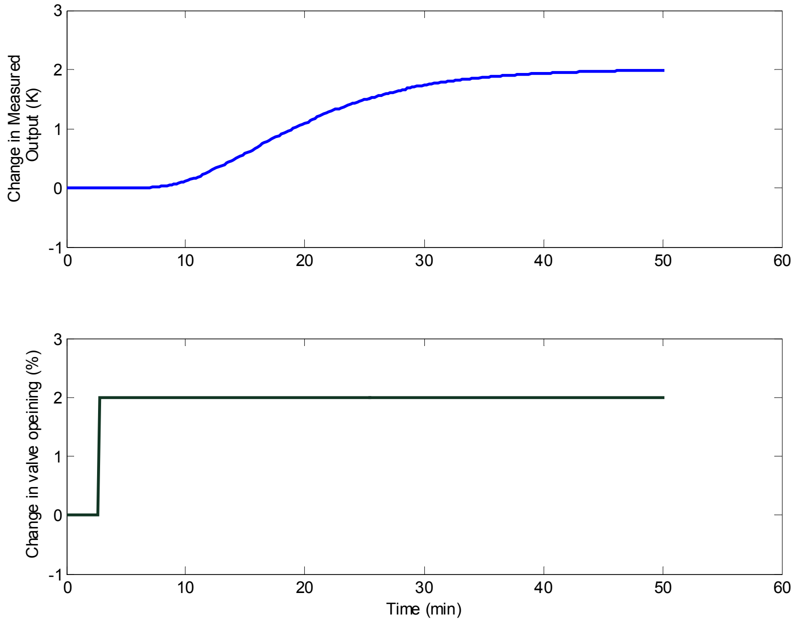


Figure 5.5

The experimental data gives us valuable information about the process. In fact, we will see in the upcoming topics that this type of information is exactly what is used for designing control systems. However, the data shows the "input-output" behavior only, and it does not provide sufficient information to enable us to reconstruct the complete process structure.

Let's see what we can conclude about the process.

- The output variable attains steady state after a step change in the input. We conclude that the process is stable and self-regulatory.
- From the shape of the output to a step, which does not oscillate, we conclude that the process is overdamped.
- The output does not change perceptibly when the input variable is first changed. This indicates a "dead time". However, we cannot be sure about the process structure that would yield this behavior. Recall that a series of first-order processes has a step response with essentially no change for an initial period; we call this "apparent dead time". So, we conclude that the process has either an actual time delay, e.g., a pipe, or a higher order, overdamped process. Naturally, a combination of dead time and time constants is also possible.

- Since the output has no "inverse response" we conclude that no negative zeros in the transfer function. Since the output does not overshoot its final value, we conclude that positive zeros are not greater than the poles. In short, the step response is smooth and monotonic in spite of any parallel paths that might exist.
- We can determine the steady-state gain from the graph, which is

$$K_p = \Delta(\text{output})/\ \Delta\ (\text{input}) \approx 1.0\ \text{K/\%open}$$

- We can determine the "speed" of the response, which we characterize using the 63% time of the response.

$$t_{63\%} = \sum_i (\theta_i + \tau_i) \approx 15.0\ \text{min}$$

As we see, we can learn a lot from the data, but we cannot describe the process exactly.

5.6 We have models for several processes which we decide to connect in the process structure shown in Figure 5.6. The input variable experiences a step change of 3.5 % open. Describe the dynamic behavior based on qualitative and semi-quantitative analysis, that is, do not simulate the process.

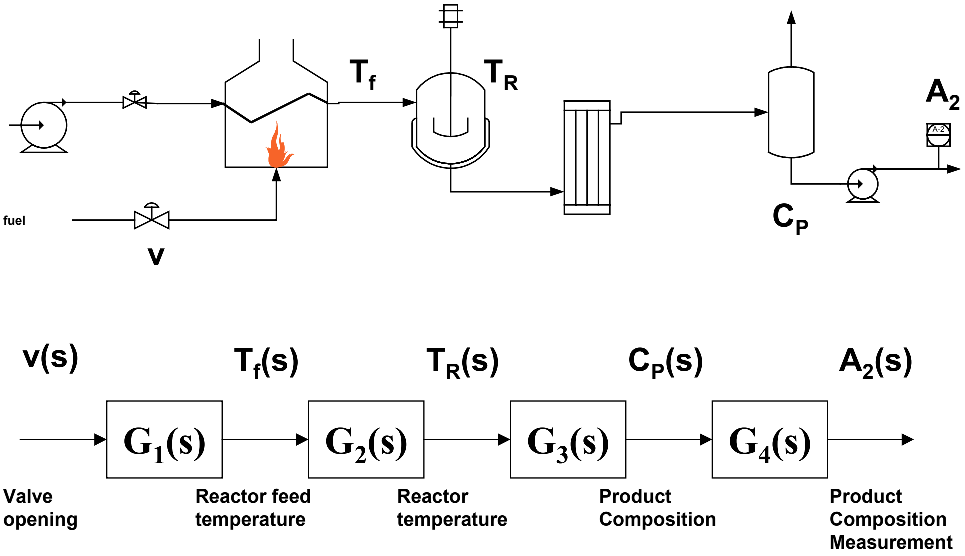


Figure 5.6

The models for the system are given in the following.

$$G_1(s) = \frac{1.2e^{-1s}}{5s+1} \qquad G_3(s) = \frac{1.5e^{-2s}}{(3s+1)(5s+1)}$$

$$G_2(s) = \frac{0.80e^{0.5s}}{(2s+1)} \qquad G_4(s) = \frac{1.0e^{-0.5s}}{(1s+1)(2s+1)}$$

We can determine a great deal about the dynamic response.

- The processes are in series; therefore the overall transfer function is the product of the individual process transfer functions.
- Each individual process is satble (negative poles), so the series is stable.
- The steady-state gain of the series is the product of the individual gains.

  Kp = (1.2)(0.80)(1.5)(1.0) = 1.44 mole fraction/%open

  From this result, we can calculate the steady-state change in the product composition for a 3.5% change in the valve opening.

  ΔA1 = 1.44*3.5 = 5.04 mole fraction

- The shape of the dynamic response can be determined in a qualitative way. First, some dead time will exist. Second, the system is sixth order and overdamped, because the roots of the denominator are all real. (Note they can be factored.)
- The "speed" of the process can be estimated from the 63% time of the step response, which is the sum of the dead times and time constants of the elements in the series.

  $$t_{63\%} \approx (1+0.5+2+2+3+5+0.5+1+2) = 17 \ \textbf{minutes}$$

We could determine an approximate first-order with dead time model using the moments method in Appendix D, but this effort is not usually warranted. We already have a good understanding of the response, and we can simulate it easily if more precise results are required.

5.7 The recycle process shown in Figure 5.7 is to be analyzed in this question.

Reaction: A → B Rate: $-r_A = kC_A$

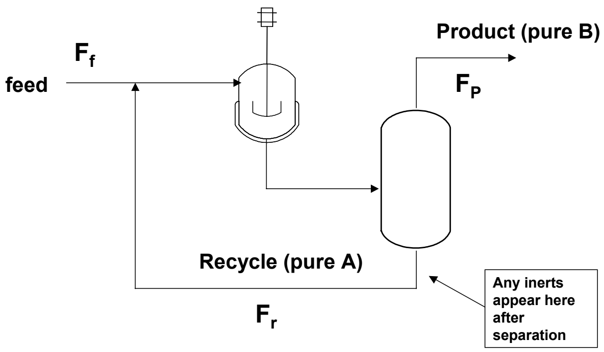


Figure 5.7

Information:

- The initial steady-state reactor conversion in the isothermal, constant-volume CSTR is 50%. Therefore, $F_f = F_R$.
- The separator has first order dynamics.

a. Determine the dynamic behavior of the concentration of an inert that enters in the fresh feed. The inert exits the separation unit in the bottoms stream that is the recycle; none leaves in the product stream. (To simplify the analysis, assume that the concentration of the inert is initially small, so that the chemical reaction and the total flow rates are not affected by changes in the inert concentration.)

b. Determine the response of the concentration of the reactant to a change in the reactor temperature that reduces the reaction rate by 10%, i.e., from 50% to 45%.

a. Qualitative analysis: We note inert material enters with the fresh feed and does not exit the process. Therefore, inert must accumulate in the process, leading to an increasing concentration. Thus, the inert composition is a non-self-regulatory variable. While the composition is initially small and might not affect the process, it will ultimately increase sufficiently to affect the reaction and separation.

We know from our Material and Energy Balances course that a recycle system should have a purge to prevent an excessive concentration of inert. Naturally, the purge can be costly due to loss of material; therefore, the purge rate is set to achieve the acceptable inert concentration.

Quantitative analysis: For the inert component, component balances are required. Note that we take advantage of the assumption that the total flows and reaction rate are not affected, which is valid when the inert concentration is very small at the initial part of the transient. The following balances can be derived.

Reactor feed concentration (mass fraction) (essentially steady-state mixing):

$$x_{fi}(s) = 0.50x_{freshi}(s) + 0.50x_{recyclei}(s)$$

Reactor outlet concentration (essentially, a mixing tank):

$$x_{reactori}(s) = \frac{1}{\tau_R s + 1} x_{fi}(s)$$

Recycle concentration (first order dynamics given in statement)

$$x_{recyclei}(s) = \frac{2}{\tau_S s + 1} x_{reactori}(s)$$

These equations can be combined to give a concentration response to a change in the fresh feed concentration. We will select the reactor feed concentration. We solve the linear equations simultaneously, by combining and eliminating variables (using methods introduced in Chapter 4).

$$\begin{aligned} x_{fi}(s) &= 0.50 x_{freshi}(s) + 0.50 x_{recyclei}(s) \\ &= 0.50 x_{freshi}(s) + 0.50 \frac{2}{\tau_S s + 1} x_{reactori}(s) \\ &= 0.50 x_{freshi}(s) + 0.50 \frac{2}{\tau_S s + 1} \frac{1}{\tau_R s + 1} x_{fi}(s) \end{aligned}$$

Solve for $x_{fi}(s)$,

$$x_{fi}(s)\left[1 - 0.50 \frac{2}{\tau_S s + 1} \frac{1}{\tau_R s + 1}\right] = 0.50 x_{freshi}(s)$$

Rearrange to yield a transfer function,

$$\frac{x_{fi}(s)}{x_{freshi}(s)} = \boxed{\frac{1}{s}} \frac{0.50(\tau_S s + 1)(\tau_R s + 1)}{\tau_S \tau_R s + (\tau_S + \tau_R)}$$

The model for the reactor feed inert concentration has an "1/s" in the transfer function. This is a "pure integrator". The quantitative analysis confirms our conclusion from the qualitative analysis.

b. <u>Qualitative analysis</u>: No reactant exits the process; therefore, all reactant must be consumed by the chemical reaction. Since the reactor temperature has decreased, the rate of chemical reaction decreases. Therefore, the initial response must be an increase of reactant in the system. Is reactant concentration also a pure integrator?

The difference is the concentration in the reactor affects the reaction rate, the consumption of reactant A and production of product B. The reactant concentration increases until the reaction rate attains its original value. Since the temperature caused a 10% decrease in reaction rate, the concentration must increase enough for the rate (kC_A) to achieve a new steady state. To increase the concentration, the recycle flow rate must increase.

The net effect is a reduction of the "single-pass" reactor conversion and an increase in the recycle flow rate, so that the "overall conversion" attains its previous value.

In practice, the best single-pass conversion depends upon side reactions and energy costs for recycle.

Quantitative Analysis: The models and analysis for this system is presented in the design example in Chapter 25. The large increase in the recycle is demonstrated. The effect is sometimes called the "snow-ball" effect for the buildup of snow on a ball as it rolls downhill on snow-covered ground. Some control designs can avoid this behavior.