Solutions for Tutorial 13
Feedback Control Performance

13.1 The process in Figure 13.1 has a single-loop feedback controller using the PID algorithm. We seek to maintain the product composition within ± 0.10 mole/m^3 of the set point for all disturbances. The feedback dynamics between the heating valve and the analyzer and the disturbance dynamics between the feed composition and the analyzer are given in the models.

$$\frac{A1(s)}{v(s)} = \frac{-0.11e^{-44s}}{(54s+1)}$$

$$\frac{A1(s)}{XF(s)} = \frac{0.50e^{-42s}}{(35s+1)}$$

A1 is product composition (mole/m^3)
v is the valve affecting the heating stream (% open)
XF is feed composition (mole/m^3)
Time is in seconds

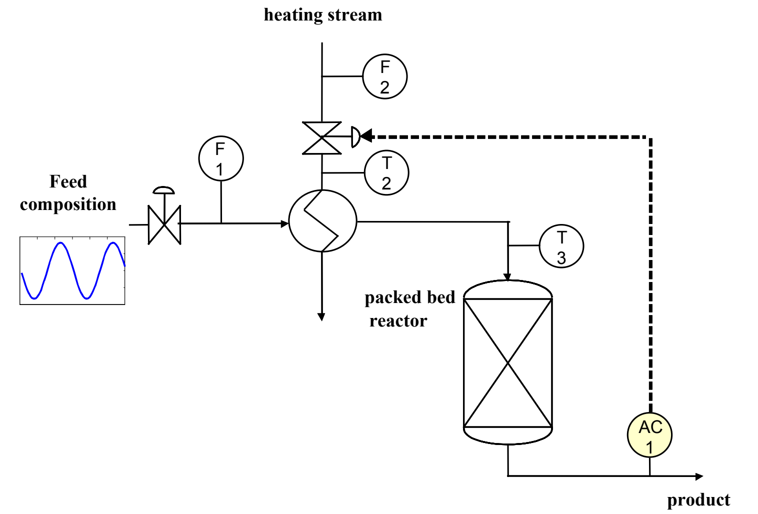


Figure 13.1

a. A feed composition disturbance occurs that can be approximated by a sine. The disturbance magnitude is 0.50 mole/m^3 and the period is 6280 s/cycle, i.e., its frequency is 10^{-3} rad/s. Without simulating, do you think that the feedback control can maintain the product composition within the desired maximum deviation?

Solution: We want to determine the behavior of the closed-loop system. Before starting, we will tune the PID controller, which is required for the quantitative calculations to check our answer. We will use the tuning correlations in the textbook.

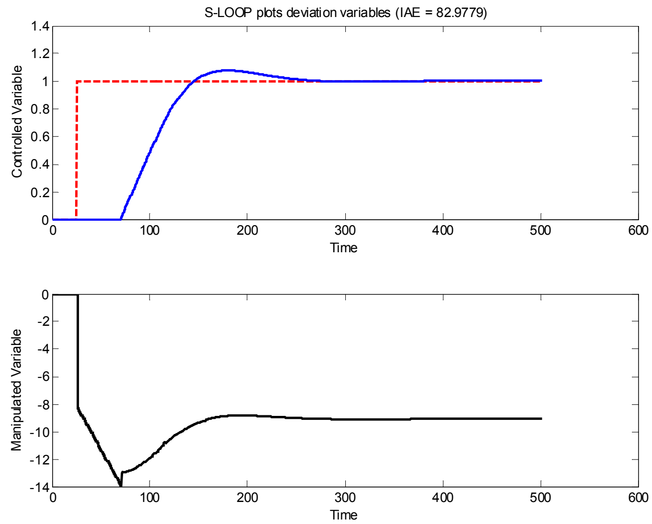
The fraction of dead time is equal to 44/98 = 0.45. From Ciancone's tuning correlation,

$K_cK_p = 0.9$ $\quad T_I/(\theta+\tau) = 0.66,$ $\quad T_D/(\theta+\tau) = 0.07$

The PID tuning parameters are:

$K_c = 0.9/(-.11) = -8.2$ %open/ mole/m^3
$T_I = 98(0.66) = 64.7$ s
$T_D = 0.07(98) = 6.9$ s

The closed-loop behavior in the time domain for a step set point change shows that the tuning is reasonable.



Now, how good is the performance for a sine input with a frequency of .001 rad/s? We know that the feedback controller will function well for disturbances at frequencies much lower than the feedback critical frequency. Also, feedback is not effective at much higher frequencies (but the process attenuates the disturbance). Near the critical frequency, the control performance will be worst.

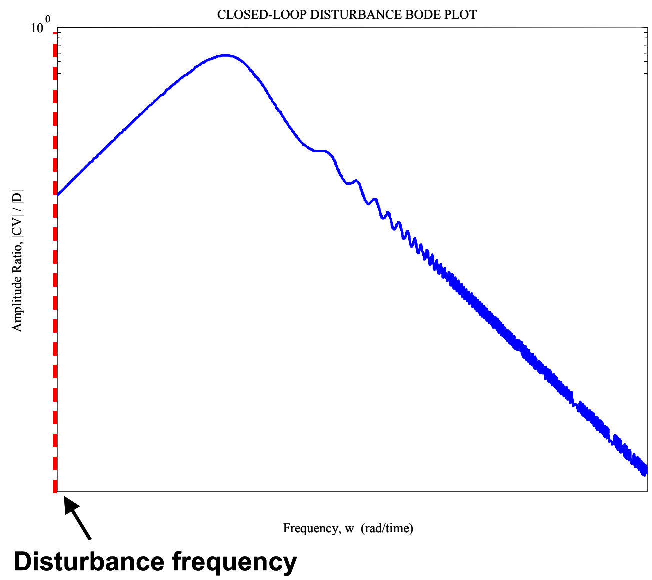
The critical frequency for this feedback loop is defined by the following equation.

$$\Phi = -180^{\circ} = \theta\omega(360/2\pi) + \tan^{-1}(\omega\tau)$$

The trial and error solution gives $\omega_c = 0.044$ rad/s.

The disturbance frequency is much small that the critical frequency of the closed-loop system. Therefore, we predict that the control performance should be good.

This qualitative analysis is confirmed by the quantitative calculation, here performed using S_LOOP. The amplitude ratio is .035; therefore, the output amplitude would be (0.5 mole/m^3/mole/m^3)(0.035 mole/m^3) = 0.0175 mole/m^3 << 0.10 mole/m^3. Therefore the control performance would be acceptable.



b. Here, we repeat part (a) with a different disturbance frequency. A feed composition disturbance occurs that can be approximated by a sine. The disturbance magnitude is 0.50 and the period is 300 s, i.e., its frequency is about .02 rad/s. Without simulating, do you think that the feedback control can maintain the product composition within the desired maximum deviation?

In this case, the disturbance frequency is near the critical frequency of the closed-loop system. Therefore, a quick estimate of the output amplitude (K_d*ΔD = 0.5*0.50 = 0.25) is greater than the maximum allowed amplitude. In fact, essentially the same answer is obtained using the frequency response above from the quantitative calculation. In this case, we predict that acceptable dynamic performance cannot be achieved. Other methods are required to improve performance, and some will be introduced in subsequent chapters.

This question demonstrated the importance of the disturbance frequency on feedback control performance. Disturbances near the critical frequency are not affected by feedback and not reduced by process time constants.

13.2 The series of first order processes in Figure 13.2 without control experiences an input disturbance that can be approximated as a sine. The input has a magnitude of 1.0 and a frequency of 0.333 rad/min. Determine the output of each system in the series, and discuss the results. Each of the systems has the same dynamic model, given in the following equation.

$$\frac{X_{i+1}(s)}{X_i(s)} = \frac{0.80}{(3s+1)}$$

time in minutes

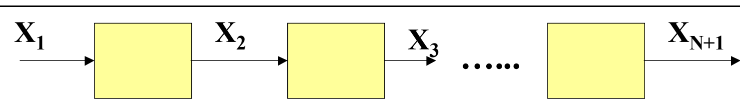


Figure 13.2.

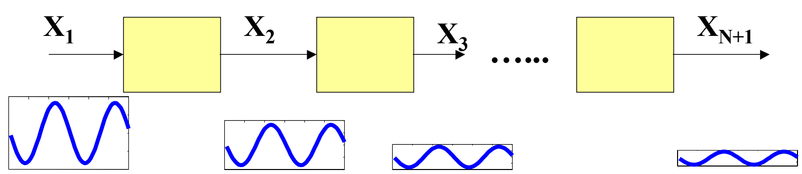
The amplitude ratio for each of the systems is given below.

$$|G(j\omega)| = \left| \frac{0.80}{\sqrt{1+(3*.333)^2}} \right| = \frac{0.80}{1.414} = 0.566$$

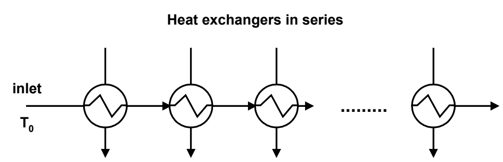
The amplitude ratio for a series process is the product of the individual amplitude ratios.

$$|G(j\omega)|^n = \left| \frac{0.80}{\sqrt{1+(3*.333)^2}} \right|^n = \left(\frac{0.80}{1.414} \right)^n = (0.566)^n$$

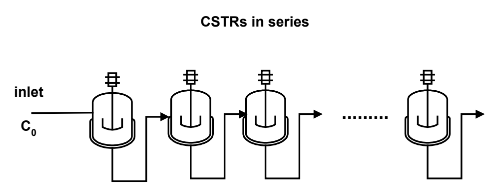
We see that the amplitude ratio for each system is less than one and that the series amplitude ratio is the amplitude ratio for single system to the nth power. Therefore, the amplitude ratio will decrease as the series has more elements.



This result is important, because we learn that a series of process (with AR<1) will reduce the effect of a periodic disturbance without control. Let's look at a couple of typical process systems.



The temperature in a series of heat exchangers or the feed composition in a series of chemical reactors will behave as we have seen for a series system.



The question demonstrated that a series of processes (with an amplitude ratio less than 1.0) can attenuate a periodic disturbance, even if no control is applied.

13.3 In the previous questions, the amplitude ratio had the same or smaller value than the disturbance gain for every system. Is this relationship true for all process systems?

The chemical reactor without feedback control in Figure 13.3 has the following transfer function, which is derived in Appendix C of Marlin (2000).

$$\frac{T(s)}{F_c(s)} = \frac{-(6.07s + 45.84)}{(s^2 + 1.79s + 35.80)} = \frac{K_p(\tau_{lead}s + 1)}{\tau^2 s^2 + 2\xi\tau s + 1}$$

with

$$K_p = -1.28 \quad K/(m^3 \text{ min})$$
$$\tau_{lead} = 0.132 \text{ min}$$
$$\tau = 0.167 \text{ min}$$
$$\xi = 0.15$$

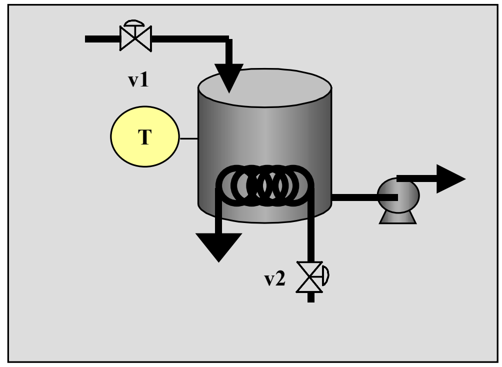
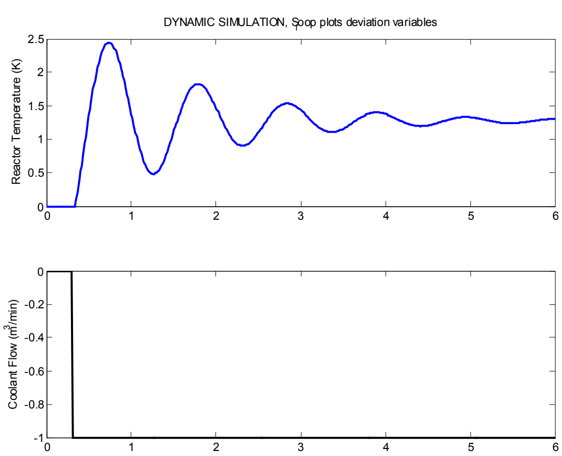


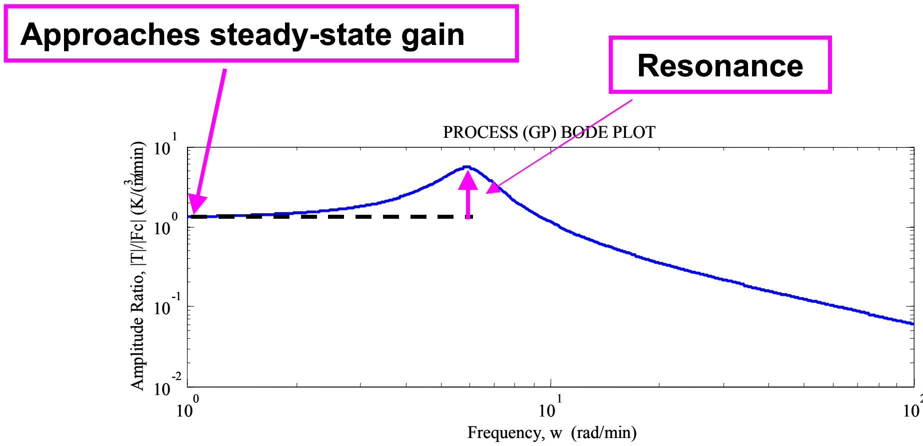
Figure 13.3

The coolant flow rate (v2) input is a sine. What is the amplitude ratio of the output to the input? (Hint: You may want to use a software package for the calculation, such as SOFTLAB or write a short MATLAB program.)

Before investigating the frequency response, let's understand the qualitative behavior of this process. We observe that the process is second order, and from Chapter 5, we know that a second order system can be overdamped, critically damped or underdamped. The underdamped systems will tend to oscillate, even if the input does not oscillate. The reactor model demonstrates that the process is underdamped, because the damping factor, $\xi = 0.15 << 1$. The figure below shows the behavior of the temperature to a step change in coolant flow, where we clearly see the oscillatory nature of the process.



Now, we evaluate the frequency response over a range of input frequencies and plot the amplitude ratio in a Bode plot. The results are given in the following figure.



We note that the amplitude ratio at very low frequencies is 1.28, which is the magnitude of the steady-state gain. (The limit of very low frequencies is steady state.) In addition, the amplitude ratio becomes small at very high frequencies, as occurs in all processes. However, we see that at intermediate frequencies, the amplitude is much greater than the steady-state value. Clearly, the system amplifies the effect of the input at frequencies near the resonance frequency. We must avoid disturbances near the critical frequency for underdamped systems.

This question showed that an underdamped system can increase the amplitude of a periodic disturbance. Note that most feedback control systems are underdamped. Therefore, disturbances near the critical frequency are highly undesirable.

13.4 In this question, we will again consider the packed bed reactor that was used in Tutorial Question 13.1. The basic information is repeated below.

$$\frac{A1(s)}{v(s)} = \frac{-0.11e^{-44s}}{(54s+1)}$$

$$\frac{A1(s)}{XF(s)} = \frac{0.50e^{-42s}}{(35s+1)}$$

A1 is product composition (mole/m^3)
v is the valve affecting the heating stream (% open)
XF is feed composition (mole/m^3)
Time is in seconds

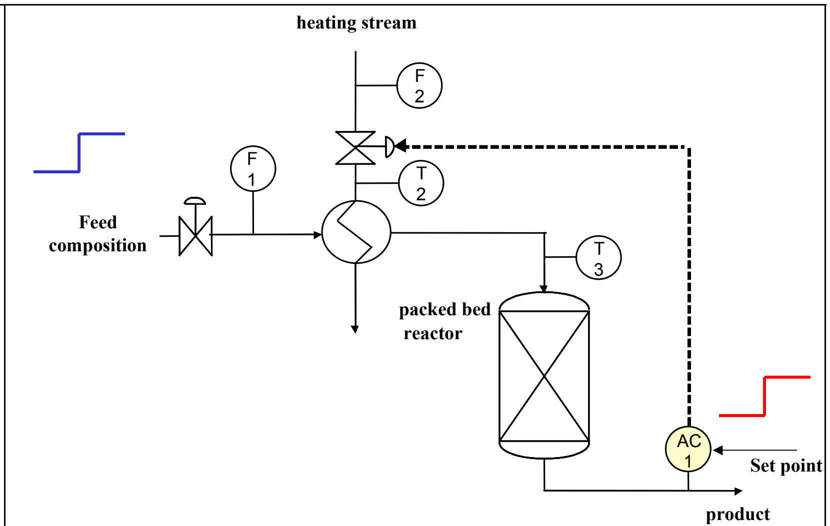


Figure 13.4

In this question, we will investigate the behavior in response to step inputs (rather than sine inputs, as was done in Question 13.1.). Each step input will be investigated individually.

a. A step disturbance occurs in the feed composition with a magnitude of 0.50 mole/m^3. We seek to maintain the product composition within 0.10 mole/m^3. Is this performance possible using the feedback control show in the figure?

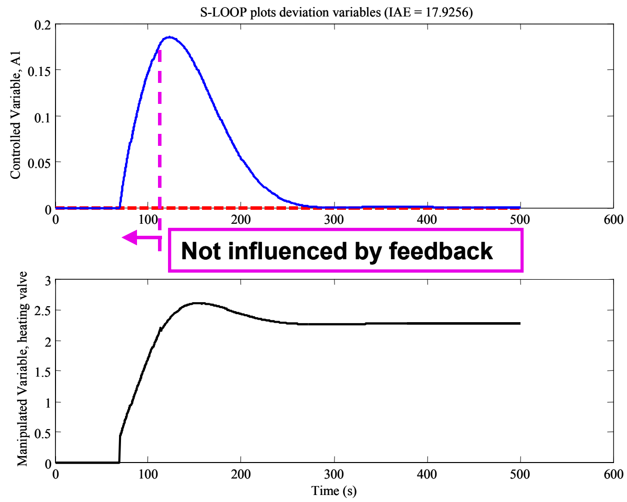
We could simulate the system to answer the question. However, let's first apply our knowledge and see if we can answer the question without simulation. The feedback controller cannot immediately influence the controlled variable, because of dead time (and inverse response, if it existed in this process). Therefore, the disturbance will not be influenced by feedback for the dead time in the feedback process.

The dead time in the feedback process is 44 seconds. The disturbance will be unaffected for 44 seconds, and the step response for those 44 seconds is calculated in the following. (Note that the disturbance dead time does not influence this calculation, because disturbance dead time just delays the time when the effect is observed in A1.)

$$\begin{aligned} A1(t) &= (\Delta XF)K_d(1-e^{-t/\tau_d}) \\ &= (0.50)0.50(1-e^{-44/35}) = 0.25(.716) = 0.179 \quad mole/m^3 > 0.10 \ mole/m^3 \end{aligned}$$

The deviation of 0.179 is the smallest possible using feedback, and it is too large! We conclude that the required control performance cannot be achieved by the process and feedback control loop. We can take steps to reduce the disturbance or evaluate some of the advanced methods in subsequent chapters (cascade, feedforward, etc.)

Let's simulate the control system to confirm our prediction. We use the PID tuning determined in the solution to Question 13.1. The results are given in the following figure, with the variables in deviation from their initial values.



We see that the maximum deviation is close the minimum calculated above.

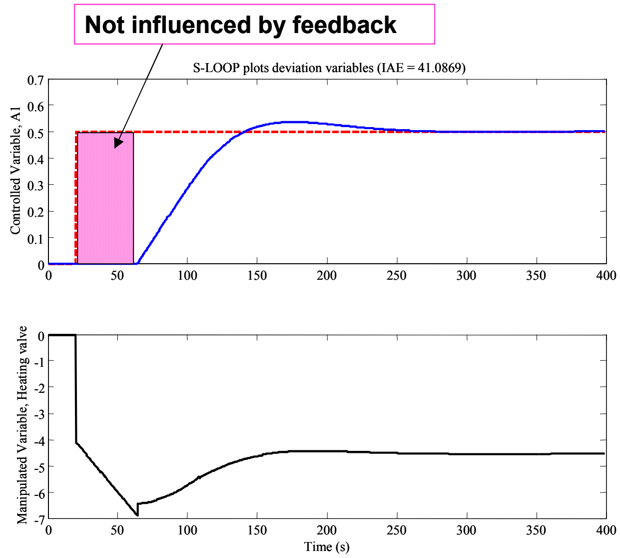
b. A step set point change is introduced to the feedback composition controller with a magnitude of 0.50 mole/m^3. We seek to change the product composition to its new value (within a small deviation) within 200 seconds. Is this performance possible using feedback control as show in the figure?

We could simulate the system to answer the question. However, let's first apply our knowledge and see if we can answer the question without simulation. The feedback controller cannot immediately influence the controlled variable, because of dead time (and inverse response, if it existed in this process). Therefore, the controlled variable will not "track" the set point change for at least the feedback dead time, and longer because of the time constant. (Note that information about the disturbance is not used in this part of the answer.)

The dead time in the feedback process is 44 seconds. The controlled variable will be unaffected for 44 seconds; then, it will respond faster than an open-loop step change because of the overshoot in the manipulated variable.

Let's evaluate the response of the controlled variable to a step in the manipulated variable (without feedback). We do this because the calculation is simple and the response of the controlled variable will be slower than for the closed-loop set point change. The step response requires one dead time plus three time constants to approach its final value; for this process the time would be (44+3*54) = 206 seconds. This is on the order of the 200 seconds required. Since the feedback response will be faster, we predict that the required control performance can be achieved.

Let's simulate the control system to confirm our prediction. We use the PID tuning determined in the solution to Question 13.1. The results are given in the following figure, with the variables in deviation from their initial values.



We see that the control performance is achieved, as predicted!

This question showed the importance of dead time on control performance. It also demonstrated that we can estimate the performance for step inputs in the time domain using simple principles about the process dynamics.

13.5 The temperature of a stirred tank heat exchanger will be controlled using a single-loop feedback PID controller. Two designs in are proposed Figure 13.5a/b. Select the control design from these two proposals that would give the better feedback performance.

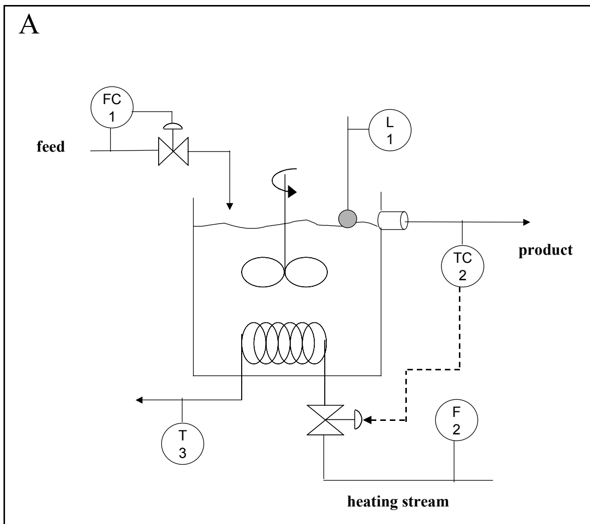


Figure 13.5a

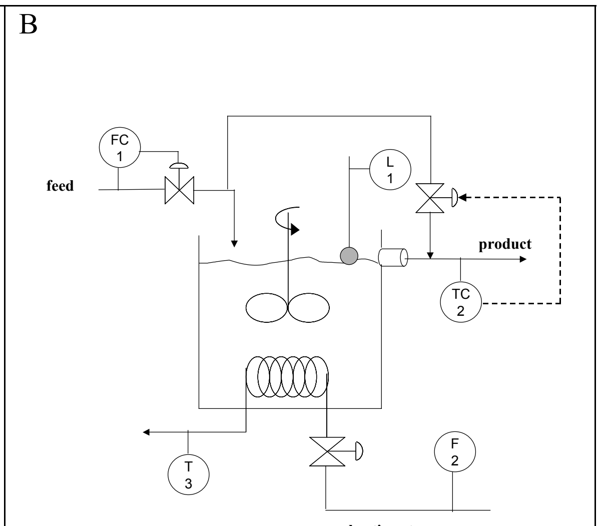


Figure 13.5b

A. We observe that the feedback path includes the valve, heat exchanger and liquid in the tank. This could be very slow, depending on the equipment designs.

B. The feedback path in this design includes the mixing point. These dynamics will be much faster than design A. Therefore, this design will provide much better feedback control performance.

Note that we have made a relatively small change to the process equipment and obtained a substantial improvement in control performance!

13.6 The temperature of a stirred tank heat exchanger will be controlled using a single-loop feedback PID controller. Two designs are proposed; Design B is the same as A except that a mass of metal is in the tank. Select the control design for these two proposals that would give the better feedback performance (faster response of the controlled variable) for a set point change in TC-2.

A B

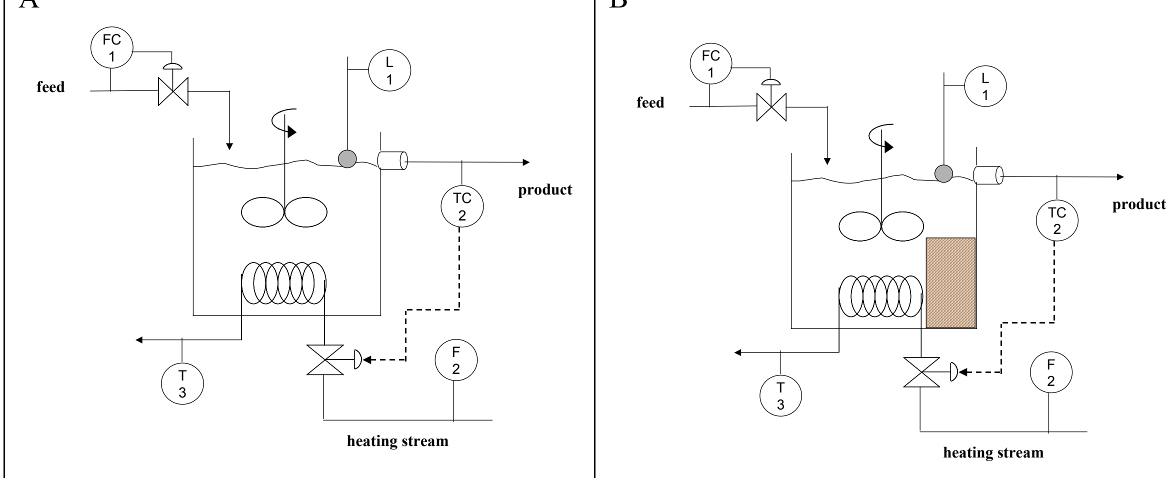


Figure 13.6

Note: This question is analogous to determining the effect of catalyst (thermal capacitance) on dynamic performance.

We observe that we have increased the “thermal holdup” in the stirred tank, because the heat capacity (energy/volume) of metal is higher than of a typical liquid. The result is slower dynamic response to the changes in coolant. Therefore, Design A, with faster feedback dynamics, would give better performance for a set point change.

The last two questions showed that comparing the feedback performance of competing designs can be achieved without simulation in limited cases by applying principles and knowing the (relative) process dynamics.

13.7 The control design in Figure 13.7 has been proposed. Three different sizes for the globe control valve have been proposed. Which of the valve sizes do you recommend and explain why?

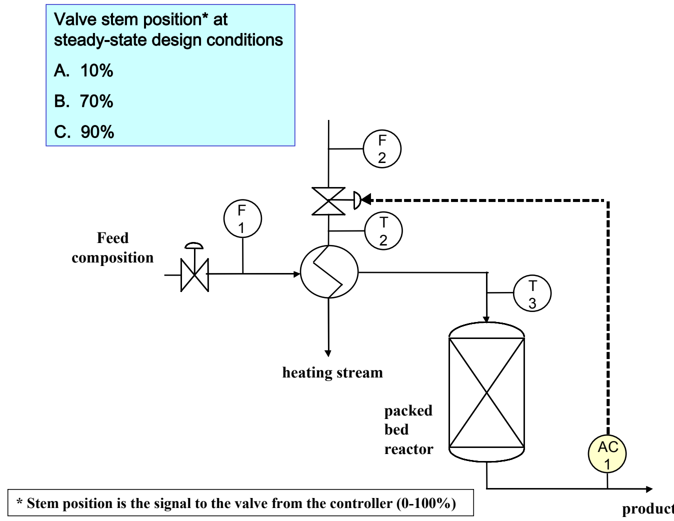


Figure 13.7

When we discuss the valve size, we mean the C_v, which is the flow rate at design conditions through the valve at 100% open. The C_v can be determined from information from valve manufacturers. The valve size increases with the pipe size for the valve.

The control equipment capacities are selected to provide good performance at the expected, design conditions and to be able to adjust the manipulated variable in response to differences from the design conditions, which can be due to the following

- Inaccuracy in the models used for design (which always exist)
- Disturbances in operation from expected conditions, e.g., feed composition, cooling water temperature or pump exit pressure
- Changes to the operating condition, for example, to produce a new product

Another important factor is the ability to change the manipulated variable with sufficient precision, i.e., the change the valve opening in small increments to have "smooth", continuous changes to the manipulated flow rate.

Let's evaluate the proposed valve sizings in light of the discussion above.

A. The valve is 10% open at design conditions. Clearly, the valve has a large capacity and could be adjusted for changes from no flow to nine times the design flow (if the relationship between flow and opening were linear). However, the valve is being operated nearly closed during expected operation. This valve would have very poor precision; small errors in the valve opening would constitute large changes in flow. This valve size is not recommended.

B. The valve is 70% open at design conditions. It can be adjusted to increase the flow rate by about a factor of two from design. Also, the precision should be good at this location in the valve opening. This valve size is recommended.

C. This valve is 90% open at design conditions. Clearly, the flow cannot be increased much; this valve has too small a capacity. This valve is not recommended.

