Solutions for Tutorial 10
Stability Analysis

10.1 In this question, you will analyze the series of three isothermal CSTR's show in Figure 10.1. The model for each reactor is the same at presented in Textbook Example 3.2, which is repeated in the figure. To simplify, we will assume that a 1% change in the valve will cause a 1 mole/m^3 change in the feed composition, C_{A0}. The effluent concentration of reactant is controlled by adjusting the pure reactant flow rate to the mixing point using a proportional-integral controller.

$$\frac{C_{A0}(s)}{v(s)} = 1 \frac{mole/m^3}{\%open}$$

$$\frac{C_{Ai}(s)}{C_{Ai-1}(s)} = \frac{0.503}{(12.4s+1)}$$

The unit of time is minutes. The sensor and final element (valve) dynamics are negligible.

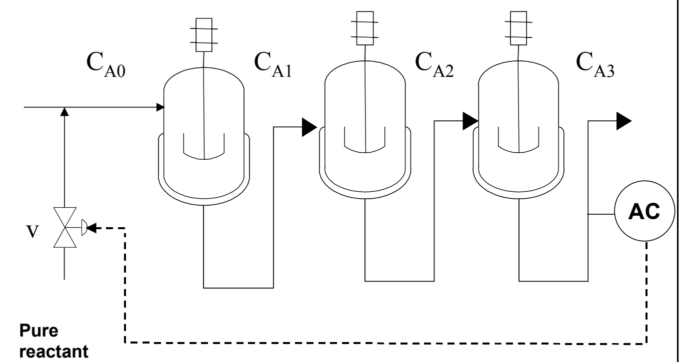


Figure 10.1

Answer the following questions. (Hint: Use the MATLAB program S_LOOP for the calculations.)

a. Determine the characteristic equation.
b. Determine the poles (roots of the characteristic equation) for a closed-loop system with a proportional-only controller for values of the controller gain (0, 15, 30, 45, 60, 75).
c. Plot the poles determined in part (b).
d. Discuss the expected dynamic behavior obtained for each of the results in part (b).
e. Determine the tuning for a PI controller using the Zeigler-Nichols method.

a. The characteristic equation is the denominator of the closed-loop transfer function. For this system, it is

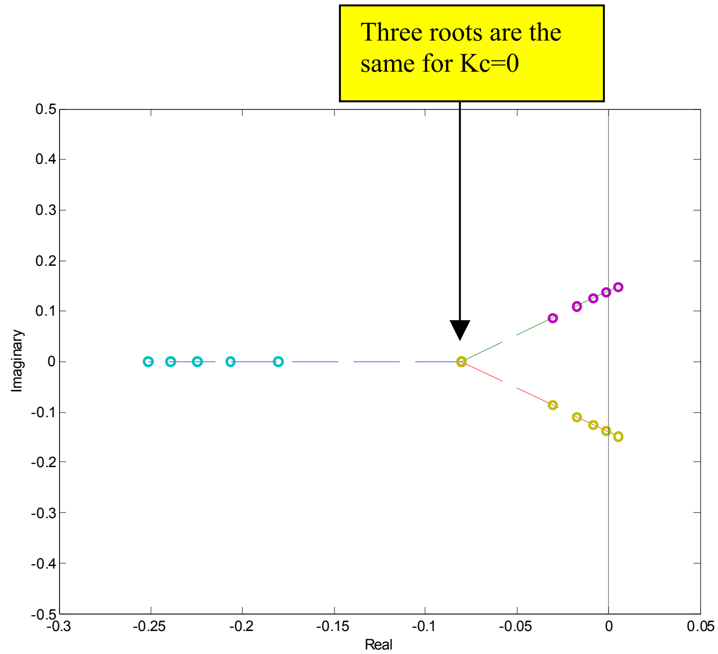
$$1 + G_p(s)G_v(s)G_c(s)G_S(s)$$

$$1 + \frac{K_C K_P}{(1+\tau s)^3} = 1 + \frac{K_C (0.503)^3}{(1+12.4s)^3} = 0$$

b&c. The characteristic equation can be rearranged to

$$\tau^3 s^3 + 3\tau^2 s^2 + 3\tau s + (1 + K_p^3 K_c) = 0$$

The results for the five values for Kc are plotted in the following figure.



Values for all three roots are plotted. We note that the system becomes unstable (the real part becomes positive) between the Kc values of 60 and 75.

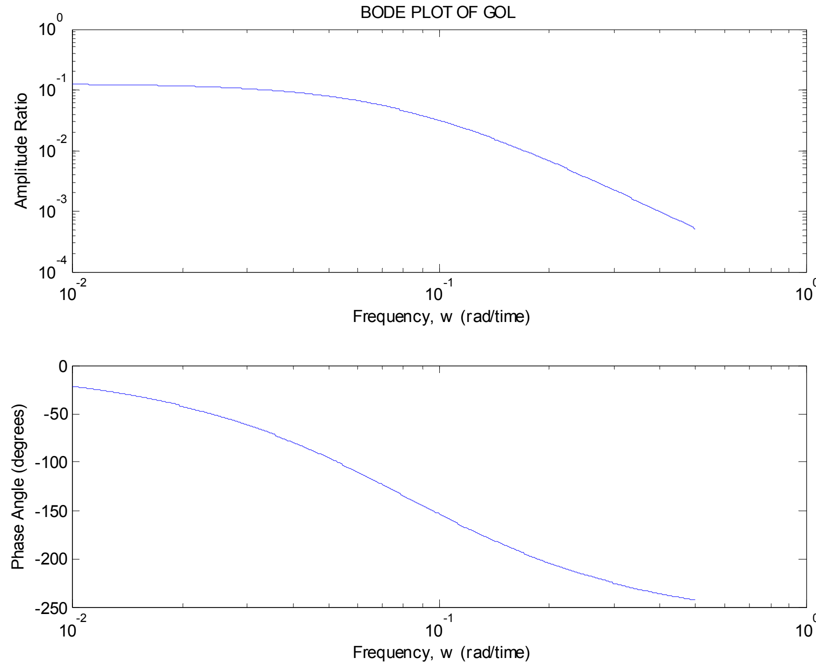
d.

- The roots are all real for Kc = 0. We expect that the behavior will be critically damped (no oscillation).
- The roots with Kc > 0 are complex. We expect oscillatory behavior, which becomes stronger as the imaginary parts become larger in magnitude.
- When the real parts are negative, the oscillations decrease in magnitude; when the real parts are positive, the oscillations increase in magnitude.

e. We apply the Zeigler-Nichols tuning method. The "open-loop" transfer function is every element in the feedback loop, with the controller being a proportional-only with Kc = 1.

$$G_{OL}(s) = \left[\frac{0.503}{12.4s+1}\right]^3$$

The frequency response is evaluated by setting s=jω and evaluating the amplitude and phase angle for various values of the frequency, ω. The resulting plot was generated using S_LOOP.

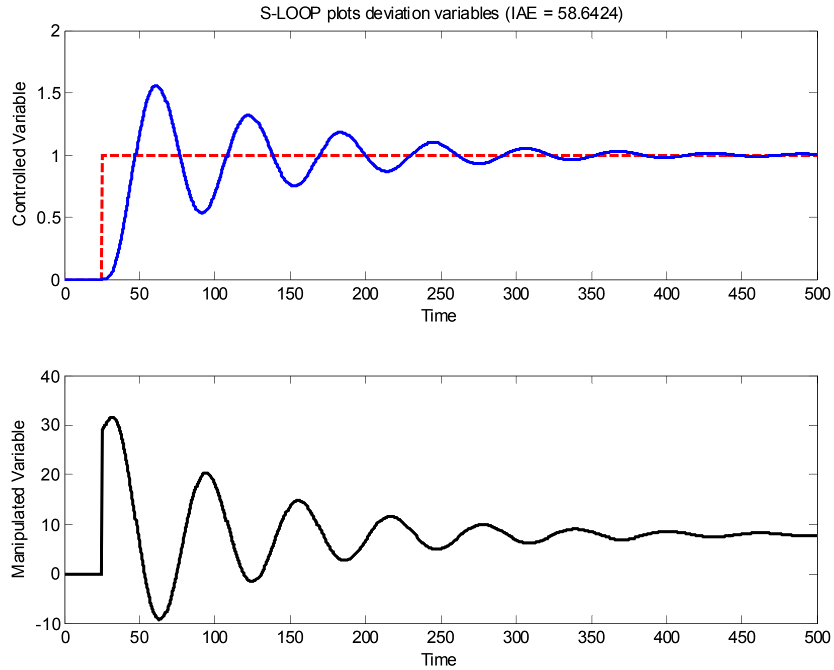


The critical frequency is 0.1396 rad/min, and the amplitude ratio at the critical frequency is 0.0159 %open/ (mole/m^3).

Ku = 1/0.0159 = 62.9 mole/m^3/%open
Pu = $2\pi/\omega c$ = 6.28/0.1396 = 45.0

Kc = Ku/2.2 = 28.6 mole/m^3/%open
TI = Pu/1.2 = 37.5 min

The dynamic behavior obtained with this tuning is given in the following figure for a step set point change.



We note that the behavior is too oscillatory and is not acceptable. We conclude (again) that the Zeigler-Nichols tuning is not the best available method. (Let's remember that they developed the concepts and procedures in the 1940's, well before digital computation.)

This question demonstrated the application of two methods for stability analysis; pole evaluation and Bode. Both methods are based on an analysis of the characteristic equation of the closed-loop system.

10.2 The results from textbook Example 10.12 give tuning for feedback control of several series of first order systems, different numbers of elements in the series. The results are repeated in Table 10.2.

Table 10.2

n	ω_c	$AR\|_{\omega c}$	K_c	T_I
1	∞	--	∞	--
3	0.35	0.122	3.72	15.0
5	0.145	0.348	1.31	36.1
7	0.096	0.484	0.94	54.5

Process: $$\frac{CV(s)}{MV(s)} = \left[\frac{1.0}{5s+1}\right]^n$$

a. Rank the results based on how aggressively each manipulates the manipulated variable from most to least aggressive.

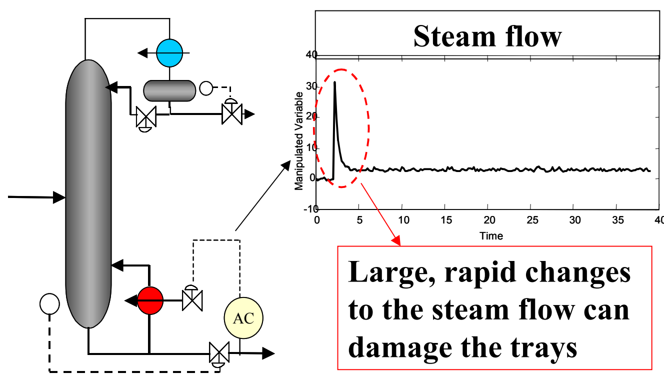
b. Discuss why limitations might exist in a real process to very aggressive adjustments.

a. We note that the controller gain decreases and the integral time increases with increasing "n". Therefore, the controller in most aggressive for small "n" and becomes progressively less aggressive as "n" increases.

We also note that for n=1, the controller gain is infinite, which is not practical. This will cause the final element to bounce between is maximum and minimum limits.

b. Very aggressive (large and frequent) changes to the manipulated variables can cause damage to equipment or cause the equipment to operate improperly. Note that this is not a general rule, but in many practical cases, the engineer must tune for moderate changes to the manipulated variable, while achieving the desired performance for the controlled variable.

An example (form Chapter 9) is given below.



This question demonstrates that good control performance requires more than stability and more than good controlled variable behavior.

10.3 Zeigler-Nichols tuning was determined for two process models in textbook Example 10.13. The results are repeated below. Perform the calculation for a third process with the following model, and discuss the results.

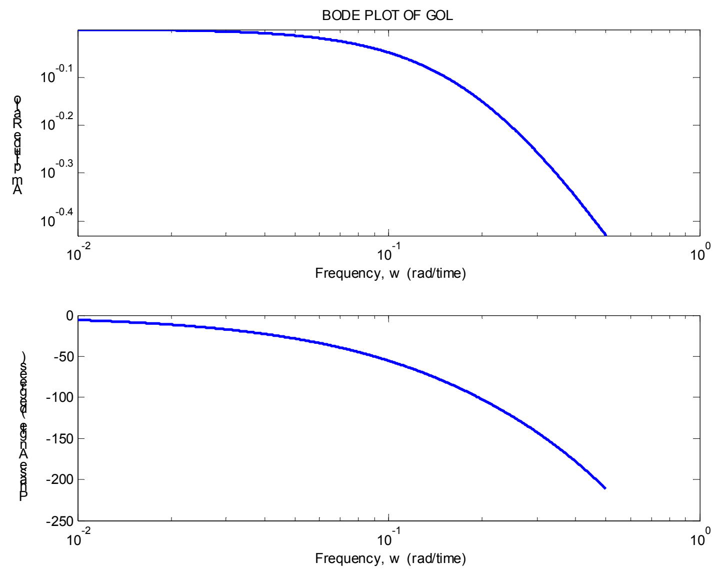
Plant A: $$\frac{CV(s)}{MV(s)} = \frac{1.0e^{-2s}}{(8s+1)}$$ Plant B: $$\frac{CV(s)}{MV(s)} = \frac{1.0e^{-8s}}{(2s+1)}$$

New Case: $$\frac{CV(s)}{MV(s)} = \frac{1.0e^{-5s}}{(5s+1)}$$

Parameter	Plant A	New Case	Plant B
ω_c	0.86		0.32
AR_c	0.144		0.84
K_u	6.94		1.19
P_u	7.3		19.60
K_c	4.1		0.70
T_I	3.65		9.8
T_d	0.91		2.45

The calculations can be performed by hand (trial and error) or using S_LOOP.

The results of the analysis are given in the following figure, where $G_{OL}(jw)$ are plotted.



The critical frequency is between 0.40554 and 0.40633

The amplitude ratio at the critical frequency is 0.44196

Ku = 1/0.442 = 2.26
Pu = 2π/ωc = 6.28/0.406 = 15.5

Kc = Ku/1.7 = 1.33
TI = Pu/2.0 = 7.75
Td = Pu/8 = 1.94

We note that the tuning is less aggressive than Plant A and more aggressive than Plant B.

This question demonstrates that the tuning becomes less aggressive as the feedback dynamics become slower, with increasing dead time and time constant.

10.4 You have tuned a PID controller using the Ziegler-Nichols method. You know that the process gain (K_P) changes due to equipment changes (e.g., heat exchanger fouling, catalyst aging) and variability in operating conditions (e.g., production rate). For purposes of this question, we will assume that only the gain changes.

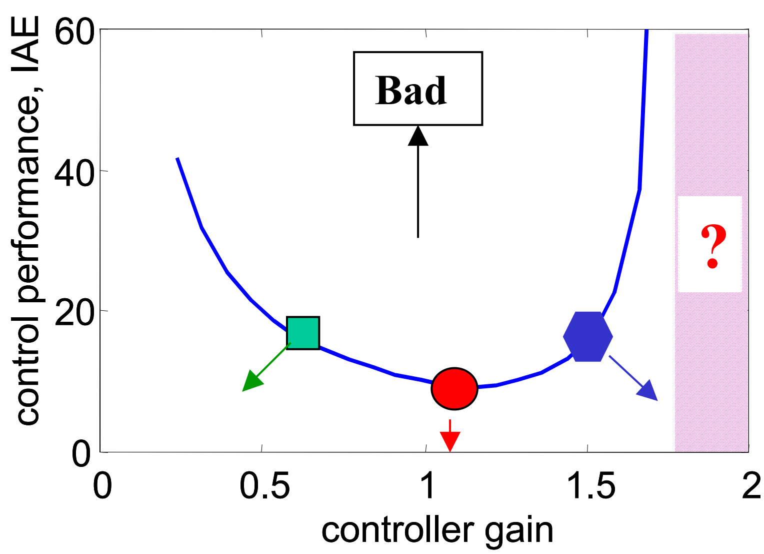
a. How large a change in process gain will cause the closed-loop system to become unstable?

b. We know that "small" or "reasonable" changes to the gain are acceptable. What is this reasonable range?

a. The process gain affects the amplitude ratio but NOT the phase lag of G_{OL}. Also, the Zeigler-Nichols method provides a gain margin of approximately 2. Therefore, we expect that an increase in the process gain of about a factor of 2 will cause instability.

b. There is not exact rule for the sensitivity because of difference objectives for the controlled variable behavior for different process applications. However, we have seen in part (a) that a factor of 2 will be unacceptable for most tuning rules.

Also, we have seen in Chapter 9 that small changes (about 25%) do not strongly influence control performance, when the controller is properly tuned. A typical result is repeated in the figure below.



This question demonstrated the importance of a "reasonable" margin from the stability limit. The engineer must understand the variation occurring in the process when deciding the appropriate margin for a specific application.

10.5 Processes have variables that are stable and unstable without process control.

a. For the distillation tower in Figure 10.5, identify two examples of variables in both categories.
b. For each variable, discuss why it is stable or unstable, as appropriate.
c. For each variable, decide whether the variable should be controlled automatically.
d. For the variables identified in part (c) as needing automatic control, select a manipulated variable. (Hint: The manipulated variables should be valves.)

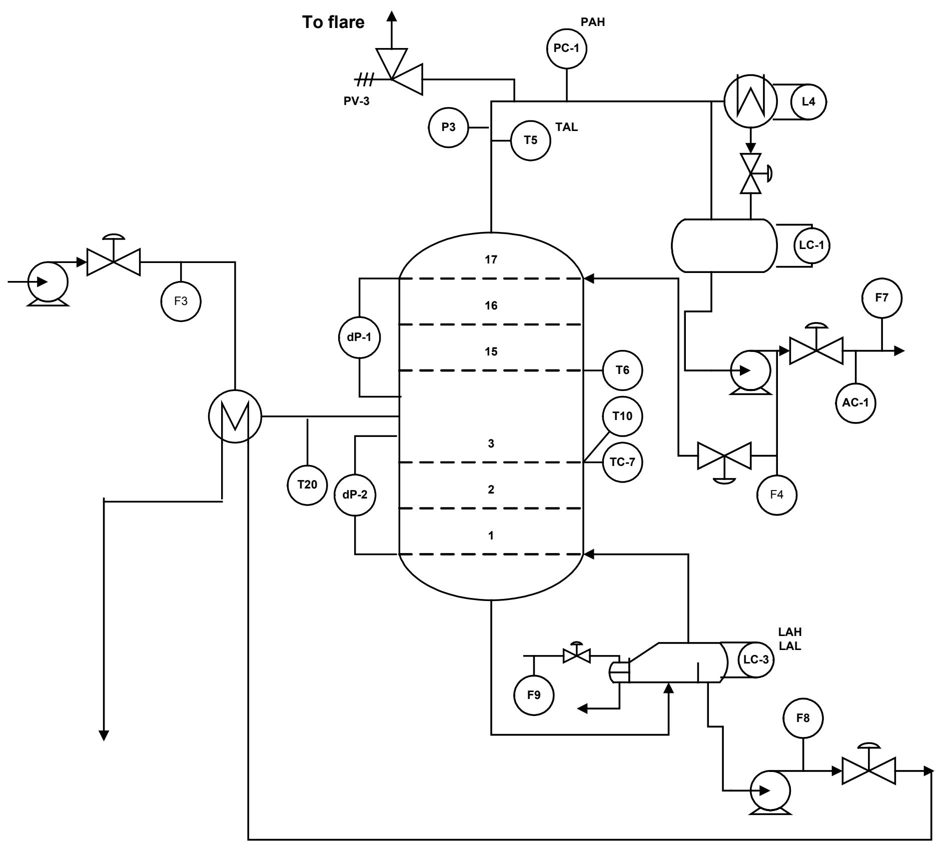


Figure 10.5

a&b.
Unstable:
L-1 which is a level with a pumped exit flow rate
L-2 which is a level with a pumped exit flow rate

These variables are unstable because the level has no effect of the flows in or out (they are NOT draining tanks). If the flows in and out are not exactly equal, the level will empty or fill completely.

Stable:
Feed temperature T-20. This is stable, because as the temperature increases, the heat exchanger duty tends to decrease. Thus, the temperature will reach an "equilibrium or steady state.
Bottoms flow F-8. This is stable because equilibrium is quickly reached between the head provided by the pump and the head required for a steady state flow.

c&d.

L-1: Yes, it must be controlled. The flow of either the reflux or the distillate product can be manipulated to control the level.
L-2: Yes, it must be controlled. The flow of the bottoms product can be manipulated to control the level.
T-20: This could be controlled, but it is not necessary. In this design, no valve is available for controlling the feed temperature. The variations in the temperature will be a disturbance to the distillation tower.
F-8: This is influenced by the valve in the liquid bottoms product pipe. However, the level L-2 uses this valve. Therefore, the flow is not controlled to a specific value; it is changed to ensure that L-2 does not empty or overflow.

This question demonstrates the analysis to identify unstable process variables. These must be controlled by feedback.

10.6 The distillation process in Figure 10.6 has two feedback analyzer controllers. Should you tune each controller using the Zeigler-Nichols method?

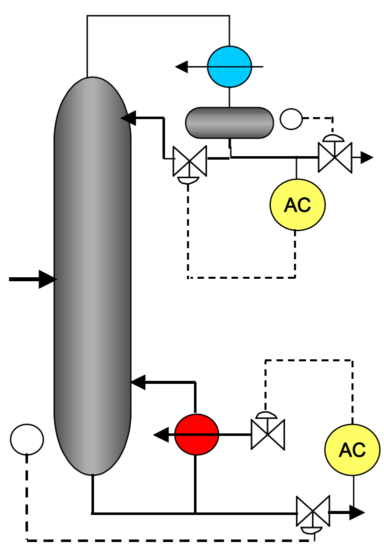
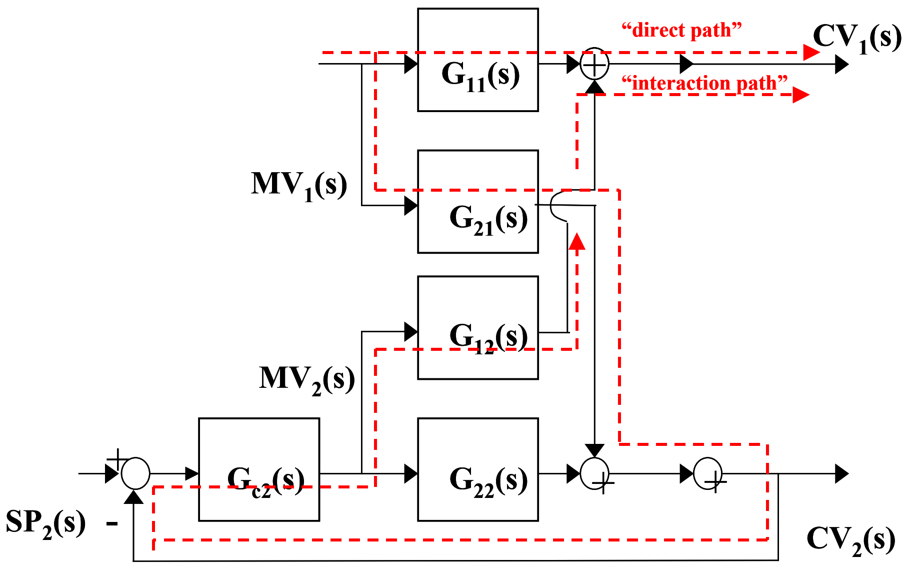


Figure 10.6

The Zeigler-Nichols method assumes that the process model, Gp(s), is the model between the valve and the measured variable to be controlled. However, in this situation, the relationship between a valve and a sensor includes not only the process, but also the "other" controller. Therefore, the Zeigler-Nichols method is not appropriate.

The situation is shown in the following figure, where the relationship between MV1 and CV1 is influence by controller Gc2.



You will learn about multiloop system in Chapter 20.

This question warns us that tuning a several multi-loop controllers is different from tuning each controller as a single-loop.

