

# Continual Improvement

CHAPTER

26

## 26.1 ■ INTRODUCTION

Decades of industrial experience have demonstrated the success of process control in maintaining selected variables near their desired values. Essentially all process plants apply automation, using feedback and feedforward principles to achieve safe and profitable production of consistently high-quality product. In general, process control is very effective when the control system has sufficient time to respond to disturbances (i.e., the feedback dynamics are fast compared with the disturbance frequency).

While process control, using the methods presented in this book, is required for regulating some process variables, the application of these methods may not be appropriate for all important variables. In some situations the best operating conditions change, and a fixed control design may not respond properly to these changes. In other situations, continuous feedback compensation can be too aggressive, leading to excessive variation in the controlled variables. Two approaches for continually improving plant operation are introduced in this chapter to address these situations. Both use the basic principle of feedback control: using outputs of a system to influence inputs to the system. However, these approaches involve very different technologies to address unique objectives. The approaches introduced in this chapter enhance the good performance achieved through process control.

**OPTIMIZATION.** Optimization methods find the *extremum*—maximum or minimum—of an objective. Generally, the objective function will be profit, which we aim to maximize. When control objectives were discussed in Chapters 2, 7 and

24, profit optimization was given less importance than safety, environmental and equipment protection, smooth operation, and product quality. Thus, these shorter-term objectives must be satisfied before we can turn our attention to profit, although the company will not survive in the long run without achieving profitable operation.

**STATISTICAL PROCESS CONTROL (SPC).** The methods presented to this point in the book can be referred to as *automatic process control (APC)*, because the control calculation is executed and the final element adjusted “automatically” as a result of the control calculation. In *statistical process control (SPC)* the process data is analyzed for opportunities for improvement, and when an opportunity exists, the data is diagnosed to ascertain an appropriate action. Thus, SPC involves statistical analysis of the real-time data, but not necessarily an action, at each execution. This additional analysis generally results in less frequent feedback actions, which can improve performance in some processes.

Both of these methods appear in the process control implementation hierarchy in Figure 25.2, which shows them as higher levels in a cascade structure. Their decisions can be implemented through lower-level process control loops. For example, optimization systems can adjust the controller set points that regulate operating conditions such as temperatures and production rates. Alternatively, the highest-level decisions may involve complex manual intervention; in these cases, the results are provided in an advisory manner to plant personnel. Examples of such decisions are a change in feed material type and decisions on regenerating catalyst. Also, some diagnostic results indicate only that a significant change in process equipment performance has occurred, and further investigation by plant personnel is required to ascertain the cause and corrective actions.

Each of these topics is quite large, and entire books have been dedicated to their coverage. This chapter introduces some basic concepts for each approach and demonstrates how each relates to process control. It is important to recognize that most plants require excellent process control, to achieve safe and smooth operation, before the approaches in this chapter can be implemented and that opportunities for optimization and monitoring often exist. Thus, the engineer is not confronted with an “either/or” decision: all approaches in the hierarchy can be implemented concurrently.

## 26.2 ■ OPTIMIZATION

The control design procedure in Chapters 24 and 25 allocates manipulated variables to achieve good dynamic performance, which is measured by the (hopefully, small) variability in key variables. Often, the number of manipulated variables exceeds the number of controlled variables. In these situations, safe operation and good product qualities can be achieved by manipulating selected process inputs that give the best control performance, and some manipulated variables can be maintained at arbitrary, constant values within an acceptable range. Alternatively, the excess manipulated variables can be adjusted to increase profit; these excess manipulated variables will be referred to as *optimization variables* (Marlin and Hrymak, 1997). Some approaches for achieving high profit with excess manipulated variables have already been introduced; for example, the variable-structure controls in Chapter 22 provide the means for utilizing manipulated variables in a

specified order, with the proper order based on the process economics. In this chapter, additional optimization approaches are introduced that address more complex situations, where a strategy for adjusting the excess variables is not as straightforward to determine and may change frequently. Three methods for optimizing process economics through adjusting optimization variables are discussed below and demonstrated with process examples.

## I. Process Control Design

The first step in designing optimizing controls, after the regulatory controls have been designed, is an analysis to determine the proper strategy for the optimization variables. This analysis uses models of the process or plant data to answer two important questions:

1. Do incentives exist for optimization? In some situations the profit will not vary significantly as the values of the excess manipulated variables—the optimization variables—change. When the profit does not change, the optimization variables can be maintained at constant values selected for convenient operation. When the profit is significantly different for various values of the optimization variables, the next question is evaluated.
2. Is the optimal strategy constant and simple? When the profit is sensitive to the optimization variables, the response of these variables to external changes, disturbances, and set point changes should be evaluated. In some cases, the optimal response to these external changes (i) is nearly the same for all expected operating conditions and economics and (ii) can be implemented via straightforward real-time calculations as part of the control strategy.

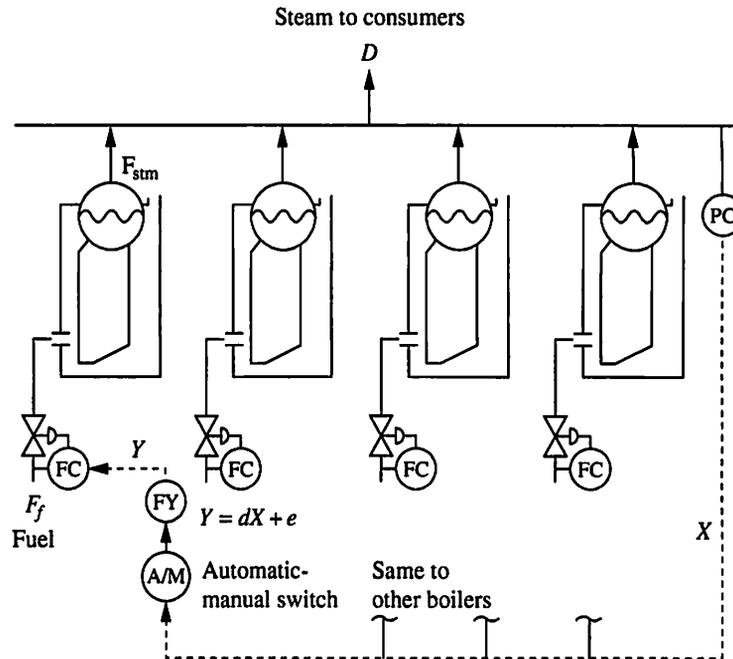
When the answers to both questions are yes, a control strategy can be designed to approximate the best performance. Examples of this approach that have already been presented include the valve position controller in Figure 22.13 and the production maximization in Figure 22.14; in these examples, the best operating conditions were close to limiting values of key variables (i.e., they were “pushing constraints”). The method for process control design introduced in this subsection may not result in as simple a policy as operating near a constraint, but the concept is the same: implementing an operating policy that has been determined to be close to the best possible. The following example demonstrates the approach for answering the two questions above and, when appropriate, building the strategy to maximize profit via control calculations that do not explicitly involve economics.

### EXAMPLE 26.1.

Steam is used in most process plants for power, driving turbines, and heat transfer. To satisfy the large and variable plant demands, many process plants have their own boilers and steam distribution networks. Typically, the boilers are arranged as shown in Figure 26.1, with all boilers providing steam to a single pipe, termed a *header*, from which all consumers are supplied. The total steam demand can be provided by any combination of individual boiler productions that sums to the total demand. The boiler productions are often termed *loads*, expressed in units of fraction of the maximum steam from one boiler. This convention is used in the example, with all boilers having the same maximum and the total consumer steam

### Methods For Optimization

- I. Process control design
- II. Model-based optimization
- III. Direct search


**FIGURE 26.1**
**Multiple boiler and steam header.**

demand expressed as a multiple of the maximum possible steam production from one boiler.

The basic requirement for process control is to ensure that the steam required by the consumers is produced by the boilers; in other words, the consumers and producers of steam are "in balance" at all times. This is achieved by controlling the header pressure by adjusting the fuel to the boilers; any combination of steam productions from the four boilers that sums to the required total satisfies the basic objective. The percent efficiency for a boiler is defined as  $100 \times (\text{energy transferred to the water})/(\text{total heat of combustion})$ ; note that the energy to the water includes preheating the water, vaporization, and superheating the steam. Since the efficiencies vary as the demand changes and are different for different boilers, opportunity exists for influencing profit by using the minimum fuel, while satisfying the total demand from the steam consumers. In this example, the boiler efficiencies, from Cho (1978), are given in Figure 26.2.

Using this data, the process performance can be determined for any distribution of boiler loads at any steam production,  $D$ , which is the consumer demand. As explained in Chapter 2, the additional information required to calculate the benefits for automation is the distribution of plant operating conditions, which is here defined by the variability of the consumer demand. For this example, the demand is assumed to be uniform over the range of 0.8 to 2.5, as shown in Figure 26.3; for a real situation, this distribution would be determined based on process data.

The fuel requirements for any steam demand, which is the direct measure of economics, can be determined by the application of equation (26.1).

$$F_f = \sum_{i=1}^N \frac{(F_{stm})_i (H_0 - H_{stm})}{\Delta H_c (\eta_i / 100)} \quad (26.1)$$

where

- $F_f$  = total flow of fuel to all boilers
- $(F_{stm})_i$  = flow of steam from boiler  $i$
- $H_0$  = specific enthalpy of water to boiler

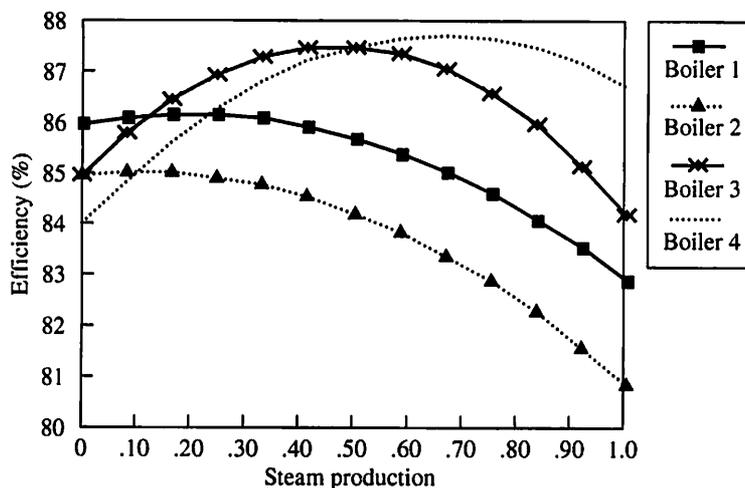


FIGURE 26.2

Boiler efficiencies for Example 26.1.

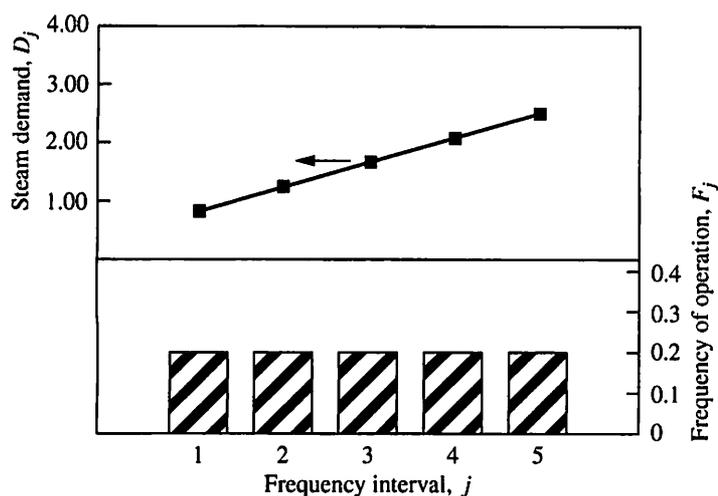


FIGURE 26.3

Data to calculate the average boiler performance.

- $H_{\text{stm}}$  = specific enthalpy of steam to the header
- $\Delta H_c$  = heat of combustion of the fuel
- $N$  = number of boilers (in this example, 4)
- $\eta_i$  = efficiency of boiler  $i$  (see Figure 26.2)

The total steam demand  $D$  is determined by the consuming process units and is variable. The best boiler operation satisfies the steam demand and minimizes the total fuel or, equivalently, maximizes the average efficiency. Also, the best operation is the average of the operations at the different demands weighted by the fuel at each. The maximization is defined mathematically in equations (26.2).

$$\max_{\{(F_{stm})_i\}_j} \eta_{ave} \tag{26.2a}$$

subject to

$$D_j = \left[ \sum_{i=1}^N (F_{stm})_i \right]_j \tag{26.2b}$$

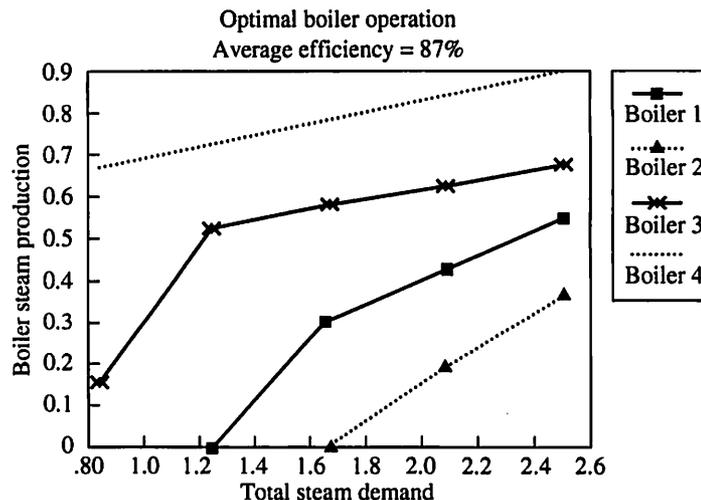
$$[\eta_i]_j = [a_i (F_{stm})_i^2 + b_i (F_{stm})_i + c_i]_j \tag{26.2c}$$

$$\eta_{ave} = \frac{\sum_{j=1}^M F_j \left[ \sum_{i=1}^N (F_{stm})_i (\eta_i) \right]_j}{\sum_{j=1}^M F_j \sum_{i=1}^N [(F_{stm})_i]_j} \tag{26.2d}$$

where  $F_j$  = frequency at interval  $j$  (0.20 for all  $j$  in this example)  
 $M$  = total number of intervals (in the example, 5)  
 $F_{stm} \geq 0.0$

The solution to this nonlinear mathematical problem requires optimization mathematics, which are not central to this introductory coverage; this topic is explained elsewhere (Edgar and Himmelblau, 1988), and good software exists, such as GAMS (Brooke et al., 1992) and SPEEDUP (Aspen Technology, 1994). Thus, the results of the numerical solution of problem (26.2) are given in Figure 26.4 without details on the optimization method used. The best operation generates the required total steam by adjusting the steam produced from all boilers in response to a change in the demand. The approach gives the highest average efficiency, 87%. This complete optimization could be implemented as part of a control strategy but would require an optimization problem (26.2) to be solved frequently in real time.

The values of the optimization variables change as disturbances occur. In Section 26.2 two questions were posed in evaluating operations optimization. First, do incentives exist for optimization? This can be answered by determining the plant performance (here the average boiler efficiency) under the standard type of



**FIGURE 26.4**  
Optimal boiler load allocation for Example 26.1.

control. This base case is taken to be a load distribution for all boilers, so that the load of each boiler at any steam demand  $D$  would be  $D/N$ . The average efficiency for this example under the “equal loading” base case would be 86%, which is 1% lower than the optimal operation. Since this could represent a substantial increase in fuel consumption, incentives exist, and the second question will be considered.

The second question involved a simple control strategy that could, at least partially, replace the complex optimization calculations for real-time implementation. Since simplicity is always a goal—although not at the expense of poor product quality or significant loss in profit—an alternative approach to achieve partial optimization is evaluated. The simple alternative is to maintain the boiler loads at constant ratios, with the values of the constant ratios selected to give good (but suboptimal) economic performance. This design problem—which is solved only once, during design, to give parameters to be used in the real-time calculations—is the same as equations (26.2a) through (26.2d), but with the addition of equations (26.2e) for boilers  $i = 2, N$  and interval  $j = 1, M$ :

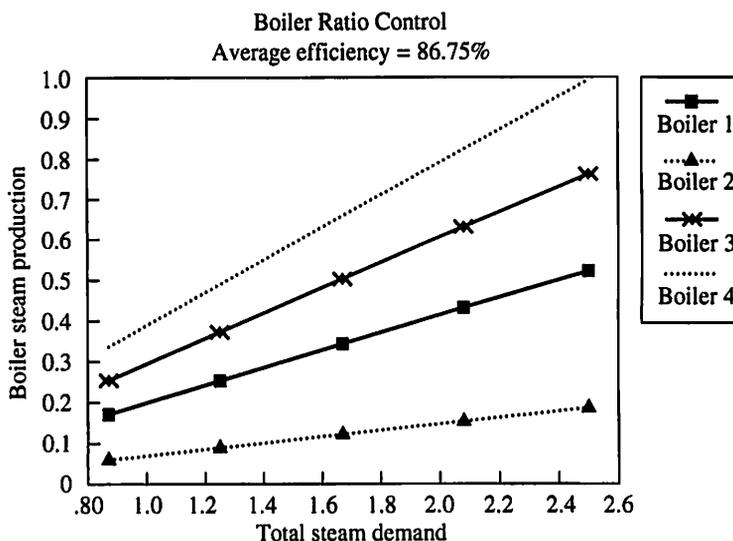
$$[(F_{\text{stm}})_i]_j = R_i [(F_{\text{stm}})_1]_j \quad (26.2e)$$

The solution of equations (26.2a) through (26.2e) determines the best values for the load ratios at each demand  $D_j$ . Note that  $R_i$  is the ratio of the steam from the  $i$ th boiler to the steam from the first boiler and that, once determined, this ratio does *not change* when the total steam demand changes. Thus, the resulting control strategy involves very simple calculations.

The solution of this problem is given in Figure 26.5. As expected from the optimal results, the ratios are selected to have a high steam production from the more efficient boilers. The average efficiency from this much simpler approach is only 0.25% less than the exact optimum for the wide range of operating conditions in Figure 26.3. Considering the likely accuracy of the boiler efficiency curves, this difference does not seem to be significant, and the simpler ratio control design would usually be selected. The ratio control could be implemented in a manner that would not influence good performance of the very important pressure controller. As shown in Figure 26.1, the pressure controller output influences every boiler fuel flow directly, and the controller output is modified to allow a ratio to be adjusted. The coefficients in the ratio calculation,  $d_i$  and  $e_i$ , would be determined from Figure 26.5 and would not be adjusted in real time.

An important lesson from this example is that tracking the best operating conditions does not always require extensive real-time calculations. The proper control calculations can be ratio control (this example), constraint pushing using signal selects, split range, or valve position controller. The correct design often requires careful process analysis to give a structure that closely follows the best operation in real-time control calculations.

This first approach, using a control design to approximate optimal operation, is appropriate when the control calculation need not change with time. For example, the ratios in Example 26.1 do not change as long as the efficiency curves for the boilers do not change with time. The result is a simple method that does not calculate or estimate the profit as part of the control calculation. In contrast, the next two approaches can respond to changes in plant performance by using process measurements in the calculation of profit, at the cost of much greater complexity.

**FIGURE 26.5**

The best ratio boiler load allocation for Example 26.1.

## II. Model-Based Optimizing Control

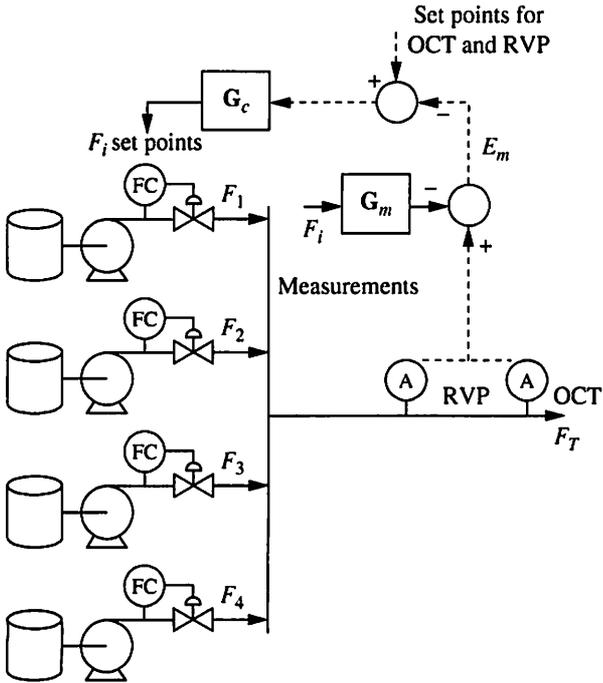
This second approach can be used when incentives exist for adjusting the optimization variables but the method for optimization cannot be implemented in a straightforward strategy such as constraint pushing or ratio control. In this approach a mathematical model of the process is used to calculate the best operating conditions for the current situation, and inevitable model errors are corrected (at least partially) using feedback measurements. Many technologies are available for real-time, model-based optimization. One of the simpler and frequently employed model-based approaches is introduced in the next example; it uses a linear model and a simple feedback updating method. When linear models are adequate, the model-based optimization can use the highly reliable linear programming solution of the optimization problem. When the feedback is introduced by adjusting the “bias” term in the linear model, the optimizing controller can be formulated in the model-predictive structure.

### EXAMPLE 26.2.

In some cases, linear models can represent a process with satisfactory accuracy for the purpose of optimization of single process units. An industrially important control problem is the blending of several materials into a product mixture, with the control objectives to achieve the specified production rate and to maintain the product qualities within their limits. In this example, several hydrocarbon components are blended to produce gasoline. The product qualities, octane number (OCT) and vapor pressure (RVP), are important for the performance of the gasoline in an internal-combustion engine (Stadnicki and Lawler, 1985). The component flows can take any values from zero to the maximum amount available.

This process is shown in Figure 26.6, and the linearized model is

$$(\text{RVP})F_T = \sum_{i=1}^L r_i F_i \quad (26.3a)$$



**FIGURE 26.6**

**Blending process with optimizing, model-predictive controller.**

$$(\text{OCT})F_T = \sum_{i=1}^L o_i F_i \quad (26.3b)$$

$$F_T = \sum_{i=1}^L F_i \quad (26.3c)$$

where

- OCT = product octane
- $o_i$  = component octane
- RVP = product vapor pressure
- $r_i$  = component vapor pressure
- $F_T$  = product flow
- $F_i$  = component flow
- $L$  = number of component flows (4 in this example)

In this example, the same model structure is used to represent both the true plant and the model used for control (i.e.,  $G_m$  in Figure 26.6). The parameters in the controller model are *not* identical to those of the plant; these differences always occur in practice due to model error.

Note that the dynamics are so fast that the process is essentially at steady state, so the controller model is algebraic [ $G_m(s) = \mathbf{K}_m$ ]. The controller in the model predictive structure involves an inverse of the process model. However, the process and the process model have more manipulated than controlled variables; four manipulated flows and only two controlled product qualities. In this situation many combinations of the manipulated-variable values can satisfy the controlled-variable values. This flexibility can be capitalized upon not only to satisfy

the controlled-variable bounds in equation (26.3), but also to maximize profit by using the lowest-cost components. This flexibility is advantageous, but it leads to a mathematical problem that offers more challenge than taking the inverse of a square matrix. The statement of the problem to be solved by the *controller*  $G_c$  is

$$\max_{F_i} \text{profit} = V_T F_T - \sum_{i=1}^L V_i F_i \quad (26.4a)$$

subject to

$$(\text{RVP}_{\min}) F_T \leq \sum_{i=1}^L r_i F_i + F_T (E_m)_{\text{RVP}} \leq (\text{RVP}_{\max}) F_T \quad (26.4b)$$

$$(\text{OCT}_{\min}) F_T \leq \sum_{i=1}^L o_i F_i + F_T (E_m)_{\text{OCT}} \leq (\text{OCT}_{\max}) F_T \quad (26.4c)$$

$$F_T = \sum_{i=1}^L F_i \quad (26.4d)$$

$$0 \leq F_i \leq (F_i)_{\max} \quad (26.4e)$$

where  $V_T$  = value of the product  
 $V_i$  = value of each component  
 $E_m$  = feedback correction defined in equations (26.5), which would be zero if no feedback were implemented

Mathematical problems of this structure—linear equations that include both equalities and inequalities—are well known in applied mathematics as *linear programming* (Best and Ritter, 1985). The solution to this problem gives the values of the four manipulated variables (flows) that satisfy all equations under “subject to” and also maximizes the profit. The number of equations that are equalities at the solution is the number of original, strict equalities (26.4d) and the number of inequalities ( $\leq$  or  $\geq$ ) that are at their limits at the solution. This forms the set of equations to be solved by adjusting the same number of manipulated variables. In this case, the solution contains one equality (26.4d) and two inequalities due to limits on the product quality (26.4b and 26.4c). Thus, three manipulated flows must be adjusted to values that satisfy the equalities. Since four flows exist, one flow is not specified, and linear programming theory demonstrates that this “excess” optimization variable must be at either its upper or lower limit, depending on which limit results in the highest profit.

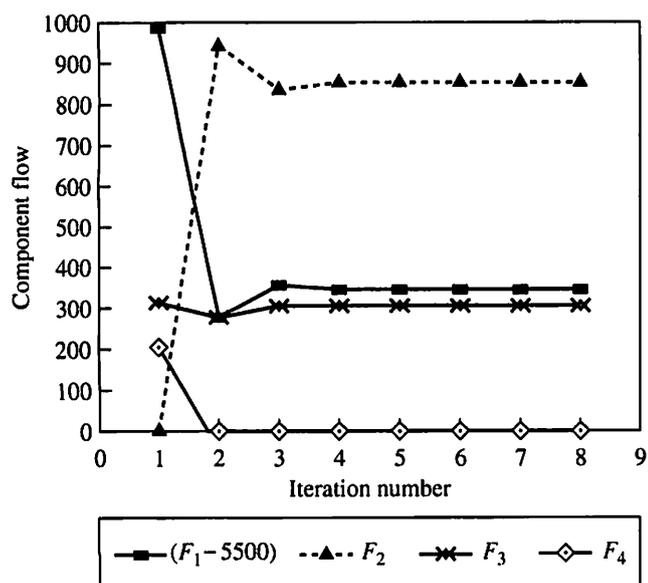
Efficient computer programs are available to solve the linear program in equation (26.4), which is shown as  $G_c$  in Figure 26.6. If no feedback were included, the model would be used in a feedforward prediction of the correct flows to optimize the operation. The feedback control system in Figure 26.6 uses measurements of the product qualities to correct the model. Many possible methods can be used to correct the model, and in principle, all coefficients ( $o_i$  and  $r_i$ ) could be adjusted when sufficient data is available. In this example, only the simplest feedback is considered, in which the difference between the measured value of the product quality and the model prediction is used to correct the model “bias” term. This is essentially the same type of feedback used in the model predictive controllers in Chapters 19 and 23. Thus, the  $E_m$  terms in equations (26.4) are

$$\begin{aligned} (E_m)_{\text{RVP}} &= \text{RVP}_{\text{meas}} - \text{RVP}_{\text{model}} \\ (E_m)_{\text{OCT}} &= \text{OCT}_{\text{meas}} - \text{OCT}_{\text{model}} \end{aligned} \quad (26.5)$$

with the subscript “meas” indicating the measured values in the product stream.

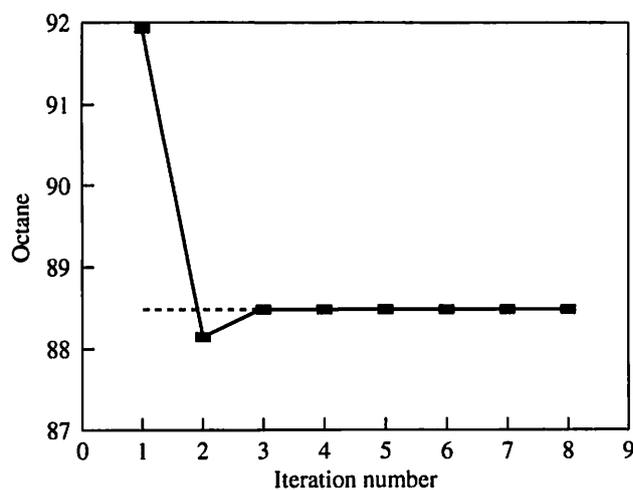
While this type of feedback was shown to provide zero steady-state offset for steplike disturbances for the controllers in Chapter 19, it is *not guaranteed* to provide exact tracking of the true plant optimum for all situations of model errors, although it may under some conditions. Conditions for its success are given by Forbes and Marlin (1994).

In this example, the model used in the controller differs from the true plant performance, as would essentially always occur. The component qualities are given for the true plant and the controller model in Table 26.1. The dynamic response for this case under closed-loop control, with the model update equations (26.5) and the optimization problem (26.4) solved every controller execution, is given in Figures 26.7 through 26.9. In Figure 26.7, the component flow rates are shown for each controller iteration. The first iteration was performed without feedback ( $E_m = 0$ ),



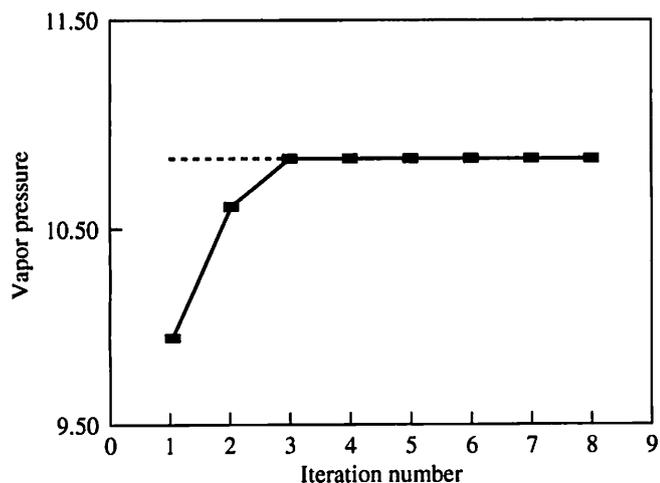
**FIGURE 26.7**

Component flow rates during feedback control.



**FIGURE 26.8**

Response of octane under closed-loop optimizing control.



**FIGURE 26.9**

Response of vapor pressure under optimizing control.

TABLE 26.1

Table of data for plant behavior and controller model

System	Property	Component				Product	
		$F_1$	$F_2$	$F_3$	$F_4$	High	Low
Model	Octane	88	64.5	92.5	98	—	88.5
	Vapor pressure (psi)	5	14	138	5	10.5	—
Plant	Octane	91.8	64.5	92.5	96.5	—	88.5
	Vapor pressure (psi)	4	12	138	7	10.5	—
Model	Value (\$/bbl)	34	26	10.3	37	33	
Model and plant	Maximum flow bbl/d	12000	6500	3000	7000	7000 (fixed)	

1 bbl (barrel) = 0.159 m<sup>3</sup>; psi = 6.89 kPa

so these results are a feedforward prediction of the best operation. After each controller execution, feedback measurements were taken and used to calculate the corrected biases  $E_m$  to be used by the controller for the next iteration. By the completion of the eighth iteration, the control system, using the feedback model corrections, achieved operating conditions that maximize profit in the true plant. The actual measured product qualities are shown in Figures 26.8 and 26.9. Both qualities should be within their upper and lower limits and, at the optimum, arrive at a limit—the upper limit for vapor pressure and the lower limit for octane, because this operation maximizes profit. Note that the qualities violate their limits during the transient responses in spite of the controller containing explicit equations for these limits, because the model errors are large enough to lead to significant, although temporary, violations of product quality limits in this example.

In general, many decisions must be made in designing and implementing a model-based real-time optimizer; some of these are model structure, parameters to be updated, measurements used for updating, and the updating calculation (e.g., least squares). Some guidance on these decisions is provided by Forbes and Marlin (1994) and Krishnan et al. (1993). Industrial experience indicates great benefit for real-time optimization (e.g., Fatora et al., 1992; Larmon, 1977; Yang and Waldman, 1982). The best experiences are reported for plants with accurate models and good measurements, so that the feedback model updating leads to accurate representations. Also, substantial improvements occur more often in complex plants with many variables and changing conditions, where control structures, such as operating close to the same constraint, are not likely to yield the highest profit.

### III. Direct Search

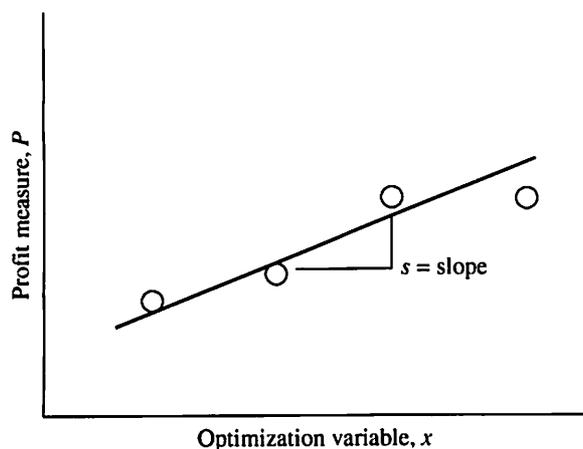
This third approach can be used when incentives exist for adjusting the optimization variables, but the strategy for optimization cannot be implemented in a straightforward strategy such as constraint pushing or ratio control, and accurate models do not exist. In these situations, a very simple, locally accurate model of the process

is determined *empirically* from plant data. This model is used to determine the direction in which changes in the manipulated variables will increase profit. The plant operating conditions are then changed a small amount in this direction, and a new, updated model is evaluated. The direction for optimization is determined again from plant data, and another step is taken.

This iterative approach has been used for many years to study plant behavior and determine improved operating conditions. When the experiments are time-consuming and expensive, effort must be made to reduce the duration of the study; then, only a few experiments are performed and careful statistical evaluations are used to determine whether further improvement is likely and, if so, which direction is the best. Infrequent application of this concept in studies or “campaigns” is usually termed *evolutionary operation*, a term coined by Box and Draper (1969), who provided procedures, guidelines, and statistical tests for this periodic approach. While the periodic approach minimizes disturbances to the plant resulting from its designed experiments, it cannot track the best operation when it changes frequently.

The concept of building a locally accurate model for determining the direction of optimization can be extended to real-time, feedback control. Many algorithms are possible, and one of the simplest is discussed here (Bozenhardt, 1986). The concept is shown in Figure 26.10, where the last few values of the optimization variable and calculated profit are plotted. Recall that the true plant profit is never known exactly; thus, an estimate of profit must be calculated from plant measurements. The direction in which the optimization variable should be changed to increase the calculated profit can be determined from the data in the plot. One method for determining the direction is to fit some of the most recent data with a straight line by least squares (Box et al., 1978). The slope of the line gives the correct direction (i.e., whether the optimization variable should be increased or decreased). The expression for the slope when there is one optimization variable is

$$S = \frac{\sum_{i=1}^{N_p} (P_i - P_{ave})(X_i - X_{ave})}{\sum_{i=1}^{N_p} (X_i - X_{ave})^2} \quad (26.6)$$



**FIGURE 26.10**

Using past data to determine the search direction.

where  $N_p$  = number of points used in calculating the slope  
 $P_i$  = profit at point  $i$   
 $P_{ave}$  = average profit (in  $N_p$  data points)  
 $S$  = slope  
 $X_i$  = optimization variable value at point  $i$   
 $X_{ave}$  = average value of the optimization variable (in  $N_p$  data points)

In this method, the optimizing controller makes a change in the optimization variable equal to  $\Delta X[\text{sign}(S)]$ , with the step size  $\Delta X$  a fixed value independent of the magnitude of the slope. Note that this algorithm can be extended to more manipulated variables by modifying the expression for the slope. The following parameters appear in this algorithm and their selection and tuning are demonstrated in the next example.

**CALCULATED PROFIT.** The calculated variable should be directly related to plant profit and should be relatively insensitive to measurement noise and process disturbances.

**OPTIMIZATION VARIABLE(S).** The manipulated variables that yield excellent feedback control of safety-related variables and product quality should be allocated to these higher-priority tasks. The additional manipulated variable(s) that influence profit can be adjusted slowly to improve profit.

**NUMBER OF PAST DATA POINTS.** Past data provides a filter that makes the slope less sensitive to measurement noise; for this purpose, a large number would be good. However, too long a memory has two disadvantages. First, long memory gives importance to very old data that might not represent the current plant performance. Second, long memory requires many points on the “other side” of the maximum before the slope changes sign, which leads to large oscillations about the optimum operating point.

**STEP SIZE.** The step size should be small so that the change does not significantly influence important controlled variables, such as product quality. However, the step size should be large enough to cause a measurable change in the profit calculated from noisy plant measurements.

**EXECUTION PERIOD.** The approach to direct search described in this section requires the plant to achieve steady state between executions for measured values to represent the profit properly. Thus, the minimum execution period must be long enough for the process to achieve steady state: approximately the dead time plus four time constants for a first-order-with-dead-time process. Other approaches have been investigated that estimate parameters in a dynamic model and use the steady-state gain to determine the best direction (e.g., Bamberger and Isermann, 1978; Garcia and Morari, 1981).

**CALCULATED DIRECTION.** This method bases the direction on the slope. It would be possible to fit a higher-order curve to the data; however, the use of

process measurements in calculating the profit estimate introduces noise into the method, which usually leads to unreliable estimates of coefficients of the higher-order terms.

**EXAMPLE 26.3.**

The steady-state operation of the chemical reactor in Figure 26.11 is to be optimized in response to unmeasured disturbances. The profit is maximized by achieving the highest possible concentration of product B in the reactor effluent.

Information: The chemical reactions are

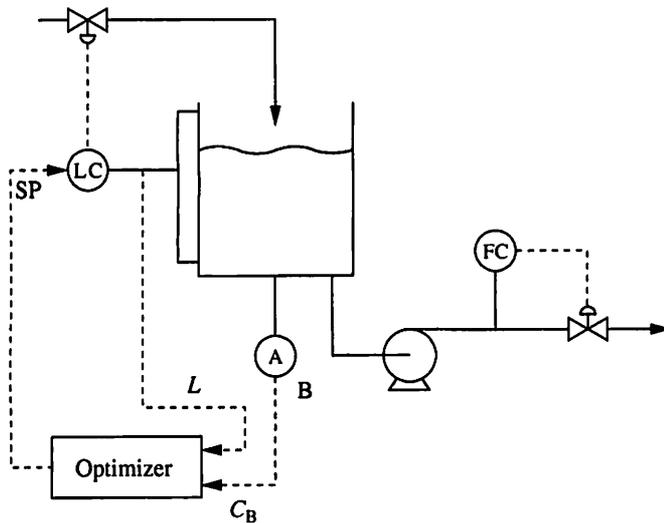


where

1. The rate expressions  $-r_A = k_{A0} \exp(-E_{A0}/RT)$  and  $r_C = k_{C0} \exp(-E_{C0}/RT)$  with nominal values of  $k_{A0} = 17748.5 \text{ min}^{-1}$ ,  $k_{C0} = 643,048 \text{ min}^{-1}$ ,  $E_{A0}/R = 3000 \text{ K}$ , and  $E_{C0} = 4000 \text{ K}$ .
2. The temperature is constant at 330 K.
3. The heat of reaction, heat transfer, and work are negligible.
4. The volume is constant and the contents are well mixed.
5. The flow rate is  $2.65 \text{ m}^3/\text{min}$ .

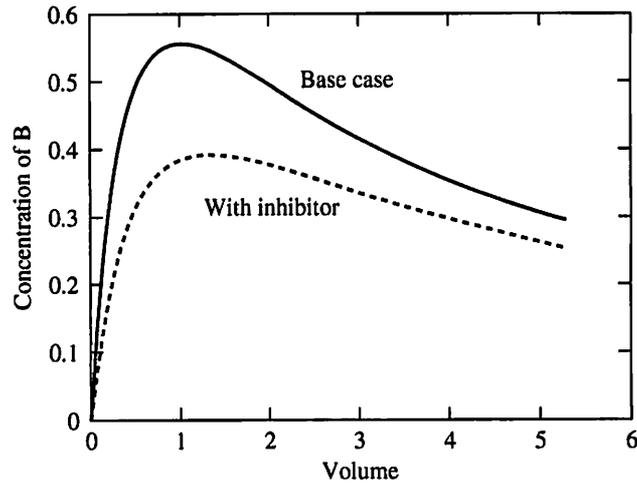
An optimum concentration of B ( $C_B$ ) exists because too low a concentration of the desired product B is not optimal, and a large concentration of B leads to excessive losses of B to undesired byproduct C. The conversion is influenced by the residence time in the reactor; therefore, the manipulated variable for this reactor is selected to be the volume of liquid in the reactor.

The optimum operating condition for the parameters in this example is  $V = 1.0 \text{ m}^3$ , which gives  $C_B = 0.556$ . However, the plant is subject to disturbances that require us to frequently change the operations (level) to obtain the highest  $C_B$  possible at the time. For this example, the disturbance involves occurrence of an inhibitor that decreases the rate of the first reaction;  $k_{A0}$  is decreased from 17,748.5 to  $10,000 \text{ min}^{-1}$  when the inhibitor is present. The steady-state behavior of the



**FIGURE 26.11**

**Stirred-tank reactor with direct-search optimizing control.**


**FIGURE 26.12**

Concentration of B for two situations in Example 26.3.

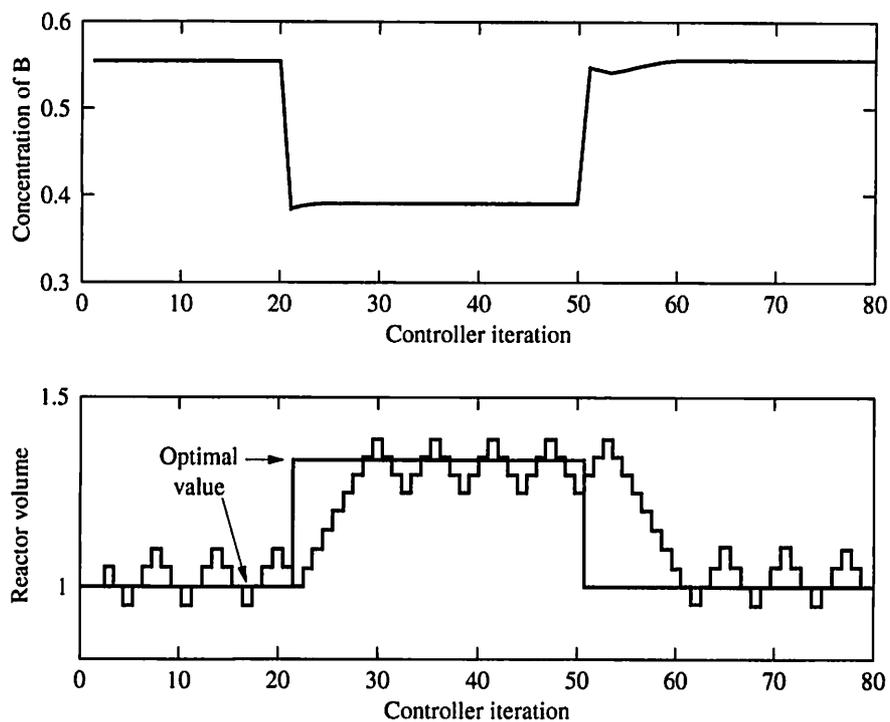
reactor is shown in Figure 26.12 for two situations, no inhibitor and inhibitor present. Naturally, many other disturbances are possible, and the real-time optimization approach should respond well to all disturbances.

The direct search optimization approach is applied to the reactor problem using the following parameter values:

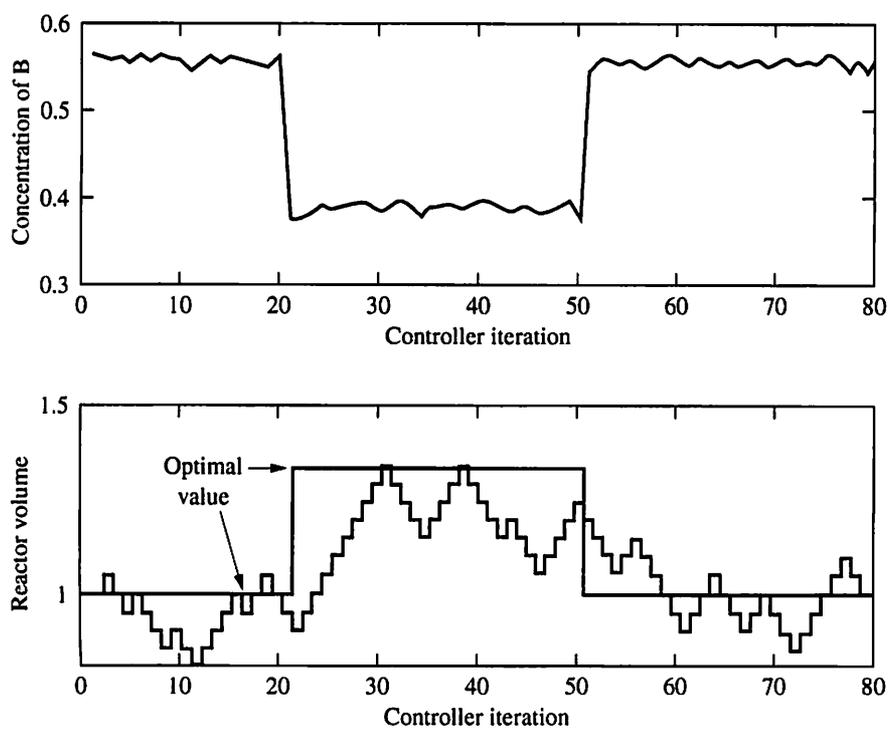
Profit measure	= $C_B$
Optimization variable	= $V$ (with $T$ , $C_{A0}$ , and $F$ constant)
Number of points in memory	= 3
Step size ( $\Delta V$ )	= $0.05 \text{ m}^3$
Execution period	= to achieve steady-state
Calculated direction	= slope from equation (26.6)

The performance of the direct-search optimization for the ideal situation, a plant without measurement noise, is shown in Figure 26.13. At controller iteration 20 the inhibitor in the feed increases in a step, and at iteration 50 it returns to its original value of 0.0. As a result of this disturbance, the concentration  $C_B$  decreases; then the search method adjusts the reactor volume  $V$  to achieve the maximum concentration of B for the current situation. Note that the optimum volume is shown in the figure only to aid in evaluating the performance of the optimizing controller; the optimum volume would normally not be known and was *not* used by the search algorithm.

The performance of the direct search for a realistic situation, in which the measurement of  $C_B$  includes noise, is given in Figure 26.14. The same scenario is involved in this data. As expected, the optimization performance is not as good, with some "wandering" around the optimum, but the algorithm was successful in changing the optimization variable in the proper direction and about the correct magnitude. Again, the true optimal value of the volume was *not* used by the direct-search controller.

**FIGURE 26.13**

Direct-search optimization without measurement noise.

**FIGURE 26.14**

Direct-search optimization with measurement noise.

## 26.3 ■ STATISTICAL PROCESS CONTROL (SPC)

Automatic process control (APC) using feedforward and feedback principles identifies a deviation from desired operation (i.e., from the set point or points) and makes an immediate adjustment in a manipulated variable. Thus, automatic process control does *not eliminate* the cause of poor operation (i.e., the disturbance); the adjustment is selected to *compensate for the effects* of the disturbance and maintain the controlled variable at its desired value. Since the sources of disturbances have not been affected, the APC approach leaves the process susceptible to future disturbances from the same source. In contrast, statistical process control (SPC) has as a goal the *identification and elimination* of disturbances. By this approach of removing the source of disturbances, the long-term effect of SPC is to reduce variability in process operation and improve product quality. Since some variability in process operations is inevitable, statistical process control alone cannot adequately control most process operations. Fortunately, SPC and APC can provide complementary improvements and can be applied to the same process to improve the overall performance.

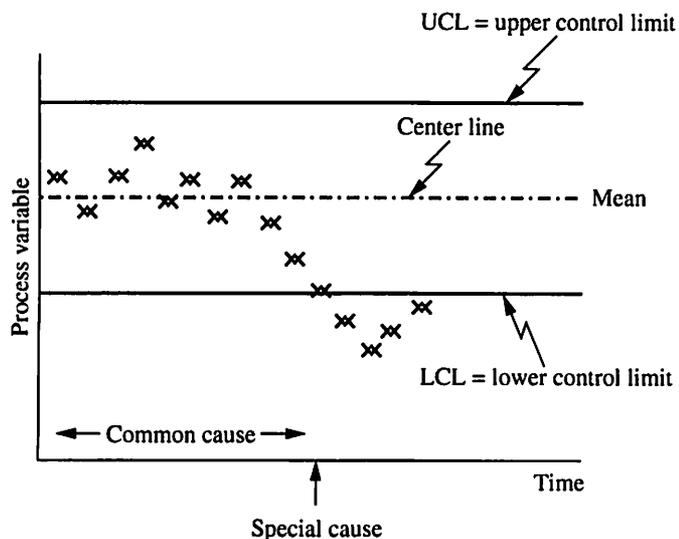
Statistical process control identifies deviations in process performance using real-time measurements. The base-case performance is established, not from a fundamental mathematical model, but rather from experience; thus, empirical data is used in establishing the typical variability in process variables. This variability results from many (small) disturbances and sensor noise, which are considered to be unavoidable. This typical variability is referred to as *common-cause*, which results in consistent variability over time. As each new set of process data is collected, it is evaluated by comparison with the common-cause variability, and the possibility of a significant change in process operation is evaluated. Significant deviation from the common-cause variability would then result from a disturbance that is not typical; this is referred to as a *special (or assignable) cause* of variability. If a change has occurred, the process is diagnosed to determine the proper corrective action. The corrective action may be as simple as adjusting a final control element, or it might be as involved as changing the source of feed material or catalyst to prevent the cause of the disturbance.

Automatic process control compensates for deviations from set point. In contrast, statistical process control has the goal of identifying and eliminating causes of variability in key process variables.

Statistical process control is now demonstrated by way of its best-known method.

### Shewhart Chart

The analysis of the process data to quickly and easily recognize changes in process performance is facilitated by the *Shewhart chart*, shown in Figure 26.15. The Shewhart chart provides a visual display of recent process data of a single measurement along with limits representing the typical, common-cause variability. The limits are determined empirically from “good” process operation and are



**FIGURE 26.15**  
Shewhart chart.

typically set to include 99.7% of the data; if the data is normally distributed about its mean, the limits are located  $\pm$  three standard deviations from the mean. The limits are referred to as the *upper and lower control limits* (UCL and LCL). Comparing a measured value with these limits is essentially a statistical hypothesis test on whether the mean of the variable has changed; this test could be calculated in a standard manner, although the clarity provided by the visual display of data with the limits increases the appreciation of the effects of variability (Montgomery, 1985). Also, modifications are available for variables with nonnormal distributions (Jacobs, 1990).

When the process is experiencing typical variability, a situation often referred to as “in the state of statistical control,” most data will be within the limits. Although there is variation of the measurement within these limits, this variation is accepted as inevitable and no action is taken, whereas automatic process control makes a feedback compensation for any nonzero error. If the measured value exceeds the limits, the SPC approach requires a diagnosis to determine the special or assignable cause and requires the implementation of the appropriate corrective action.

**EXAMPLE 26.4.**

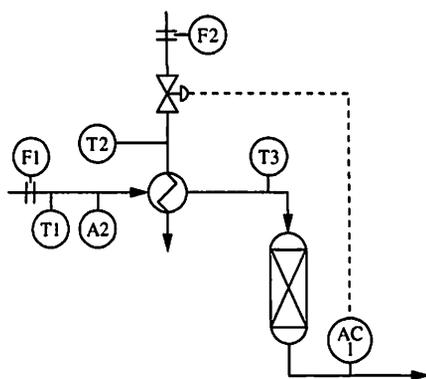
Reconsider the chemical reactor in Example 26.3 without the optimizer. The liquid level is controlled and the concentration of component B is measured online. Describe how the process could be monitored using a Shewhart chart.

The concentration of B is the key indicator of process performance and can be plotted on a Shewhart chart. Historical data, not shown, has been used to establish the common-cause variability and the control limits for the concentration. Some data are plotted in Figure 26.15 for this example. In the initial data, the concentration remains within the action limits, although it varies due to the common-cause disturbances: small changes in the level, flow, reactor temperature, and feed concentration. At a time indicated by an arrow, the concentration of B deviates from its usual range and remains outside this range for an extended time, which indicates

a special-cause disturbance has occurred. In this example, the source of the disturbance is not obvious from the data, so additional diagnosis would be required. For example, the measures of the key process variables could be checked for errors, the reactor temperature could be determined, and the feed composition could be measured. As noted in Example 26.3, the inhibitor concentration is an important factor in the process performance and could be determined by laboratory analysis. If the inhibitor concentration has caused this deviation, as is likely for such a large disturbance, the underlying source of the disturbance should be determined; for example, the reason could be contamination in storage or poor quality from a supplier. Whatever the cause, the corrective action should not only eliminate the current disturbance but also prevent future occurrences. Notice that the optimization results in Example 26.3 can only give the best performance with a given level of inhibitor, which can represent a substantially lower concentration of B; only eliminating the disturbance can restore this process to its desired high concentration of B.

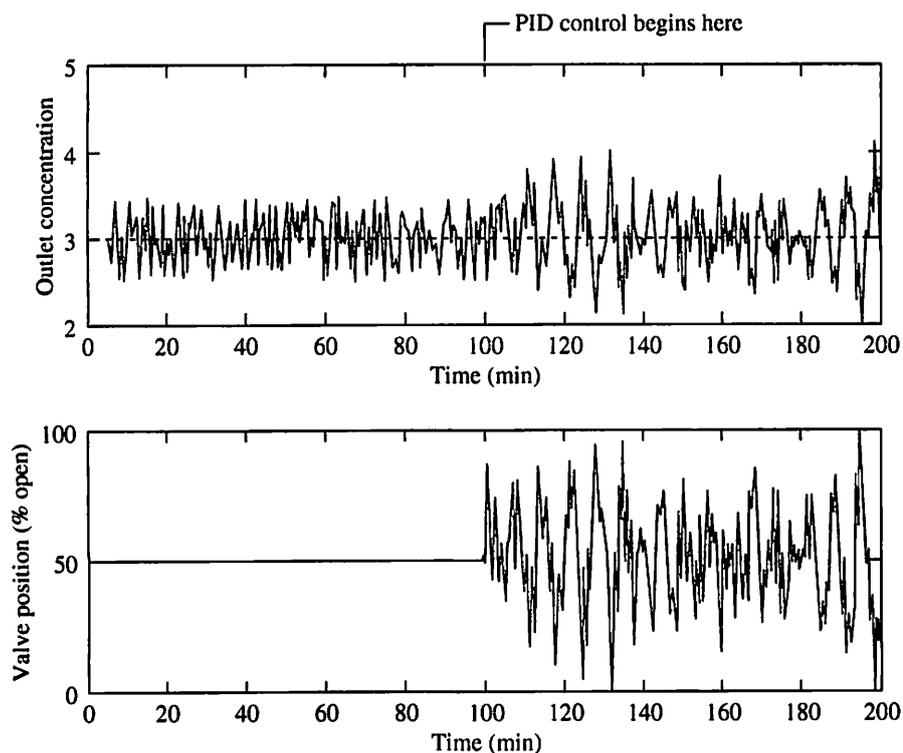
### Reducing Variability

The distinction between APC and SPC can be clarified and the strengths of each can be demonstrated by considering two examples which could involve the same process, but experiencing different disturbances. Consider the packed-bed chemical reactor in Figure 14.11. The objective is to maintain the concentration in the effluent measured by the sensor at a desired value, and concentration can be influenced by adjusting the heating medium valve in the reactor preheat exchanger. The performances of this process with and without feedback control are considered for two different scenarios.

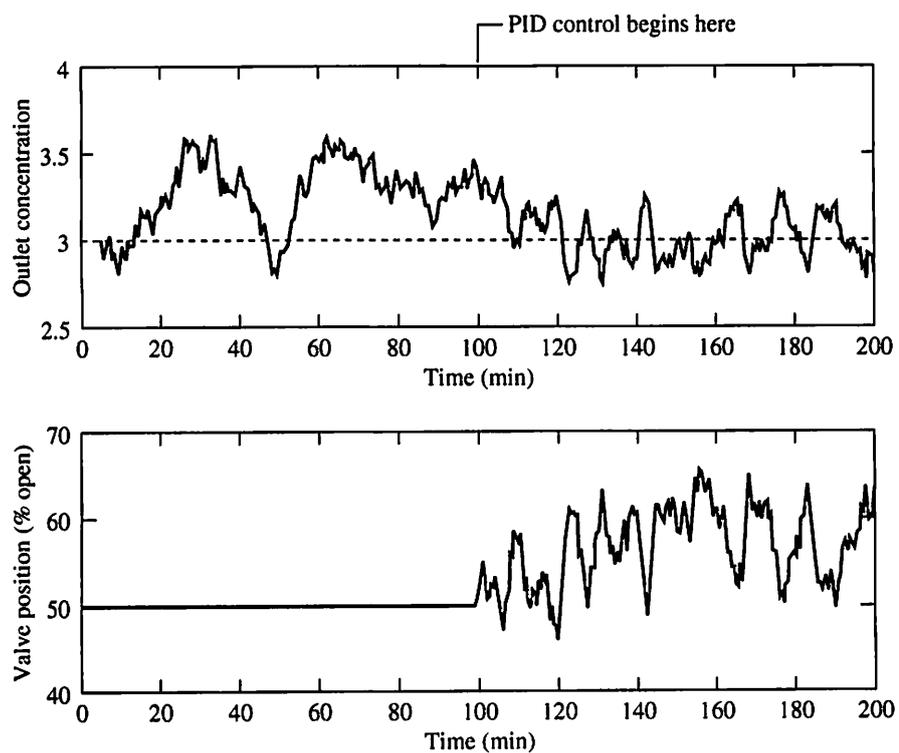


**SCENARIO I.** For this scenario, the initial data is given without any feedback action in Figure 26.16. The cause of the variation for Scenario I is essentially random, uncorrelated noise about the constant mean value. For example, this could occur if (1) no (significant) disturbances occur in the reactor operating variables and (2) the sensor experiences a random error each time a sample is analyzed. In this situation, the proper operating policy for this common-cause variability is to make no adjustment to the valve, since the current error cannot be corrected by the adjustment and the current deviation does not provide an indication of the future deviations. As shown in Figure 26.16, implementing a standard proportional-integral feedback control calculation will *increase* the variability in the product quality. Thus, the SPC approach provides better performance for regulating the reactor in Scenario I.

**SCENARIO II.** In this scenario, the initial data is given without any feedback action in Figure 26.17. The variation for Scenario II is due not only to random sensor error but also to slower-changing disturbances in some process input variables such as feed composition and heating medium temperature. We can observe that the variability of the product composition appears correlated in time; that is, the composition includes a slow drift along with some random noise. In this situation, the current deviation provides an indication of the likely future deviations, and

**FIGURE 26.16**

Dynamic response for Scenario I, in which feedback degrades performance.

**FIGURE 26.17**

Dynamic response for Scenario II, in which feedback degrades performance.

the feedback dynamics are fast enough that adjustments in the valve can compensate for the slowly changing disturbances. Thus, automatic process control is appropriate, as shown in Figure 26.17, which shows a *decrease* in the variability when a proportional-integral feedback controller is implemented. Thus, the APC approach provides better performance for regulating the reactor in Scenario II.

The comparison of the performance of SPC and APC for these two scenarios demonstrates that both have many applications. When the variability without feedback compensation is nearly random, so that feedback corrections cannot compensate for the deviations, an SPC approach is appropriate. When the variability without feedback compensation is due to slowly varying disturbances, APC can be effective in reducing the variability. For further discussion on this point, see MacGregor (1990).

### Variability of the Manipulated Variable

Another distinction between APC and SPC stems from the frequency of corrective actions taken. APC involves an action every time the controller is executed; thus, it must be possible to adjust the final element without disrupting the process operation, which is possible with standard control valves. As a result, APC reduces the variance of the controlled variable while increasing the variance of the manipulated variable. This situation is sometimes described as “moving” the variability from the important controlled variable to the less important manipulated variable, as demonstrated in Figure 26.16. This situation has been discussed previously and has been shown in Figures 7.8, 7.9, 13.18, 23.10, and 24.19.

In contrast, SPC involves infrequent adjustments—only when the measurement exceeds the control limits. This is advantageous for systems in which the cost of the control action is considerable. Examples of costly adjustments are changing the reactor catalyst, changing the feed material, and stopping and adjusting machinery. Since the special-cause disturbances occur infrequently and the action limits are set to result in few “false alarms” (only 3 in 1000), the SPC approach, when applied to appropriate scenarios, reduces the adjustments in the manipulated variable required to maintain the controlled variable within the upper and lower control limits.

This perspective suggests an approach for diagnosing process performance for variables that are under PID feedback control. In situations with effective feedback, the controlled variable may not deviate greatly from its set point, although significant disturbances occur. However, the occurrence of these disturbances can be determined by monitoring the manipulated variable, because it must be adjusted to compensate for disturbances.

### Process Capability

The discussion to this point has addressed the variability of key process variables; now, the requirements of the market are added to the considerations. In particular, the comparison of the variability (here, assumed normally distributed) with the required minimum variability is an important factor in evaluating the success of the process operation. The comparison of actual with required variability is termed

the *process capability*, defined as follows:

$$\text{Capability index} = C_p = \frac{\text{USL} - \text{LSL}}{6\sigma} \quad (26.7)$$

$$C_{pk} = \min \left[ \frac{\text{USL} - X_m}{3\sigma}, \frac{X_m - \text{LSL}}{3\sigma} \right] \quad (26.8)$$

where

- $C_p$  = process capability index
- $C_{pk}$  = process capability index
- USL = upper specification limit on acceptable variation in product variable
- LSL = lower specification limit on acceptable variation in product variable
- $X_m$  = mean value of the variable
- $\sigma$  = standard deviation of the actual variability of the product quality

The variable  $C_p$  is meaningful when the target for the product specification is the mean of the range. The  $C_{pk}$  is meaningful when the target is not the mean of the range. The best situation occurs when the variability of the process is small compared with the variability allowed in the market:

- $C_{pk} \ll 1$  Considerable “off-specification” material is produced
- $C_{pk} \approx 1$  Most production satisfies specifications
- $C_{pk} \gg 1$  Nearly all production well within the specifications

The capability index is a useful measure for evaluating the current process performance against the market needs. However, continual improvement efforts should not cease when  $C_p$  and  $C_{pk}$  are greater than 1.0.

The reduction of variability should be a continual effort. The goals include the reduction in number of times special causes occur and the reduction of the common-cause variability.

The producer of the highest-quality product often can increase total sales or profit margins, and experience has shown that the lower-quality producers often cannot sell their products.

## 26.4 ■ CONCLUSIONS

Two approaches for continual process improvement have been introduced in this chapter. Optimization is appropriate when the operating profit changes significantly because of frequent disturbances and there are available manipulated variables that can be adjusted to increase the profit without degrading the product quality. These variables tend to be set points of the underlying regulatory process controls. Thus, optimization generally functions as the highest level in a cascade control structure.

Statistical process control has as its goal the reduction of variability, primarily in the key product qualities. In contrast to automatic process control, statistical

process control involves actions that address the root cause of the disturbance. By diagnosing and eliminating these causes, the number and severity of future disturbances are reduced, and the process performance is improved.

These approaches have merely been introduced in this chapter. The reader is encouraged to refer to the References and Additional Resources for further information. These methods can provide substantial improvement when applied continually to a process that is operating under excellent automatic process control.

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Montgomery, D., *Introduction to Statistical Quality Control*, Wiley, New York, 1985.

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Yang, C., and B. Waldman, "On-Line Optimization Boosts Ethylene Profits," *Oil and Gas J.*, 80, 104–109 (September 1982).

## ADDITIONAL RESOURCES

Many examples of process control designs that improve the economic performance of processes are given in

Liptak, B., *Optimization of Unit Operations*, Chilton Books, Radnor, PA, 1987.

A complete discussion of empirical methods for estimating response surfaces is given in the following reference:

Box, G., and N. Draper, *Empirical Model Building and Response Surfaces*, Wiley, New York, 1987.

An introduction to the standard methods in statistical process control is provided in the following references, which tend to provide methods for the parts manufacturing industries. They also give some guidance on managing the process diagnosis procedures.

Grant, E., and R. Leavenworth, *Statistical Quality Control*, McGraw-Hill, New York, 1988.

Ishikawa, K., *What Is Total Quality Control? The Japanese Way*, Prentice-Hall, Englewood Cliffs, NJ, 1985.

Oakland, J., *Statistical Process Control, A Practical Guide*, Wiley, New York, 1986.

The process industries involve fast sampling of processes with relatively slow dynamics. In this situation, each data point can be correlated with points in the recent past. Although the general approaches in classical SPC are directly applicable, many of the specific methods and quantitative approaches must be modified. The following references provide insight into the special needs of the process industries.

Box, G., and T. Kramer, "Statistical Process Monitoring and Feedback Adjustment—A Discussion," *Technometrics*, 34, 251–267 (see also the extensive discussions on pages 268–285) (1992).

Harris, T., and W. Ross, "Statistical Process Control Procedures for Correlated Observations," *Can. J. Chem. Eng.*, 69, 139–148 (1991).

Hunter, S., "The Exponentially Weighted Moving Average," *J. Qual. Techn.*, 18, 203–210 (1986).

MacGregor, J., "On-Line Statistical Process Control," *Chem. Eng. Prog.*, 84, 21–31 (1988).

The concepts of SPC can be extended to multivariable processes, although the direct, independent monitoring of many independent variables via Shewhart charts would be tedious and difficult to interpret. An alternative method is described in

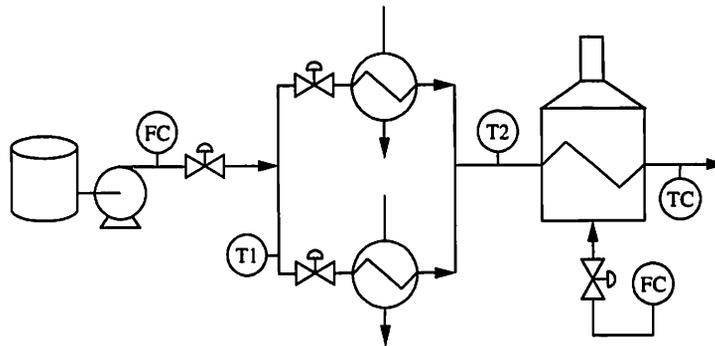
Kresta, J., J. MacGregor, and T. Marlin, "Multivariate Statistical Monitoring of Process Operating Performance," *Can. J. Chem. Eng.*, 69, 35–47 (1991).

## QUESTIONS

**26.1.** Design an optimizing control strategy for the process in Figure Q26.1 to satisfy the following objectives:

- (1) Tight control of the flow rate leaving the furnace via the coil
- (2) The coil outlet temperature (TC) maintained close to its set point
- (3) The total fuel consumption minimized

Design a control strategy that achieves these objectives. Clearly define the measurements, calculation of the performance function, and the control algorithm and explain how interactions among the strategies will be considered.



**FIGURE Q26.1**

**26.2.** Discuss the key elements of the single-stage refrigeration circuit in Figure Q26.2.

- (a) Design regulatory controls for this system that satisfy the demands of the consumers. Two consumers are shown as a heat exchanger (temperature controller) and a condenser (pressure controller).
- (b) Add necessary controls to minimize the energy consumption (i.e., minimize the steam consumption) while satisfying the consumers' demands. You may add sensors and add and delete valves.

**26.3.** The plant has byproducts that can be used as fuel or must be discarded with no value. Thus, all excess fuel should be consumed, if possible. Design a control strategy that provides good coil outlet temperature control and that consumes all possible excess fuel for the fired heater in Figure Q26.3. Note that (1) the two fuels have different compositions and (2) the excess fuel availability can change quickly and by large magnitudes.

**26.4.** In some plants, incentives exist to supply heat to the process via one (or a few) large, efficient fired heaters. The energy is transferred to consumers

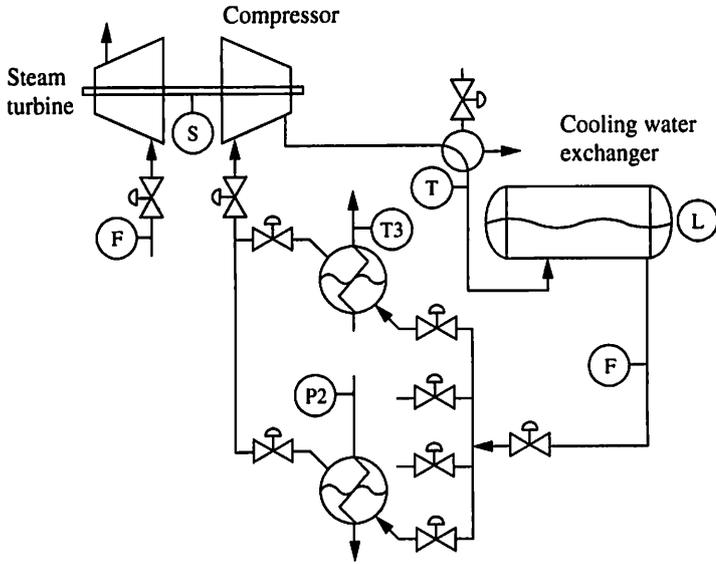


FIGURE Q26.2

throughout the plant via an oil stream with good heat transfer, heat capacity, and thermal stability properties. Design a control strategy for the process in Figure Q26.4 that satisfies the following objectives, listed in order of decreasing importance.

- (a) Control T3 and T4.
  - (b) Control T6 and T7.
  - (c) Determine the best value for the fired-heater outlet temperature, i.e., the value that satisfies (a) and (b) at minimum fuel.
  - (d) Recover as much energy as possible at the highest temperature.
- You may add sensors and add or delete piping and valves.

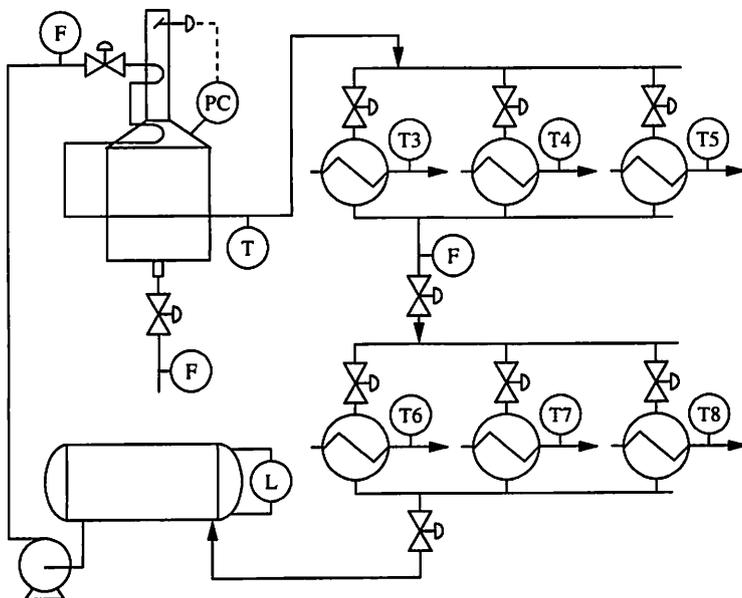


FIGURE Q26.4

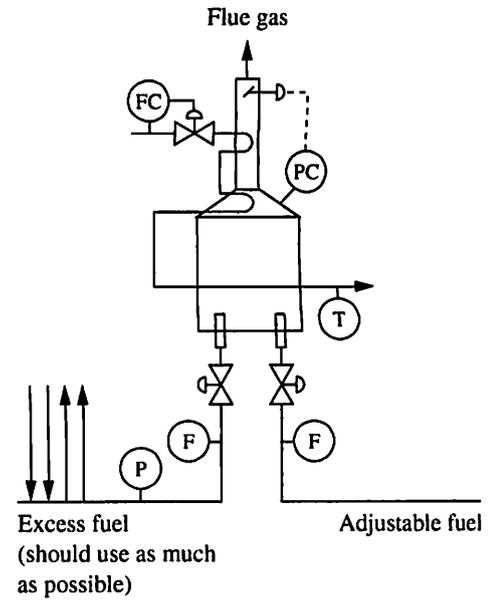
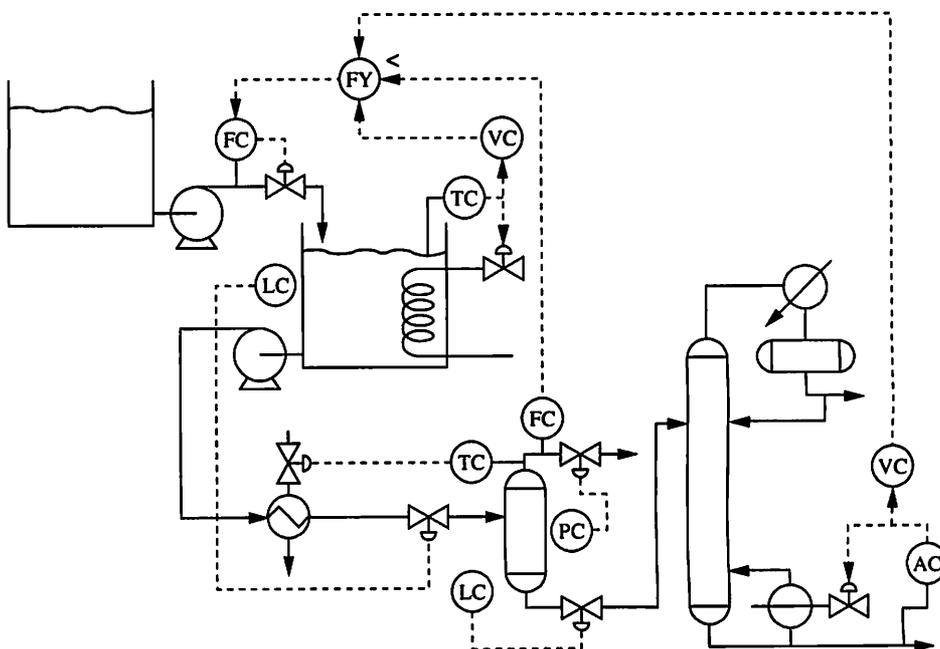


FIGURE Q26.3

- 26.5.** The control design in Figure Q26.5 is proposed for maximizing the production rate in a chemical plant. The likely equipment limitations are the maximum reactor heating, the maximum flow of vapor from the flash, and the maximum reboiler duty in the distillation tower. The proposed design may not function well because of the long dynamics. Suggest enhancements that would ensure that (a) the maximum vapor flow from the flash is not exceeded and (b) the product quality in the distillation tower would be controlled close to its set point.



**FIGURE Q26.5**

- 26.6.** Derive the general equation for the direct search algorithm in Section 26.2 for any number of manipulated variables. Also, discuss potential drawbacks with the proposed method when applied to processes with more than one optimization variable.
- 26.7.** The dynamic plots in Figure 26.13 have the iteration numbers as the abscissa. Determine an appropriate time between iterations for this process.
- 26.8.** Discuss additional considerations that should be included in a real-time boiler optimization as presented in Section 26.2. How could each consideration be integrated into the mathematical statement of the optimization?
- 26.9.** Some Shewhart charts include warning limits, which are between the mean and the control limits. Discuss (a) the interpretation one could place on a single violation (or several sequential violations) of the warning limits, (b) reasonable values for the warning limits, and (c) types of actions which could be based on these limits.
- 26.10.** The equations for the process capability used in Section 26.3 were based on normally distributed data. Describe a test of a data set to decide whether the data is normally distributed.

- 26.11.** The Shewhart chart detects changes in mean via deviations beyond the control limits.
- (a) Discuss the interpretation of several simultaneous data points above (or below) the mean, but within the control limits.
  - (b) Devise additional rules that could be used in conjunction with the standard Shewhart chart.
  - (c) Specify all assumptions required for the rules in (b) to be appropriate and when these assumptions are likely to be satisfied.
- 26.12.** The Shewhart chart uses the data to identify a change in mean. Propose a different chart that could identify a change in the variability, as measured by the standard deviation or variance.
- 26.13.** Often, the variable used in the Shewhart chart is an average of several samples taken at the same time from the process. Discuss the advantages and disadvantages of using the average of several samples rather than a single measurement.
- 26.14.** A mixing tank after a process can, in some cases, reduce the effect of process variability prior to providing the product to the customer. Discuss the effects of product mixing on the following processes. Specifically, would the mixing reduce the variability important to the customer when the mean of the production is correct but the variance of the material entering the tank is too large?
- (a) The bottoms product of a benzene-toluene distillation tower is mixed in a tank. The customer is interested in the percent benzene impurity.
  - (b) The ball bearings from a manufacturing plant are mixed in a bin. The customer is interested in the diameter of each ball bearing.
- 26.15.** Discuss the differences between the control limits (UCL and LCL) and the specification limits (USL and LSL).