Multiloop Control: Performance Analysis

CHAPTER 21

21.1 INTRODUCTION

Multiloop process control systems were introduced in the previous chapter, where some important effects of interaction on steady-state and dynamic behavior were explained, and a quantitative measure of interaction—the relative gain—was presented. This understanding of interaction is now applied in the analysis of multiloop control performance and design. Three main facets of control performance analysis are presented and applied to the design of multiloop systems. The first is loop pairing: deciding the controlled and manipulated variables for each single-loop controller in a multiloop system. The second facet is controller tuning to achieve the desired performance, as well as to maintain stability. The third facet involves enhancements to the PID control calculations that can improve control performance while retaining the simplicity of the multiloop control strategy in selected applications.

As in the single-loop case, the first step is to define control objectives thoroughly. The main aspects of multivariable control performance are presented in the following list. Several are the same as for single-loop systems; however, items 2, 5, and 6 are new, and item 4 can assume even greater importance.

1. *Dynamic behavior of the controlled variables.* The control system should provide the desired control performance for expected disturbances and set point changes. The performance can be defined by any appropriate measures presented in Chapter 9 (e.g., IAE and decay ratio).
2. *Relative importance among controlled variables.* The multiloop control structure should be compatible with the relative importance of various controlled

variables, since some controlled variables may be very important and should be maintained close to their set points, while others may not be as important and can be allowed to experience larger short-term deviations.

3. *Dynamic behavior of the manipulated variables.* Feedback control reduces the variability in the controlled variables by adjusting manipulated variables; however, the variability in the manipulated variables should not be too large.
4. *Robustness to model errors.* The control system should be robust so that it performs well in spite of inevitable modelling errors. As with single-loop systems, this objective requires that feedback controllers be tuned to ensure stability and give the best feedback performance possible for the expected model errors. In addition, we shall see that some multivariable control systems are highly sensitive to model errors and can be applied only when models are very accurate.
5. *Integrity to controller status changes.* Each controller should retain reasonable performance for its basic objectives, even if performance is somewhat degraded, as changes occur in the automatic/manual status of interacting loops.
6. *Proper use of degrees of freedom.* The control system should be able to adapt itself to the degrees of freedom available in the process, which can change when a manipulated variable cannot be adjusted (e.g., because it reaches a physical limit). This topic is addressed in Chapter 22.

It would be possible to arrive at the best design by simulating all possible loop pairings and enhancements. However, simulating the numerous candidate designs would be a time-consuming task, especially since the controllers in every candidate would have to be tuned. In addition, such a "brute force" simulation technique would provide little insight into improving performance through changes in process equipment, operating conditions, or control structure.

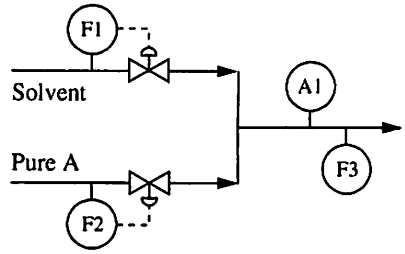
The approaches presented here are selected because they address the most important issues and generally require less engineering effort than simulating all possibilities. Because these methods build on the results of the previous chapter, it will be assumed that all systems considered are controllable. The new analysis method for each major design decision is addressed in a separate section of the chapter; then, some advanced topics are introduced. Finally, a flowchart is provided to clarify the integration of major analysis steps in reducing potential candidate designs and making decisions for multiloop systems. The hierarchical analysis method eliminates candidates with a minimum of engineering effort and results in one or a few final designs. Because of assumptions in some of these methods, the final design selection may still require simulation, but of only a few candidates. Before the methods are covered, a few motivating examples are presented to highlight some important issues that distinguish multiloop from single-loop performance.

21.2 ▧ DEMONSTRATION OF KEY MULTILOOP ISSUES

In this section, four important multiloop issues are introduced through process examples that show the key effects of interaction on the dynamic performance of multiloop control systems. These issues were selected because they often influence control design for process units and they are unique to, or assume heightened significance for, multiloop systems. The analysis methods to address these issues are provided in subsequent sections of this chapter.

EXAMPLE 21.1. Operating conditions

The first issue is the effect of operating conditions on multiloop control performance, which is introduced through consideration of the blending process in Figure 20.2. We begin by considering the same operating conditions previously considered in Table 20.5, which are repeated in Table 21.1 as the base case. For these operating conditions, the product is very dilute (5% A). Thus, changing the flow rate of component A by a small amount affects the product composition significantly while affecting the total product flow only slightly. This qualitative analysis was substantiated by the quantitative tuning analysis in Example 20.10, which leads to the recommendation of the pairing for the base case in Table 21.1.



Next, we investigate whether a different pairing is recommended for an alternative operating condition that involves a very concentrated product (95% A). In this operation, the product concentration is more sensitive to the flow of the solvent than to the flow of component A, as it was in the base case. The tuning for proportional-integral controllers is determined by the guidelines for 2 × 2 systems with one fast and one slow loop. For this alternative case the loop pairings A_1-F_1 and F_3-F_2 provide better control, because the tunings for the controllers in this configuration are not dependent on the automatic/manual status of the other controller. From this example, we can conclude:

> The proper control loop pairing depends on the operating conditions of the process.

Thus, it is not possible to specify a single control design for each unit operation, like blending or two-product distillation. Even though units may appear similar, at least with respect to equipment structure, their operating conditions and the resulting dynamic responses must be considered.

TABLE 21.1

Effect of operating conditions on multiloop performance of the blending system

Operating condition	**Set points** A_1	F_3	**Relative gain** λ_{A1-F2}, λ_{F3-F1}	λ_{A1-F1}, λ_{F3-F2}	**Pairing:** A_1-F_2, F_3-F_1	**Pairing:** A_1-F_1, F_3-F_2
Base case	0.05	100	0.95	0.05	*Recommended* The controller tuning is essentially the same for single-loop and multiloop control.	*Not recommended* The controller tuning depends strongly on the status of the interacting loop.
Alternative case	0.95	100	0.05	0.95	*Not recommended* The controller tuning depends strongly on the status of the interacting loop.	*Recommended* The controller tuning is essentially the same for single-loop and multiloop control.

EXAMPLE 21.2. Transmission interaction

The previous analysis selected the controller pairing that reduces transmission interaction. In fact, the best controller pairings for the two examples are consistent with selecting the multiloop pairings that yield relative gain values closest to 1.0, as verified by the relative gain values in Table 21.1. Given this result, it is tempting to assume that the multiloop control with relative gains closest to 1.0 always gives the best performance. This example demonstrates that this assumption is *not always valid* and that a more complete analysis is required.

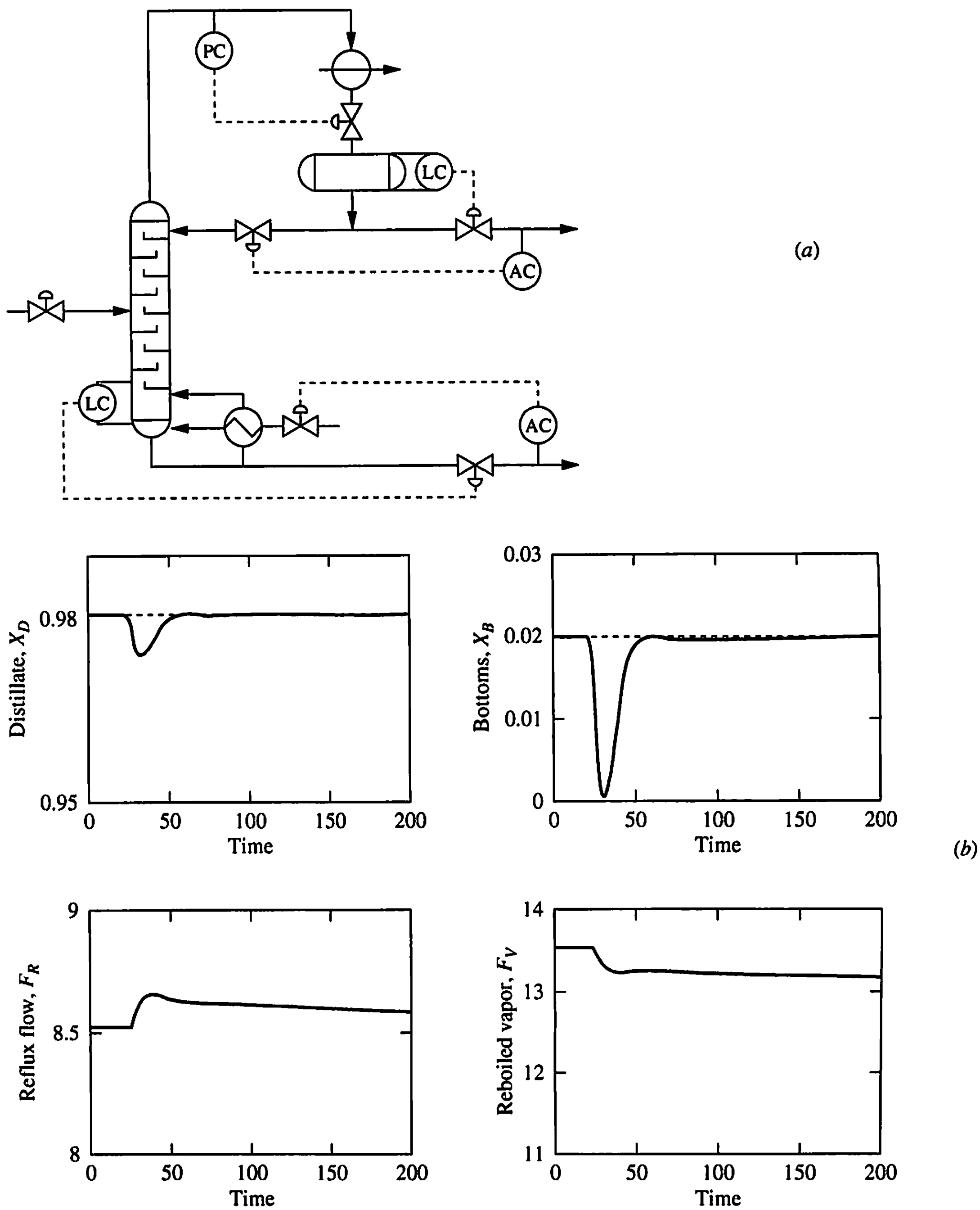


FIGURE 21.1

Energy balance distillation control: (*a*) schematic diagram; (*b*) transient response to a change in light key in feed of −0.04.

This example consists of the two-product distillation tower separating a binary feed considered in Example 20.2. Both top and bottom product compositions are of equal importance, and the major disturbance is a change in feed composition. Two regulatory loop pairings, which differ only in how the distillate and reflux flow rates are manipulated, are considered. The first, shown in Figure 21.1*a*, has the distillate manipulated to control the overhead drum level and the reflux manipulated to control the top product composition; this is called *energy balance* and was considered in Chapter 20. The second, shown in Figure 21.2*b*, has the distillate and reflux pairings interchanged; this is called *material balance* and is introduced

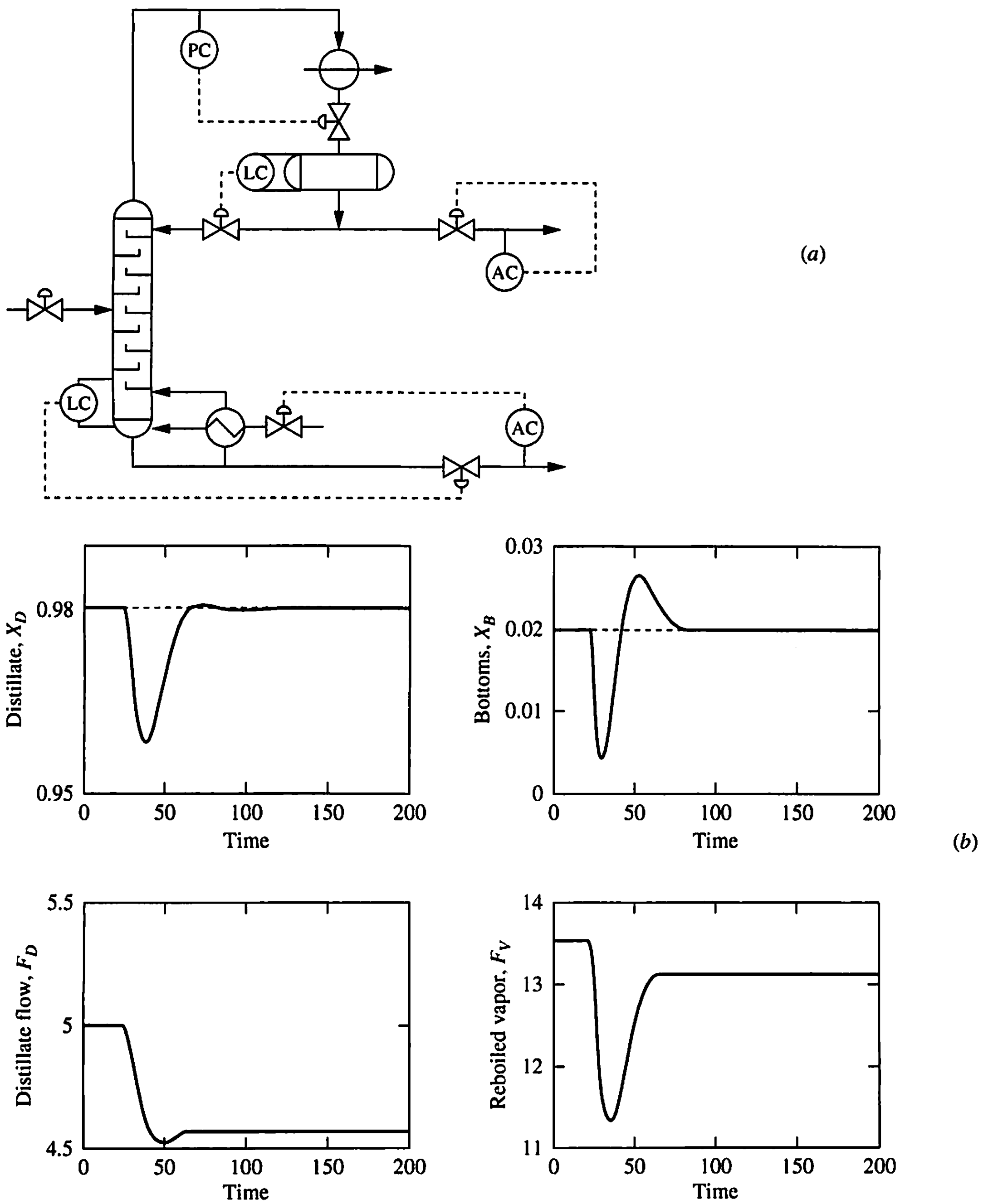


FIGURE 21.2

Material balance distillation control: (*a*) schematic diagram; (*b*) transient response to a change in light key in feed of −0.04.

TABLE 21.2

Tuning and performance data for distillation dynamics

		Energy balance	Material balance
λ_{XD-FB}		6.09	
λ_{XD-FD}			0.39
K_{CD}		10.4	−9.35
T_{ID}		9.0	10.0
K_{CB}		−6.8	−68.7
T_{IB}		6.1	6.7
Feed			
composition	IAE_{XD}	0.17	0.45
disturbance	IAE_{XB}	0.35	0.31
($\Delta x_f = -0.04$)			
	IAE_{XD}	0.35	0.0585
SP_{XD}	IAE_{XB}	0.34	0.0456
disturbance			
($\Delta SP_{XD} = 0.005$)			

here for the first time. It is important to recognize that the steady-state responses of these two systems are identical because the process equipment, controlled variables, and manipulated variables are the same. Only the transient behavior is different. The linear transfer functions, including 2 min analyzer dead times, for the two systems follow.

Energy balance.

$$\begin{bmatrix} X_D \\ X_B \end{bmatrix} = \begin{bmatrix} \dfrac{0.0747e^{-3s}}{12s+1} & \dfrac{-0.0667e^{-2s}}{15s+1} \\ \dfrac{0.1173e^{-3.3s}}{11.75s+1} & \dfrac{-0.1253e^{-2s}}{10.2s+1} \end{bmatrix} \begin{bmatrix} F_R \\ F_V \end{bmatrix} + \begin{bmatrix} \dfrac{0.70e^{-5s}}{14.4s+1} \\ \dfrac{1.3e^{-3s}}{12s+1} \end{bmatrix} X_F \qquad (21.1)$$

Material balance.

$$\begin{bmatrix} X_D \\ X_B \end{bmatrix} = \begin{bmatrix} \dfrac{-0.0747e^{-2s}}{10s+1} & \dfrac{0.008e^{-2s}}{5s+1} \\ \dfrac{-0.1173e^{-2s}}{9s+1} & \dfrac{-0.008e^{-2s}}{3s+1} \end{bmatrix} \begin{bmatrix} F_D \\ F_V \end{bmatrix} + \begin{bmatrix} \dfrac{0.70e^{-5s}}{14.4s+1} \\ \dfrac{1.3e^{-3s}}{12s+1} \end{bmatrix} X_F \qquad (21.2)$$

Tuning for these control systems can be determined by the methods in Chapter 20. The results are reported in Table 21.2.

The transient responses for well-tuned feedback control in response to a feed composition upset are given in Figures 21.1*b* and 21.2*b*, and the control performances are summarized in the IAE values in Table 21.2. Based on the total IAE values (0.52 for energy balance and 0.76 for material balance), the performance of the energy balance control design is better than the material balance controller for the feed composition disturbance—in spite of the fact that the interaction, as measured by the relative gain, is much further from 1.0 for the energy balance controller pairing. Thus, we conclude:

The best-performing multiloop control system is not always the system with the least transmission interaction (i.e., with relative gain elements closest to 1.0).

This result should not be surprising when one considers the closed-loop transfer function for a multiloop system, derived in Chapter 20 and repeated here.

$$\frac{\mathrm{CV}_1(s)}{D(s)} = \frac{\left[G_{d1}(s) - \dfrac{G_{d2}(s)G_{12}(s)G_{c2}(s)}{[1 + G_{c2}(s)G_{22}(s)]}\right][1 + G_{c2}(s)G_{22}(s)]}{\mathrm{CE}(s)} \tag{21.3}$$

with

$$\mathrm{CE}(s) = 1 + G_{c1}(s)G_{11}(s) + G_{c2}(s)G_{22}(s) + \frac{G_{c1}(s)G_{11}(s)G_{c2}(s)G_{22}(s)}{\lambda_{11}(s)}$$

The dynamic response depends on all elements in the transfer function, so both numerator and denominator must be considered, especially in multivariable systems. However, the relative gain appears only in the denominator, whereas the disturbance transfer function appears in the numerator. This result is a bit disappointing, since the design of multiloop systems would have been relatively easy if the pairing were determined completely by the relative gain. Transmission interaction is important and must be considered, but a simple pairing method based entirely on the relative gain is not always correct.

EXAMPLE 21.3. Disturbance type.

A further important question concerns the performance of candidate controls for different disturbances. Specifically, is it true that one candidate control pairing performs best for all disturbances? This issue is investigated by extending the study of the two distillation controller pairings for a different disturbance: a set point change to the distillate controller. The dynamic responses for a set point change in the top composition controller of +0.005 mole fraction, with the other set point and all disturbances constant, are given in Figure 21.3*a* and *b*. The results, summarized in Table 21.2, show that the total IAE values are 0.69 for energy balance and 0.104 for material balance. In this case, the material balance system performs better. Note that an attempt to "speed" the sluggish response of the energy balance system through tighter controller tuning will lead to instability.

From this example we conclude:

The relative performance of control designs and the selection of the best design can depend on the specific disturbance(s) considered.

This result seems reasonable when considering the following closed-loop transfer function for the set point change:

$$\frac{\mathrm{CV}_1(s)}{\mathrm{SP}_1(s)} = \frac{G_{c1}(s)G_{11}(s) + G_{c1}(s)G_{c2}(s)[G_{11}(s)G_{22}(s) - G_{12}(s)G_{21}(s)]}{\mathrm{CE}(s)} \tag{21.4}$$

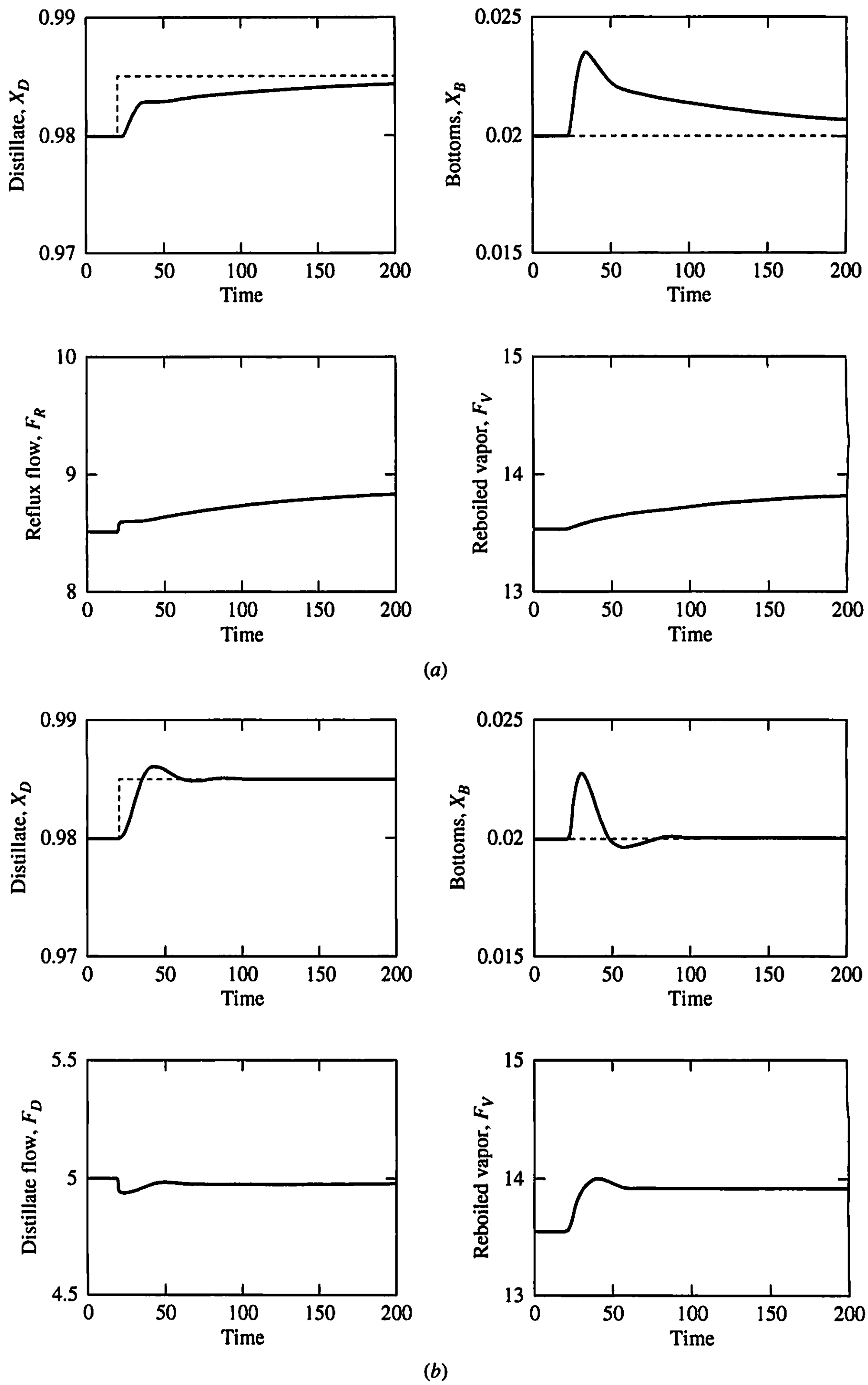


FIGURE 21.3

Transient response of distillation control to +0.005 distillate light key set point change: (*a*) energy balance design; (*b*) material balance design.

The characteristic equation is unchanged from equation (21.3), but the transfer function numerator is different for different disturbances, and thus the control performance could be different. The result again demonstrates the difficulty with having a single, standard design for a unit operation, because the types of disturbances a unit most often experiences depend on the entire plant design.

EXAMPLE 21.4. Interactive dynamics.

The examples covered to this point involved interactive systems in which the transmission interaction is not faster than the "direct" transfer function between the manipulated and controlled variables. Assuming that the controller is paired according to $CV_1(s)$–$MV_1(s)$, the systems studied to this point have had

$$G_{11}(s) \quad \text{faster than} \quad \frac{G_{21}(s)G_{c2}(s)G_{12}(s)}{1 + G_{c2}(s)G_{22}(s)}$$

A particularly difficult control challenge can occur when the transmission interaction is faster than the direct process response. As an example, two systems are considered; they have the same steady-state gains, but system B2 has fast transmission dynamics, whereas system B1 has similar dynamics for all transfer functions in the process model. In Example 20.9, system B1 has been shown to have "well-behaved" closed-loop dynamics and to be easily tuned.

System B1.

$$\begin{bmatrix} CV_1(s) \\ CV_2(s) \end{bmatrix} = \begin{bmatrix} \dfrac{1.0e^{-1.0s}}{1+2s} & \dfrac{0.75e^{-1.0s}}{1+2s} \\ \dfrac{0.75e^{-1.0s}}{1+2s} & \dfrac{1.0e^{-1.0s}}{1+2s} \end{bmatrix} \begin{bmatrix} MV_1(s) \\ MV_2(s) \end{bmatrix} \qquad (21.5)$$

System B2.

$$\begin{bmatrix} CV_1(s) \\ CV_2(s) \end{bmatrix} = \begin{bmatrix} \dfrac{1.0e^{-3.0s}}{1+2s} & \dfrac{0.75e^{-0.1s}}{1+2s} \\ \dfrac{0.75e^{-0.1s}}{1+2s} & \dfrac{1.0e^{-0.1s}}{1+2s} \end{bmatrix} \begin{bmatrix} MV_1(s) \\ MV_2(s) \end{bmatrix} \qquad (21.6)$$

System B2 has the same steady-state gains but very different dynamics. To first acquire some understanding of this system, the dynamic response is determined for a step change in $MV_1(t)$ with only the controller for variable 2 in automatic; this is the process reaction curve for the process $MV_1(t)$–$CV_1(t)$ with the other controller in automatic. The dynamic response in Figure 21.4*a* shows an inverse response, because the fast transmission effect produces an initial negative response before the slower diagonal $[G_{11}(s)]$ effect produces a positive steady-state response.

It is important to recognize that the structure of a multiloop system with interaction ensures that parallel paths exist; the parallel paths include the direct transfer function and transmission interaction, as shown in Figures 20.7 and 20.8. These parallel paths do not always create complex feedback dynamics such as inverse response or initial overshoot, but the possibility always exists. In system B2 the interactive path is faster and has an effect opposite to the direct effect, leading to the initial inverse response.

A process with an initial inverse response is usually difficult to control; thus, interaction with *fast transmission dynamics* can result in poor control performance. As an example, the control response of system B2 to a set point change in CV_1 with PI tunings that yield minimum ($IAE_1 + IAE_2$) is given in Figure 21.4*b*. (Again,

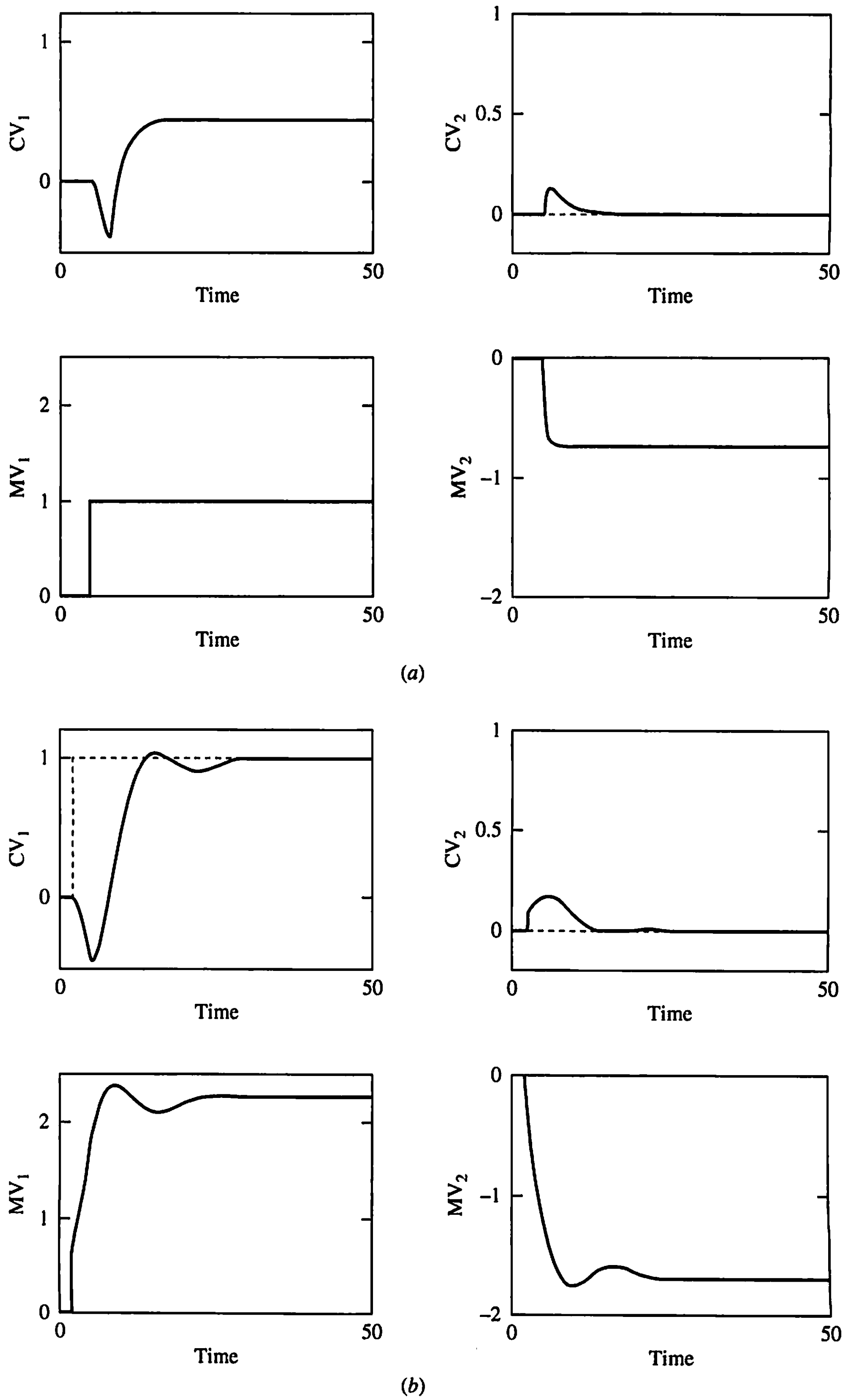


FIGURE 21.4

System B2: (*a*) Process reaction curve of MV_1–CV_1 with other loop closed; (*b*) multiloop transient response to set point change in CV_1.

TABLE 21.3

Effect of dynamics on multiloop performance

Case	K_{c1}	T_{I1}	K_{c2}	T_{I2}	IAE_1	IAE_2	$IAE_1 + IAE_2$
B1: Uniform interactive dynamics (Figure 20.11)	1.23	1.76	0.89	1.06	3.46	2.46	5.92
B2: Complex interactive dynamics (Figure 21.4*b*)	0.71	3.00	4.00	2.97	9.80	1.27	11.07

this simple measure of control performance is selected for comparison purposes only.) The feedback controller cannot eliminate the initial inverse response, which results in a relatively long time during which $CV_1(t)$ is far from its set point.

The tuning and performance for systems B1 and B2 are compared in Table 21.3. This example clearly demonstrates the importance of interactive dynamics; recall that both systems B1 and B2 have the same steady-state interaction, but system B2 has poorer performance.

This example demonstrates:

> Multivariable systems with strong interaction and fast transmission dynamics can result in complex dynamic responses, involving inverse response or large overshoot, which can degrade control performance.

The examples considered in this section have demonstrated that the design of a multiloop control system is a challenging task, involving more complex issues than single-loop systems, and that the process dynamic responses, operating conditions, disturbances, and extent of interaction must all be considered. The next three sections present methods for considering these issues when making the three main multiloop decisions: loop pairing, tuning, and enhancements.

21.3 □ MULTILOOP CONTROL PERFORMANCE THROUGH LOOP PAIRING

Loop pairing—the selection of controlled and manipulated variables to be linked through single-loop controllers—is an extremely important design decision. For the distillation examples in Figures 21.1*a*, the two possible pairings are (1) X_D–F_R and X_B–F_V and (2) X_D–F_V and X_B–F_R. However, for a system with more manipulated variables, the number of potential designs becomes very large; in fact, Loop Pairing

Integrity
Dynamics
Performance
Range

the number of initial candidates for a process with n manipulated and controlled variables is n factorial ($n!$). For example, there are 125 candidates for a five-controller, five-manipulated-variable distillation system in Figure 21.1*a* when the product compositions, pressure, and levels are considered! Clearly, the number of candidates must be reduced significantly, or the analysis task will require an enormous effort to evaluate all candidates. In this section, four separate analyses are described for eliminating clearly unacceptable pairing candidates and evaluating the remainder for likely performance. These analyses would be applied only to process designs that have been verified to be controllable and to have an adequate operating window. Also, the four analyses are employed sequentially, with only those candidates passing the prior steps evaluated at the next step.

Integrity

An important factor to be considered in multiloop control design is the performance of the system when a fault or limitation occurs. Here, a fault is assumed to involve a sensor or final element so that a control loop ceases to function; we will be considering the situation *after* a fault has been recognized and the loop with the fault has been *taken out of service*. The resulting situation is the same when one (or more) controller is placed in the manual status, so that it no longer adjusts the manipulated variable. In such circumstances, interaction influences the stability and performance of the remaining closed-loop control system. We would like the system to have integrity.

> A system has **integrity** if, after one or more loops are placed in manual, the remaining closed-loop system can be stable without changing the signs of any feedback controller gains remaining in automatic.

Some very useful results regarding integrity can be determined from the relative gain.

NEGATIVE RELATIVE GAIN. If a control loop (with integral mode) is paired using manipulated and controlled variables that have a *negative* relative gain element λ_{ij}, one of the following situations must exist (McAvoy, 1983; Grosdidier et al. 1985).

1. The multiloop system is unstable with all controllers in automatic.
2. The single-loop system ij is unstable when all other controllers are in manual.
3. The multiloop system is unstable when the ijth controller is in manual and all other controllers are in automatic.

Since all three situations are undesirable, the general conclusion is that single-loop designs should avoid pairings with negative relative gains, whenever possible. Only when essential, fast feedback dynamics can be achieved only by pairing on a negative relative gain should this design be considered. Industrial experience has shown that good designs with loop pairings on a negative relative gain occur *very* *infrequently.* An industrially important example of pairing on a negative relative gain is described by Arbel et al. (1996).

ZERO RELATIVE GAIN. When the relative gain, λ_{ij}, is *zero* for a pairing, the steady-state gain of the pairing $CV_i(t) - MV_j(t)$ is zero when the other loops are open, that is, the process gain $K_{ij} = 0$. Since no causal relationship exists, the single-loop controller cannot function. However, the multiloop system can function because of the causal relationship through the interacting process and the interacting controller. The causal interaction relationship is demonstrated with equation (20.13), which gives the transfer function between $CV_1(s)$ and $MV_1(s)$ for a 2 × 2 system with loop 2 in automatic.

$$CV_1(s)/MV_1(s) = \cancelto{0}{G_{11}(s)} - G_{12}(s)G_{21}(s)G_{c2}(s)/[1 + G_{c2}(s)G_{22}(s)] \quad (20.13)$$

Clearly, a nonzero causal relationship exists between $MV_1(s)$ and CV_1 when process interaction occurs $[G_{12}(s)G_{21}(s) \neq 0]$ and the interacting controller is in automatic $[G_{c2}(s) \neq 0]$ to create a feedback loop via the interaction path. Therefore, successful operation of a control loop paired on a zero relative gain depends on the status of the interacting loop. Pairing on a zero relative gain should be implemented only when essential, fast feedback dynamics are achieved. Industrial experience indicates that this situation is not common, but occurs *occasionally.*

In both of these cases, proper functioning of a control loop requires that the adjustments from other controllers be implemented at the final elements, which would not be satisfied if an interactive controller (1) were in manual or (2) had its output saturated at the upper or lower bound. It is not uncommon for these situations to occur, at least temporarily, and thus, multiloop control designs with relative gains less than or equal to zero could often fail to provide stable feedback regulation. To prevent these failures, a real-time computer program could be prepared to continuously monitor the control system and change controller gains and automatic/manual statuses depending on the condition of all controllers in the multiloop system.

To summarize this discussion on integrity:

- Pairing a control loop on negative or zero relative gain should be avoided, if possible; such a pairing is implemented only when essential, significant dynamic advantages can be gained by this design and by no other reasonable process or control modifications.
- When a control design has a loop paired on a negative or zero relative gain, a program should be executed in real time to monitor the interacting loops and either warn the operator or take automated actions to prevent unstable systems when the status of an interacting loop changes from automatic to manual.

To discuss a process with conventional and zero relative gain pairing, we begin by considering the fired heater process in Figure 21.5. The process fluid flows through a pipe (termed a *coil*) and is heated by radiant and convective heat transfer from the combustion of fuel. The variables to be controlled are the process

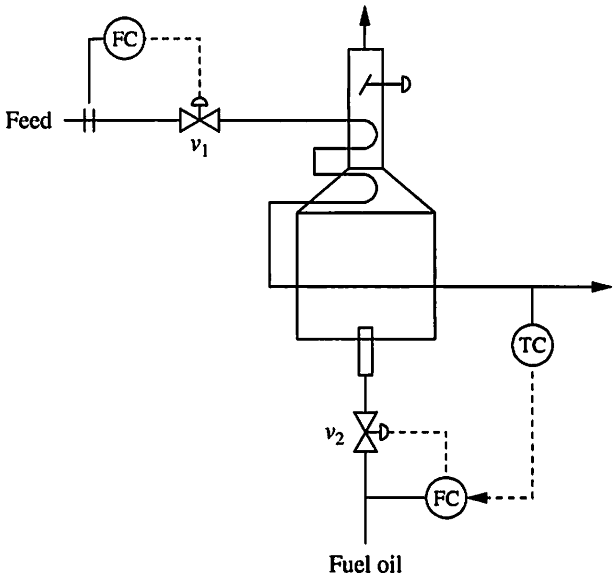


FIGURE 21.5

Furnace multiloop control pairing on variables with $\lambda > 0$.

fluid flow rate and the process fluid outlet temperature, and the two manipulated valves are in the process fluid (v_1) and fuel (v_2) lines. When no feedback controllers are present, the process fluid flow rate is influenced directly only by v_1, and the outlet temperature is influenced by both v_1 and v_2. Thus, the 2×2 gain matrix has a zero, and as shown in Chapter 20, the relative gain array has ones in the diagonal elements and zeros in the off-diagonal elements. There is only one pairing with nonzero relative gain values, and this pairing is shown in Figure 21.5, which is the *common* loop pairing used in most industrial designs.

The guideline for eliminating pairings on nonpositive relative gains conforms to theory and common industrial practice; however, there are a *few* cases where the rule is violated and pairings with zero relative gains are used. These unconventional designs are employed, in spite of their recognized drawbacks, to achieve specific advantages—typically, very fast feedback dynamics for a particularly important controlled variable. An example of an exception is given in Figure 21.6. In this case, the tight control of the coil outlet temperature is very important, and the dynamic response between the process flow valve v_1 and the temperature can be very fast when the fluid residence time in the coils is short. Since the open-loop gain between valve v_2 and the process fluid flow is zero, the proper functioning of the flow controller in this case requires the operation of the temperature controller. This design is used industrially only when the temperature is of especially great importance, feed flow control need not be controlled tightly, and other steps to improve control performance are not possible or are extremely costly.

Dynamics

If one or a few controlled variables are much more important, the control loop pairing should be selected to give good performance for the most important variables. As demonstrated in discussions on single-loop control, control performance is

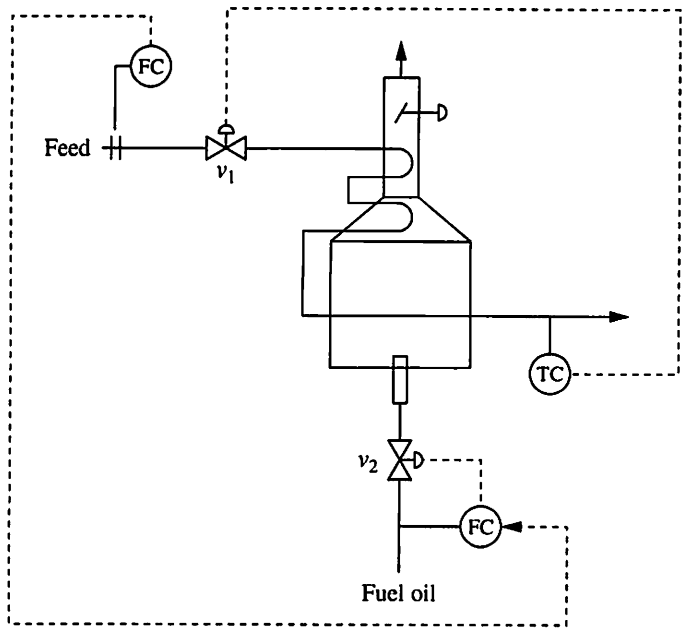
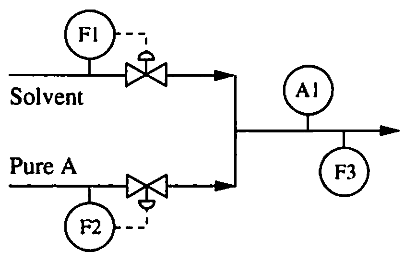


FIGURE 21.6
Furnace multiloop control pairing on variables with λ = 0.

much better when the feedback process dynamics involve a fast process with small fraction dead time. Thus, the second loop-pairing guideline is stated as follows:

> Very important controlled variables should be paired with manipulated variables that provide fast feedback dynamics with small dead times and time constants and negligible inverse response.

As an example of this guideline, consider the simplified system in Figure 21.7 in which two gases are mixed, as might occur where the heating value of the mixed gaseous fuel stream is to be controlled. The sources of the feeds are a gas stream L (lower heating value) and a vaporizer for the stream H (higher heating value). The controlled variables are the pressure and the composition in the pipe after mixing, and both manipulated variables affect both controlled variables. Generally, the pressure is of greatest importance, because variations could lead to unsafe conditions; short-term composition variations, while not desirable, can be more easily tolerated. Therefore, the pressure is controlled by manipulating the fast-responding gas feed, while the composition is controlled by manipulating the more slowly responding vaporization process. Since the pressure is most important, this pairing would be used as long as the gas feed valve has the flexibility range to control pressure—in other words, as long as it does not go fully opened or closed in response to disturbances—regardless of the interaction effects on the composition.



EXAMPLE 21.5.

Evaluate the two possible loop pairings for the blending example process with base-case conditions in Table 21.1 according to the relative gain and dynamic responses.

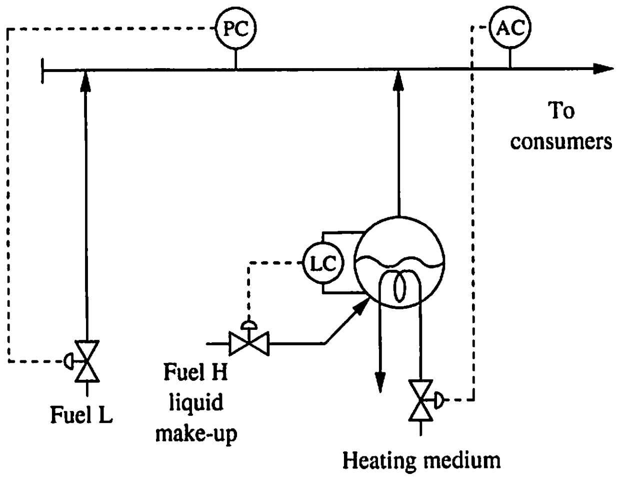


FIGURE 21.7

Fuel gas control system with key pressure variable paired with fast manipulated variable.

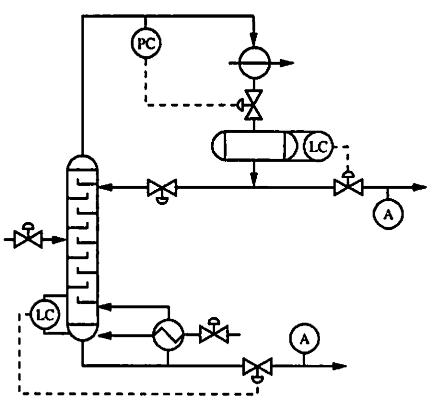
The relative gain array for the blending process with dilute product (5% A) can be evaluated from the steady-state gains to be

Relative gain array:

	F_1	F_2
A_1	0.05	0.95
F_3	0.95	0.05

Since none of the elements is less than or equal to 0.0, both possible pairings are allowed based on the first guideline. Also, the data reported in Example 20.10 show the same dynamic responses for both pairings, since the dominant dynamics are due to the sensors. Therefore, neither pairing has an advantage regarding dynamics. Finally, since the two guidelines do not exclude either pairing, the results in Table 21.1 give strong evidence for preferring the A_1–F_2 and F_3–F_1 pairing, since the tuning of each controller does not depend on the automatic/manual status of the other.

EXAMPLE 21.6.



Evaluate the two possible composition control loop pairings for the distillation example in Figure 20.3 according to the relative gain and dynamic responses.

The relative gain array can be evaluated from the steady-state gains in equation (20.24), giving

Relative gain array:

	F_R	F_V
X_D	6.09	−5.09
X_B	−5.09	6.09

Since only the pairing X_D–F_R and X_B–F_V has positive relative gains, only this pairing is allowed by the first guideline; this is the design in Figure 21.1*a*. The loop dynamics for the allowed pairing are not slower, and are even slightly faster, than the disallowed pairing, which indicates that there is no significant disadvantage to this design based on feedback dynamics.

Performance Measure

The third analysis addresses the remaining candidate pairings, involving controllable systems with positive relative gains, similar feedback dynamics, and controlled variables of equal importance, by investigating the control performance for specific disturbances. If only a few candidates remained at this point, one could simulate the systems for the important disturbance(s) to select the best design, as was done for the distillation tower in Examples 21.2 and 21.3. Here a shortcut method is outlined that provides a quick estimate of control performance and is useful in reducing the pairing candidates that can yield good control performance. Equally important, it provides insight into the effects of disturbances, specifically how interaction can be favorable or unfavorable in multiloop control (Stanley et al., 1985). The approach is introduced for 2 × 2 systems; however, it can be extended to higher-order systems (Skogestad and Morari, 1987*a*). In spite of its advantages, the method does not provide a definitive recommendation, because of the assumptions required; thus, some care is required in its application, and the results may have to be verified through dynamic simulation.

The method takes advantage of a simple estimate of control performance that can be determined directly from the closed-loop transfer function. The control performance measure used here is integral error, which can be obtained directly by using the following relationship (see Appendix D):

$$\int_0^\infty E(t)\,dt = \lim_{s\to 0}\int_0^\infty E(t)e^{-st}\,dt = E(s)\big|_{s=0} \tag{21.7}$$

This relationship demonstrates that the integral of a variable, specifically the error, can be obtained from the transfer function of a stable system *without solving for the complete transient response* (Gibilaro and Lee, 1969). Naturally, much detailed information about the transient response is lost, but a useful single measure of control performance is easily obtained. A large integral error indicates poor performance and a pairing candidate that should be eliminated. A small integral error *can* result from good performance, and the pairing should be retained for further evaluation. However, large positive and negative errors occurring during the transient could cancel in this calculation (this is not the IAE!), so a small value of integral error does not definitely prove good control performance. Thus, the final selection requires further evaluation, such as a simulation, to determine the transient behavior.

The closed-loop disturbance response transfer function for a 2 × 2 system is given in equation (21.3). The relationship in equation (21.7) can be applied to equation (21.3) with $D(s) = 1/s$, resulting, after some rearrangement, in

$$\left[\int_0^\infty E_1(t)\,dt\right]_{\text{ML}} = \left[\int_0^\infty E_1(t)\,dt\right]_{\text{SL}} (f_{1,\text{tune}})(\text{RDG}_1) \tag{21.8}$$

where Integral error under multiloop control $= \left[\int_0^\infty E_1(t)\,dt\right]_{\text{ML}}$

Integral error under single-loop control $= \left[\int_0^\infty E_1(t)\,dt\right]_{\text{SL}} = \dfrac{K_{d1}(T_{I1})_{\text{SL}}}{K_{11}(K_{c1})_{\text{SL}}}$ (21.9)

Detuning factor for multiloop control $= f_{1,\text{tune}} = \dfrac{(K_{c1}/T_{I1})_{\text{SL}}}{(K_{c1}/T_{I1})_{\text{ML}}}$ (21.10)

$$\textbf{Relative disturbance gain} = \text{RDG}_1 = \lambda_{11}\left(1 - \frac{K_{d2}K_{12}}{K_{d1}K_{22}}\right) \tag{21.11}$$

The multiloop control performance calculation in equation (21.8) is arranged to be the product of three factors so that separate facets of multiloop control are represented in each factor: (1) a factor for the single-loop performance, (2) a factor for tuning adjustment, (3) a factor accounting for interaction and disturbance. The first factor represents the single-loop performance that would be achieved if the other control loop were not in operation (e.g., in manual). This term again demonstrates that aspects of single-loop control performance, which are summarized in Chapter 13, also influence the controlled variables in a multiloop system. For example, fast feedback dynamics and small disturbance magnitudes are beneficial in multiloop systems.

The final two factors represent the change in control performance due to the multiloop structure. The detuning factor f_{tune} represents the effects of detuning the PI controllers for multiloop control. The values of the multiloop tuning constants can be estimated using methods in Chapter 20 or alternative methods cited in the references. By applying the tuning method recommended in Chapter 20 for 2×2 systems with equal dynamics for the two loops, the detuning factor can be determined from the relative gain, as shown in Figure 21.8. Since the relative gain in most properly designed control systems is greater than about 0.7, the correlation shows that the detuning factor is usually bounded between 1.0 and 2.0 for 2×2 systems (Marino-Galarraga et al., 1987*a*).

Thus, the effect of multivariable control is usually dominated by the third term, which is called the *relative disturbance gain*, RDG. The relative disturbance gain is the product of the relative gain and a disturbance factor. Recall that the relative gain is an inherent property of the *feedback* process, independent of the type of disturbance. In contrast, the RDG depends on the type of disturbance; for

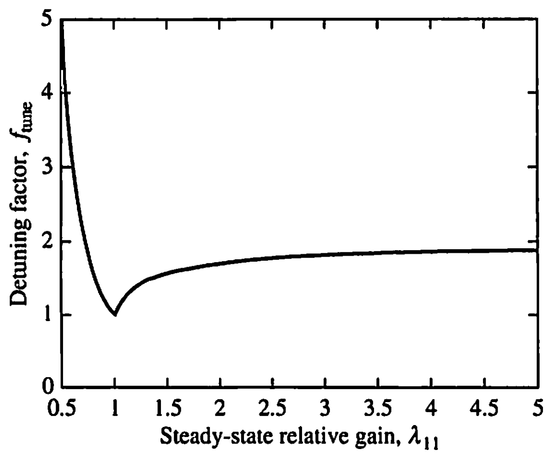


FIGURE 21.8

Correlation between detuning factor f_{tune} and relative gain for 2×2 system with equal input-output dynamics.

example, it has different values for feed composition and set point changes to a distillation tower.

The influence of the RDG is first analyzed from a mathematical, then a process point of view. The RDG is the product of two values, and its magnitude is small when control performance is good. The first factor is the relative gain; if the relative gain has a large value, its contribution will be to degrade control performance, because the integral error will tend to increase. The second factor represents the effect of the disturbance type, and because it is the difference of two values, it can have a magnitude ranging from zero to very large. A small magnitude of this factor indicates that the multiloop performance could be much better than the single-loop performance. This situation would occur when the term $(1 - K_{d2}K_{12}/K_{d1}K_{22})$ has a value near zero, which is interpreted as favorable interaction. The other result, with a large disturbance contribution and much poorer multiloop performance, is also possible and is interpreted as unfavorable interaction.

The combined effects of inherent process interaction and disturbance type determine the dominant difference between single-loop and multiloop control performance. These effects are reflected in the magnitude of the relative disturbance gain (RDG).

This clearly demonstrates that multiloop control performance can be better or worse than single-loop performance for some disturbances.

A key element in determining the effect of interaction in multiloop systems is the manner in which a disturbance affects both controlled variables, sometimes referred to as the "direction" of the disturbance. Thus, it is worthwhile considering the basis for favorable interaction. Favorable interaction occurs when controller 2, in correcting its own deviation from set point, makes an adjustment that improves the performance of controller 1, $CV_1(t)$. The net effect must consider the effects of the disturbances on both controlled variables (K_{d1} and K_{d2}), the manipulation taken to correct the $CV_2(t)$ deviation (characterized by $1/K_{22}$) and the interaction term (K_{12}). All of these parameters are in the interaction factor of the relative disturbance gain.

EXAMPLE 21.7.

For the distillation towers in Figures 21.1 and 21.2, evaluate the relative disturbance gain and provide an interpretation of the effect of interaction on the control performance of the distillate composition, X_D, for a disturbance in the feed composition.

The effect of interaction on control performance is predicted by equation (21.8), and the calculations are summarized in Table 21.4 for both distillation control designs. This analysis predicts that the energy balance performs better for feed composition disturbances, because its sum of values of $f_{tune} \times \text{RDG}_i$ for the two compositions is smaller than for the material balance system. This conclusion is confirmed by the simulation results in Figures 21.1*b* and 21.2*b* and in Table 21.2.

TABLE 21.4

Summary calculations of predicted control performance for the distillation tower in Examples 21.7 and 21.8

Data and calculated variable		Energy balance design in Figure 21.1*a*		Material balance design in Figure 21.2*a*	
		X_D	X_B	X_D	X_B
K_{FR}		0.0747	0.1173		
K_{FD}				−0.0747	−0.1173
K_{FV}		−0.0667	−0.1253	0.008	−0.008
λ		6.09		0.39	
f_{tune}		2.0		5.0	
Feed	K_d	0.70	1.3	0.70	1.3
composition	RDG	0.071	0.94	1.11	0.06
disturbance	$f_{\text{tune}} \cdot$ RDG	0.14	1.88	5.55	0.30
Set point	K_d	1.0	0.0	1.0	0.0
change (X_D)	RDG	6.09	*	0.39	*
	$f_{\text{tune}} \cdot$ RDG	12.2	*	1.53	*

* Predicted $\int E\,dt$ is finite, although RDG is infinite, due to cancellation of K_{d2} (which is zero) in numerator and denominator.

The physical interpretation of the favorable interaction is considered here for the control design in Figure 21-1a. The initial effect of **decreased** light key in the feed (before the effects of the analyzer controllers) results in the top and bottom products having too **little** light key. In response, the bottom controller **decreases** the heating flow rate, i.e., reboiler duty. This change by the bottom controller has the effect of **increasing** the fraction of light key in the top product, exactly what the top controller is doing itself! The top controller must also take action by increasing the reflux; however, the (reinforcing) interaction from the bottoms controller improves the overall control performance. Therefore, the energy balance control pairing has favorable interaction and good multiloop performance for the top controller in response to a feed composition disturbance. The reader should repeat this thought experiment for the material balance system to confirm that the interaction is unfavorable for X_D.

EXAMPLE 21.8.

For the distillation towers in Figures 21.1 and 21.2, evaluate the relative disturbance gain for a change in the distillate composition controller set point and select the better design for X_D.

The analysis method, summarized in Table 21.4, correctly predicts that the material balance performs better for set point changes in the distillate controller, as was found by simulations in Figure 21.3*b*. Note that equation (21.3) can be used to represent a set point change by setting $G_{d1}(s) = 1.0$ and $G_{d2}(s) = 0.0$, and in this case the RDG_1 is equal to λ_{11}.

In summary, equation (21.8) provides the basis for estimating the major effect of multiloop control on the performance of each controlled variable. The information required to perform this calculation involves process gains in the feedback path K_{ij} and the open-loop disturbance gains K_{di}, which can be easily determined from a steady-state analysis. One should consider the likely errors in the values of the gains, as well as in the simplifications in linearizing the process model, when interpreting the results. Small differences (10–20%) in predicted integral error should be considered within the accuracy of the information, and the candidate loop pairings should be considered indistinguishable.

This subsection introduced the consideration of disturbance type, which should be considered in all analyses of multiloop systems. However, it is necessary to repeat a caution concerning the use of the integral error, which can be small because of cancellations of large positive and negative errors. Thus, while large values of $|\text{RDG}||\Delta D|K_d|$ definitely indicate poor control performance, small values do not necessarily indicate good performance. The best recourse to determine the effects of complex dynamics at this time is to perform a dynamic simulation. Note that the procedures described here are useful in substantially reducing the number of candidates for simulation, as well as providing insight into the importance of disturbance type (or "direction") on control performance.

Control Range

The method for determining controllability in Chapter 20 is valid for the linearized model at the point of linearization. For most processes that are not highly nonlinear, the results can be extended in a region about the point. However, there is no guarantee that the results can be extrapolated, especially when a manipulated variable encounters a constraint while attempting to make the change required by the controller. The method for identifying difficulties with range in achievable steady-state behavior is to determine the operating window of the process. Even if all steady states are feasible, manipulated variables may reach limits during transients; dynamic simulation would be required to determine the importance of a temporary saturation of a manipulated variable.

This section demonstrated a stepwise method for evaluating candidate multiloop control designs:

1. Use the relative gain to eliminate some pairings which lack integrity.
2. Use dynamic models to select pairings with fast dynamics for important variables.
3. Use approximate control performance analysis—the relative disturbance gain (RDG)—for specific disturbances to evaluate systems with controlled variables of equal importance.

Note that step 1 requires only steady-state information, which means that it is easy to perform with limited modelling information. Also, steps 2 and 3 require approximate dynamic information to identify where major differences in feedback dynamics are present. This approximate dynamic modelling information is also generally easy to obtain. If the effects of interactive dynamics are not easily predicted, so that the methods here cannot provide conclusive recommendations, the final design could be simulated to determine its performance.

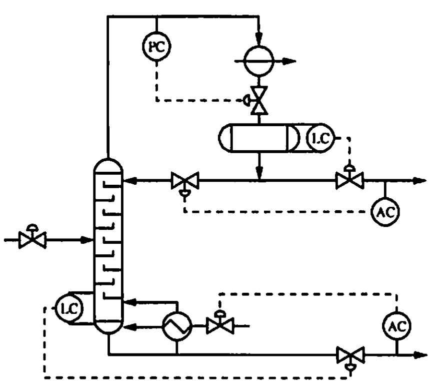
21.4 ▣ MULTILOOP CONTROL PERFORMANCE THROUGH TUNING

The tuning of PID feedback controllers should be matched with the control objectives. Prior to tuning, the first steps presented in the previous section should be applied, to eliminate inappropriate pairings by the use of the relative gain and to select pairings with fast feedback dynamics for the important controlled variables. In all cases, controllers for the most important controlled variables should be tuned tightly. The tuning of the controllers of lesser importance depends on the type of interaction present: favorable or unfavorable.

For systems with *unfavorable* interaction, as predicted by the relative disturbance gain, the effect of interaction degrades the performance of other loops; this degradation can be reduced through judicious controller detuning, consistent with the control objectives. Thus, the controllers for the important variable(s) would be tuned tightly, as close as possible to single-loop tuning. To ensure stability and prevent unfavorable interaction, the controllers for the less important variables would usually be detuned.

If the interaction is *favorable*, as indicated by a small relative disturbance gain, interaction improves the performance of other loops and should be maintained by proper tuning. In this case, the interacting loop, even if not of great importance itself, should be tuned as tightly as possible to enhance the favorable interaction.

There are no exact guidelines for how the less important controllers should be tuned. When interaction degrades control performance, a starting approach is to tune the important loops close to their single-loop values and detune the less important loops by decreasing their controller gains. Normally, all feedback controllers would retain an integral mode to return the controlled variables to their set points (albeit very slowly for some variables) after disturbances. When both are to be tightly tuned, the method in Chapter 20 would give initial values. An example of how differences in control performance in the same process can be induced through different tuning is given in the results in Table 20.2.



EXAMPLE 21.9.

The effects of tuning the composition controllers on the control performance of the energy balance distillation control design in Figure 21.1*a* are investigated. For this example (only), the distillate product composition is assumed to be much more important than the bottoms composition, so the bottoms composition will be allowed to experience larger short-term variation about its set point. Since no strict guidelines exist for this tuning, the extent of detuning used in this example represents exploratory results.

The effects of tuning, as determined by simulating the entire response, are given in Table 21.5. For a set point change in X_D, the interaction is unfavorable, as demonstrated by the large magnitude of $\text{RDG} \cdot f_{\text{tune}}$ (12.2) in Table 21.4. Therefore, tight tuning of the distillate composition controller, along with detuning the bottoms loop, reduces interaction and improves the performance of the distillate composition controller (reducing the IAE from 0.71 to 0.35). As expected, the variation in the bottoms composition (IAE) increased as the bottoms controller was detuned.

For the feed composition disturbance, the interaction is favorable, as demonstrated by the small magnitude of $\text{RDG} \cdot f_{\text{tune}}$ (0.14) in Table 21.4. Therefore, the

TABLE 21.5

The effects of tuning on performance for Example 21.9

	Tuning				Performance	
Input change	K_{cXD}	T_{IXD}	K_{cXB}	T_{IXB}	IAE_{XD}	IAE_{XB}
Set point,	10.4	9.0	−6.8	6.1	0.71	0.68
($\Delta SP_{XD} = 0.01$)	20.75	9.0	−3.4	6.1	0.35	1.37
Feed composition,	10.4	9.0	−6.8	6.1	0.17	0.35
($\Delta X_B = -0.04$)	10.4	9.0	−2.0	6.1	0.36	1.18

control performance in the case with both controllers tightly tuned has better distillate composition performance (IAE of 0.17) than the case with the bottoms controller detuned (IAE of 0.36), since detuning reduces the *favorable* interaction.

The discussion in this section and the results of Example 21.9 reinforce the importance of considering the effects of the disturbances in control design and tuning.

> Multiloop tuning should be chosen to retain favorable interaction and to reduce unfavorable interaction.

21.5 ▫ MULTILOOP CONTROL PERFORMANCE THROUGH ENHANCEMENTS: DECOUPLING

When the previous analyses are complete, it is possible to arrive at a design with two (or more) equally important controlled variables, which may not have the desired performance even with the best pairing and tuning. Often, the limiting factor is unfavorable interaction, which is indicated by a large magnitude of the relative disturbance gain (|RDG|). When poor control performance stems from unfavorable interaction, a potential solution involves reducing interaction through an approach called *decoupling*, which has the theoretical ability to improve performance in some loops without degrading performance in others.

Decoupling reduces interaction by transforming the closed-loop transfer function matrix into (an approximate) diagonal form, in which interaction is reduced or eliminated. There are at least three different decoupling approaches: (1) altering the manipulated variables, (2) altering the controlled variables, and (3) retaining the original variables but altering the feedback control calculation. Each is presented briefly in this section.

Manipulated Variables

The first decoupling approach involves changing the control structure to affect different manipulated secondary variables in a cascade structure, with the same final elements. This approach will be introduced by reconsidering the blending in Example 20.1, in which both manipulated variables influence both controlled variables. The goal is to control the same variables (A_1 and F_3) with altered manipulated variables so that the altered system's gain matrix is diagonal or nearly diagonal. This goal is usually achieved through process insight. The restructured dynamic model can be developed from equations (20.1) and (20.2) without linearizing.

$$\tau_A \frac{dA_1(t)}{dt} = \left[\frac{F_2(t-\theta_A)}{F_1(t-\theta_A)+F_2(t-\theta_A)} \right] - A_1(t) = \text{MV}_1(t-\theta_A) - A_1(t) \tag{21.12}$$

$$\tau_F \frac{dF_3(t)}{dt} = F_1(t-\theta_F) + F_2(t-\theta_F) - F_3(t) = \text{MV}_2(t-\theta_F) - F_3(t) \tag{21.13}$$

From this model it becomes clear that the two controlled variables would be independent if the manipulated variables were defined as follows:

$$\text{Manipulated variable number 1} = \text{MV}_1 = F_2/(F_1+F_2)$$
$$\text{Manipulated variable number 2} = \text{MV}_2 = F_1+F_2$$

With this modification, the system in equations (21.12) and (21.13) has been altered to two independent input-output relationships, and as a side benefit the altered system is linear. Thus, standard single-loop control methods can be used to tune the controllers in this decoupled system.

The control strategy can be implemented using real-time calculations and cascade principles, as shown in Figure 21.9, because F_1 and F_2 are measured and respond essentially instantaneously to changes in the valve positions. For example, when the mixed flow (F_3) set point is increased, the initial response of controller

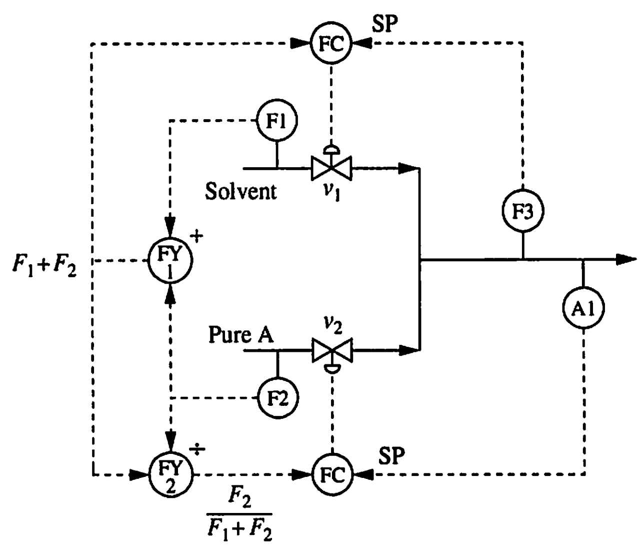


FIGURE 21.9

Manipulated-variable decoupled control of blending.

F_3 is to increase the total flow $(F_1 + F_2)$ set point; this is achieved by adjusting v_1. This changes the flow ratio and is quickly followed by an adjustment by the flow ratio controller to increase v_2 to maintain the proper ratio $F_2/(F_1 + F_2)$; this adjustment is made without feedback from the analyzer composition controller. These adjustments continue until the desired values of the total flow and ratio are achieved. By similar analysis, it can be shown that the analyzer controller output affects only the product composition, not the total flow. Thus, the interactions have been eliminated. As an added advantage, the decoupled control system is also easily understood by plant operating personnel. Naturally, the feedback controllers remain to account for small inaccuracies in the flow measurements, manipulated-variable calculations, and disturbances. Many similar strategies are used industrially to minimize unfavorable interactions and are the basis for the common water faucet design in which the total water flow and the ratio of hot to cold can be adjusted independently.

Controlled Variables

Another decoupling approach alters the controlled variables by replacing measured variables with calculated variables based on process output measurements. Again, the proper calculation is designed with knowledge of the process dynamics. As a simple example, the two-tank level control system in Figure 21.10 is considered; the levels are to be controlled by manipulating the set points of the flow controllers. If the goal were to design two decoupled controllers for maintaining the desired levels, calculated variables which yield independent equations would be sought in the basic linearized model of the process.

$$A\frac{dL'_1}{dt} = F'_{1in} - F'_1 - K_{12}(L'_1 - L'_2) \tag{21.14}$$

$$A\frac{dL'_2}{dt} = F'_{2in} - F'_2 + K_{12}(L'_1 - L'_2) \tag{21.15}$$

A decoupled system can be derived by noting that the sum of the levels depends on the sum of the manipulated variables, whereas the difference between the levels depends on the difference between the manipulated variables. This is easily shown

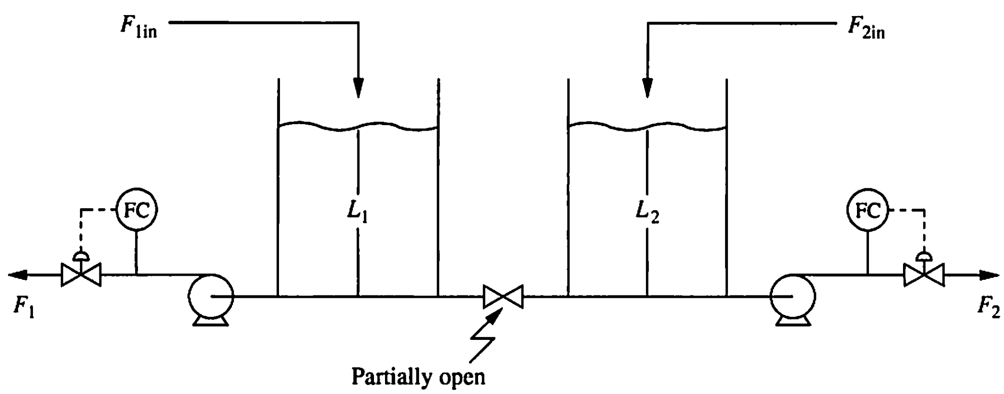


FIGURE 21.10

Level process.

by adding and subtracting equations (21.14) and (21.15) to give

$$A\frac{d(L_1' + L_2')}{dt} = (F_{1\text{in}}' + F_{2\text{in}}') - (F_1' + F_2') \tag{21.16}$$

$$A\frac{d(L_1' - L_2')}{dt} = (F_{1\text{in}}' - F_{2\text{in}}') - 2K_{12}(L_1' - L_2') - (F_1' - F_2') \tag{21.17}$$

Thus, a control design in which $(L_1 + L_2)$ and $(L_1 - L_2)$ are controlled by adjusting $(F_1 + F_2)$ and $(F_1 - F_2)$, respectively, is decoupled. Note that $(L_1 + L_2)$ is non-self-regulatory, whereas $(L_1 - L_2)$ is a first-order system. A process application of this principle to distillation reboiler level and composition control is given by Shinskey (1988).

This approach is not as widely applied as the approach based on manipulated variables, because it uses measured process *output* values in calculating the controlled variables. For this approach to function properly, all measured output variables should respond to adjustments in all manipulated variables with nearly the same dynamics so that the calculations are "synchronized." This criterion is easily satisfied for the example in Figure 21.10, because levels respond rapidly, but it is not commonly satisfied for complex units. Control designs for distillation composition using these concepts have been reported (Weber and Gaitonde, 1985; Waller and Finnerman, 1987).

Explicit Decoupling Calculations

The third approach to decoupling is to retain the original manipulated and controlled variables and alter the control calculation, while retaining the multiloop structure. There are two common implementations of this approach. The "ideal" decoupling compensates for interactions while leaving the input-output dynamic relationships for the feedback controllers unchanged from their single-loop behavior, $G_{ii}(s)$. While the concept is attractive, since controller tuning would not be affected by decoupling, experience has shown that the resulting system is very sensitive to modelling errors and generally does not perform well (Arkun et al., 1984; McAvoy 1979); thus, it is not considered further.

The "simplified" decoupling method presented here achieves a diagonal system by calculations that result in the interaction relationships between the controller outputs and controlled variables all being zero. Since it is not possible to eliminate the process interaction $G_{ij}(s)$, the decouplers are designed to provide compensating adjustments that cancel the process effects of manipulations in $\text{MV}_j(s)$ on $\text{CV}_i(s)$ for $i \neq j$ and thus yield independent, single-loop systems. The system is shown in Figure 21.11, with the decoupling transfer functions $D_{ij}(s)$ given by the following relationships:

$$\text{Decoupler:} \qquad D_{ij}(s) = -\frac{G_{ij}(s)}{G_{ii}(s)} \tag{21.18}$$

The reader may recognize the decoupler as similar to the feedforward controller, which compensates for measured disturbances; here the measured disturbance is the manipulated variable adjusted by an interacting feedback controller. The reader is referred to Chapter 15 on feedforward control for the derivation of this equation and a discussion of the possibility of the decoupler being unrealizable.

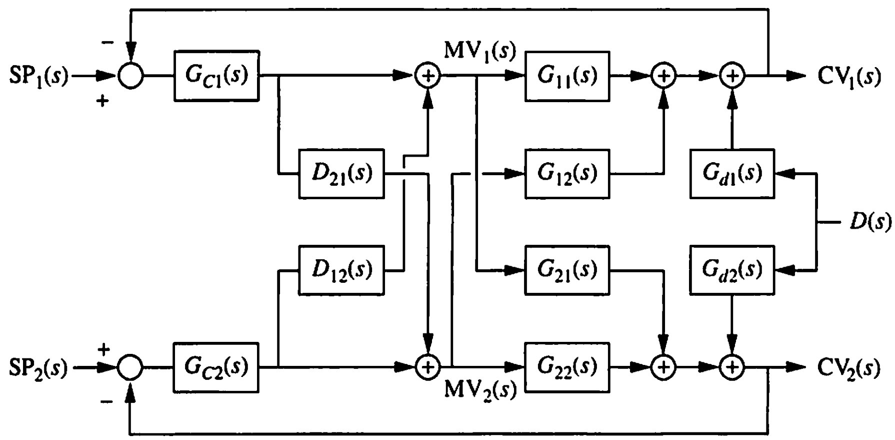


FIGURE 21.11

Block diagram of explicit decoupling.

When the process behavior can be modelled by first-order-with-dead-time transfer functions, the decoupler in equation (21.18) becomes

$$D_{ij}(s) = -\frac{K_{ij}}{K_{ii}} \frac{1 + \tau_{ii}s}{1 + \tau_{ij}s} e^{-(\theta_{ij} - \theta_{ii})s} \tag{21.19}$$

Again, this is the same form as feedforward controllers. The decoupling calculations in equation (21.19) can be implemented in digital form through the same procedures used with feedforward controllers in Chapter 15.

The explicit decoupler completely eliminates interaction only when the model is perfect. The resulting transfer function can be derived through block diagram manipulation assuming perfect decoupling, equation (21.18). The perfectly decoupled system is shown in Figure 21.12. Clearly, the "effective process" being controlled has changed because of the decoupling, and the controller tuning must be changed from single-loop values. Since the change in the "feedback process" transfer function is the inverse of the relative gain, the controller gain for the decoupled system should be taken as (approximately) the product of the single-loop controller gain, calculated using $G_{ii}(s)$, and the relative gain. This will maintain the $G_{OL}(s)$, product of the controller and the "process" $[\lambda_{11} G_{c1}(s)][G_{11}(s)/\lambda_{11}]$, nearly constant, as a first approximation.

Errors in the models used in the decouplers affect the accuracy of the decoupling and, more seriously, affect the stability of the multiloop system. The sensitivity can be determined from an analytical expression of the performance as a function of the decoupler errors. The procedure to calculate the integral error in equation (21.7) can be applied to the closed-loop transfer function for the decoupled system with modelling errors. To simplify the analysis, only the decoupler gains have errors, with ϵ_i being a multiplicative error in the decoupler controller gain, K_{Dij}. The resulting expression for the performance is

$$\int_0^\infty E_1(t)\,dt = \lambda_{11}\lambda_{\epsilon 1 \epsilon 2} \left[\frac{K_{d1} T_{I1}}{K_{11} K_{c1}}\right] \left[\frac{1}{\lambda_{\epsilon 1}} + \frac{(\epsilon_1 - 1) K_{d2} K_{12}}{K_{d1} K_{22}}\right] \tag{21.20}$$

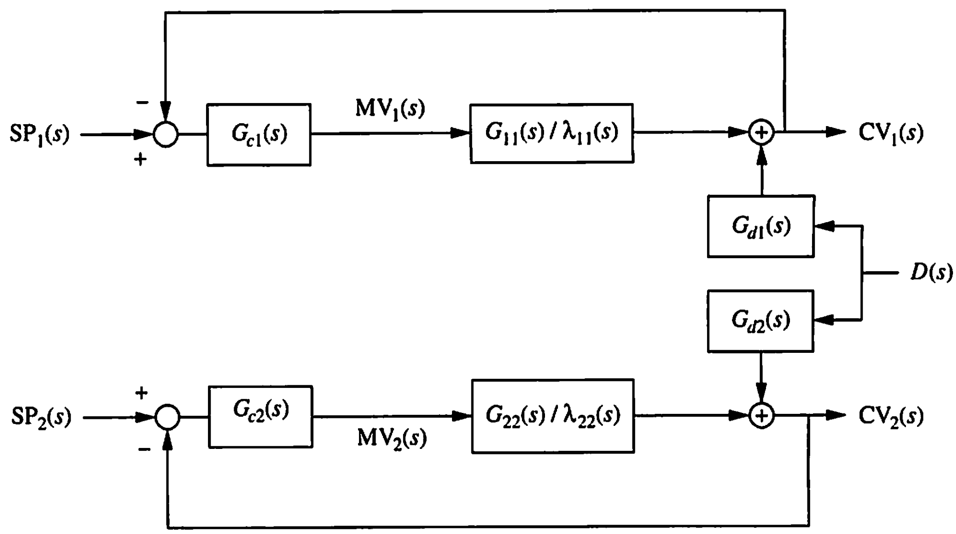


FIGURE 21.12

Consolidated block diagram explicit decoupling with perfect models. (Reprinted by permission. Copyright © 1983, Instrument Society of America. From *Interaction Analysis*.)

where $\kappa = \dfrac{K_{12}K_{21}}{K_{11}K_{22}} \qquad \lambda_{11} = \dfrac{1}{1-\kappa} \qquad \lambda_{\epsilon i} = \dfrac{1}{1-\epsilon_i\kappa} \qquad \lambda_{\epsilon 1 \epsilon 2} = \dfrac{1}{1-\epsilon_1\epsilon_2\kappa}$

$$D_{ij}(s) = -\epsilon_i \frac{G_{ij}(s)}{G_{ii}(s)} \qquad \epsilon_i = (1 \text{ for perfect model})$$

Clearly, the *error relative gain*, $\lambda_{\epsilon_1\epsilon_2}$, plays a key role. As the decoupler errors increase, this factor and the integral error can become very large and the performance very poor. For processes with relative gains significantly greater than 1, even small decoupling errors can lead to very poor performance. For example, a small (5%) model error of $\epsilon_i = 1.05$ in a decoupler applied to the distillation example with energy balance control ($\lambda = 6.09$, $\kappa = 0.836$) would increase the integral error by about 100% over perfect decoupling! Thus, caution should be used when applying decoupling, since it requires model accuracies nearly impossible to achieve for real process systems with large relative gains. Similar results have been presented by McAvoy (1979), Shinskey (1988), and Skogestad and Morari (1987*b*) using different analysis methods.

Several simplifications are possible in this decoupling approach. First, the dynamic decouplers in equation (21.18) can be approximated by the gains when this is sufficient for good control. Typically, the steady-state approximation is acceptable when $D_{ij}(s)$ has a small dead time and nearly equal lead (numerator) and lag (denominator) dynamics. Note that this simplification does not reduce the sensitivity to model gain errors shown in equation (21.20).

Also, decoupling can be simplified by using only one-way decoupling, with one $D_{ij}(s) = 0$. This approach would be applied to improve the performance of the more important controlled variable. Sensitivity analysis shows that one-way decoupling is much less sensitive to model gain errors than full decoupling, which presumably leads to its more frequent successful application in practice (McAvoy, 1979).

EXAMPLE 21.10.

Determine the performance with decoupling for the energy balance distillation control system in Figure 21.1. The disturbance is a set point change of +0.01 to the top composition controller.

The first question the engineer should ask is "Will error-free decoupling improve the control performance?" Recall that the magnitude of RDG $\cdot f_{tune}$ indicates the effects of interaction on multiloop controllers. Decoupling removes the effects of interaction, and the integral error will be the same as for a single-loop controller (i.e., with the other controllers in manual). Therefore, unfavorable interaction occurs when RDG $\cdot f_{tune} > 1.0$, and decoupling can be used in such cases to remove the unfavorable interaction. The information required is given in Table 21.4, which gives the values of 12.2 for X_D and 0.0 for X_B. Since the value for X_D is so large, decoupling should be considered.

The values for the decoupler can be determined from the linear model of the energy balance system and are as follows:

$$D_{12}(s) = 0.893\frac{10.2s+1}{15s+1}e^{-(2-3.3)s} \quad \text{(not realizable)}$$

$$\approx 0.893\frac{10.2s+1}{15s+1} \quad \text{(physically realizable)}$$

$$D_{21}(s) = 0.930\frac{10.2s+1}{11.75s+1}e^{-1.3s}$$

A dynamic response for this decoupled system to a set point change of 0.01 in the top composition is given in Figure 21.13*a*, and the tuning values and performance are summarized in Table 21.6. This theoretically best decoupling performance is quite good, with a much lower IAE than the multiloop case reported in Table 21.2 (energy balance), although in this example the set point change has twice the magnitude. Note that both manipulated variables changed immediately when the set point was changed. The immediate change in MV_1 is from the controller G_{c1}, while the immediate change in MV_2 is from $G_{c1}D_{12}$, so that the decoupler acts before the controlled variable X_B is disturbed. Again, the

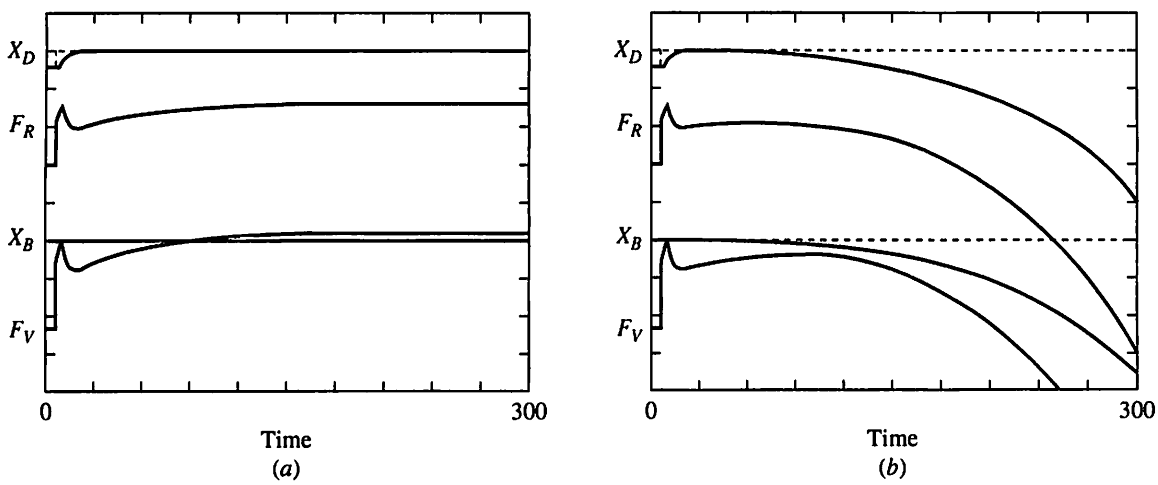


FIGURE 21.13

Explicit decoupling in distillation control, Example 21.6: (*a*) based on a perfect model; (*b*) with 15% gain errors in decouplers. (Scales: One tick = 0.02 for X_D and X_B, 0.50 for F_R, 0.30 for F_V.)

TABLE 21.6

Summary of decoupling Example 21.10

Case	K_{c1}	T_{I1}	K_{c2}	T_{I2}	K_{D12}	K_{D21}	IAE$_1$	IAE$_2$
Exact gains	60	9	−50	6.1	0.893	0.930	0.118	0.006
15% gain errors	60	9	−50	6.1	1.027	1.07	Unstable	

similarity to feedforward is apparent, because the decoupler bases an adjustment in a process input on another process input.

However, the engineer must also consider the sensitivity to modelling errors. This decoupled system will become unstable for errors of about 10% in both decoupler gains; an example with 15% errors is given in Figure 21.13*b*, which shows the instability. *No amount of detuning* (short of $K_{c2} = 0$) in the feedback controllers will stabilize this response. Although the decoupler theoretically could improve performance, it is doubtful that sufficient model accuracy is generally available to use simplified (two-way) decoupling for processes with large relative gains.

With perfect decoupling, it is theoretically possible to improve control performance by reducing unfavorable interaction through decoupling as well as to degrade control performance by misapplying decoupling to a system that has favorable interaction. Decoupling should be considered only after an analysis of the relative disturbance gain has established that interaction is unfavorable for the expected disturbances and that performance with decoupling is not extremely sensitive to model errors.

- Decoupling improves control performance only when process interaction is *unfavorable*, so favorable interaction should not be reduced by decoupling.
- The stability and performance of full decoupling can be very sensitive to model errors when the relative gain is greater than 1. One-way decoupling has much lower sensitivity to model errors.

An important observation is that greater control system complexity does not always lead to better performance!

21.6 ▫ MULTILOOP CONTROL PERFORMANCE THROUGH ENHANCEMENTS: SINGLE-LOOP ENHANCEMENTS

Many enhancements were presented in Part IV to improve the performance of single-loop control systems. These methods are also widely applied to the control of multiloop systems, as will be covered in more depth in Part VI, but a brief example is presented here to complete the methods for achieving good multiloop performance. The distillation tower in Figure 21.14 has multiloop control of the

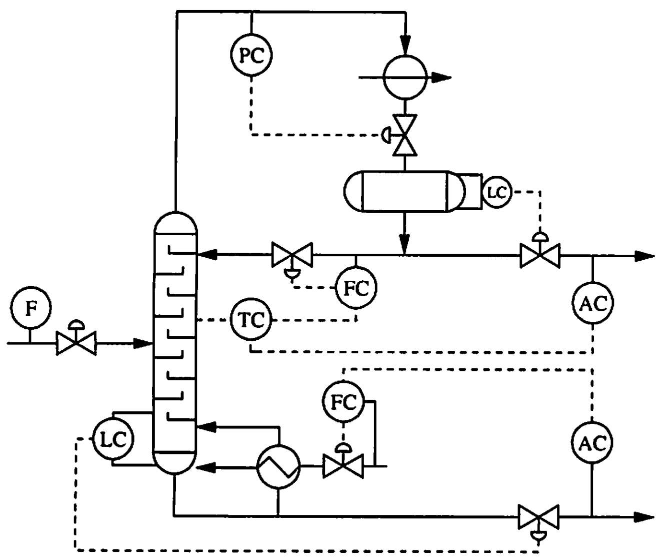


FIGURE 21.14

Multiloop distillation control with single-loop enhancements.

two product compositions. In addition, the control performance is enhanced by inferential tray temperature control, which could provide a surrogate variable for control when the top analyzer provides an infrequent feedback measurement. Also, the reboiler utility and reflux flows have cascade control to reduce disturbances that result from changes in supply pressures. Other enhancements, such as feedforward, could be included as needed.

21.7 ◻ ADDITIONAL TOPICS IN MULTILOOP PERFORMANCE

The material in Chapters 20 and 21 presents only an introduction to the advances made in meeting the daunting challenges of multiloop control. The following subsections introduce a few selected additional topics.

Regulatory Control

Examples 21.2 and 21.3 on distillation control demonstrated that the regulatory control loops influence the composition control performance. An excellent control design objective is to select regulatory designs giving manipulated variables that simultaneously reduce transmission interaction (i.e., make the relative gain close to 1) and improve the disturbance rejection capability of the system (i.e., make the magnitude of the relative disturbance gain small). An example of such an approach is the simple distillation design developed by Rhyscamp (1980), which has proved remarkably successful on two-product distillation towers (Stanley et al., 1985; Waller et al., 1988). When simple regulatory loops do not provide these advantages, calculated variables can sometimes be derived that potentially improve multiloop performance (Haggblom and Waller et al., 1990; Johnston and Barton, 1987); however, the sensitivity of these approaches to model errors has not been fully evaluated.

Integrity: Integral Stabilizability and the Niederlinski Index

As already discussed in Section 21.3, the integrity of a multiloop control system is an important property that is influenced by decisions on loop pairing. Here, a further test for acceptable closed-loop behavior is presented; like the relative gain, this test can be performed with minimal information about the system, i.e., steady-state process gains. For this test, we consider multiloop controllers with integral modes, a very common situation in practice.

> For integrity, we want the control system to have the following property, which we term **integral stabilizability:** stable control can be achieved when the signs of the controller gains are the same for (1) the single loop situation (with all other loops in manual) and (2) the multiloop situation (with all other loops in automatic).

We begin the test by arranging the steady-state process gain matrix so that the loop pairings involve the 1–1, 2–2, …, n–n diagonal elements in **K**; note that this step only changes the variable order in the model. Then, the following calculation is performed to evaluate the integral stabilizability of the plant with the proposed loop pairing:

$$\text{If } NI = \left(\frac{\det \mathbf{K}}{\prod_{i=1}^{n} K_{ii}} \right) < 0 \qquad \text{the system is not integral-stabilizable}$$

> Only control designs with the Niederlinski index $NI > 0$ should be considered further; those with $NI < 0$ should be excluded.

This test is sufficient but not necessary for *lack* of integral stability, which is unacceptable behavior. (The condition is necessary and sufficient for 2 × 2 systems.) The proof of this condition and limitations on the plant dynamics for its applicability are presented in Grosdidier et al. (1985). Further results on integrity can be found in Grosdidier et al. (1985), Chiu and Arkun (1990), Morari and Zafiriou (1989), and Campo and Morari (1994).

Loop Pairing

Some alternative guidelines for loop pairings have been published by Yu and Luyben (1986), Economou and Morari (1986), and Tzouanas et al. (1990). The selection of the final design, after many alternatives have been eliminated using methods in this chapter and references, relies on experience with similar units or dynamic simulation.

Robustness

The models used in control design never exactly match the true process behavior, and this factor would normally influence the performance of the system. While this issue could be addressed with simple assumptions and reasonable computation for single-loop systems, multiloop systems involve many more model parameters, all of which can be in error. Errors are introduced through empirical identification and as a result of changes in plant operation, such as flow rates and reactor conversions. Thus, the parameter errors in linearized models are not independent; that is, they have structure that must be considered in the analysis of robustness. The importance of robustness was discussed clearly by Doyle and Stein (1981) and is covered in Skogestad and Morari (1987*b*) and extensively by Morari and Zafiriou (1989).

Dynamics

The results of Example 21.4 demonstrated the importance of considering interacting dynamics. The frequency-dependent relative gain was introduced in the previous chapter to evaluate interaction near the closed-loop critical frequency, and it has been shown that reliance only on steady-state analysis measures can result in good designs being improperly eliminated (e.g., Skogestad et al., 1990). Any predictions of control performance using the methods introduced in this chapter should be validated with a simulation of the closed-loop response. Since the design procedures usually result in a few candidates and simulation software is readily available, this final step should take little engineering effort.

21.8 ▫ CONCLUSIONS

The main result of Chapters 20 and 21 is the evaluation of the key effects of interaction on multiloop control. All of the factors that affect single-loop control affect multiloop control in similar ways. Table 21.7 summarizes the effects of interaction on performance.

In this chapter, methods have been presented for achieving good control performance in multiloop systems through variable pairing, tuning, and simple enhancements. The methods have demonstrated that no single control performance predictor is available; for example, control strategies with relative gain values near 1.0 may not perform well for the disturbances of greatest importance. Even using the relative disturbance gain alone can lead to improper designs. For example, the pairing and tuning of a multiloop strategy can be selected to give better performance for a specific controlled variable (or variables) of particular importance over other variables of much less importance. Thus, the multiloop strategy must be selected with careful attention to the control objectives and process dynamic responses.

The flowchart in Figure 21.15 gives a procedure by which the analysis methods presented in this chapter can be applied to a 2×2 system analysis. Naturally, the control objectives must first be defined; then the necessary process information must be developed. The minimum information includes all steady-state gains as shown in Table 21.4 and some semiquantitative information on the relative dynamics between the manipulated and controlled variables is needed to select pairings based on dynamics and calculate the tuning factor. Finally, dynamic models, at least linear transfer functions and perhaps nonlinear models, are required if simulation verification is performed.

In the first step in the flowchart, the process is screened for the feasibility of multiloop control through evaluation of the controllability and operating window; if multivariable control is not possible, a different selection of variables or a process equipment modification is required. The first decision in the flowchart

TABLE 21.7

Effects of interaction on multiloop performance

Issue	Measure	Comments
Feasibility of feedback control	1. det $\mathbf{K} \neq 0$	1. Independent relationships exist between manipulated and controlled variables
	2. Specified set points can be achieved for expected disturbances	2. Manipulated variables have sufficient range; i.e., the process has sufficient capacity
Performance and integrity	1. For $n \times n$, not integral-stabilizable if $\left(\frac{\det \mathbf{K}}{\prod_{i=1}^{n} K_{ii}}\right) < 0$ For 2 × 2, not integral-controllable if $\lambda_{ij} < 0$	1. Niederlinski criterion (or RGA for 2 × 2) used to evaluate whether controllers with integral modes can stabilize both single and multiloop systems without changing sign of controller gains
	2. $\lambda_{ij} > 0$	2. Usually, pairing selected that functions in single-loop and multiloop. ($\lambda_{ij} = 0$ or $\lambda_{ij} < 0$ sometimes acceptable)
Stability and tuning	For 2 × 2, λ_{11}	Interaction influences the characteristic equation, so it influences stability. Controller tuning must be modified for single-loop, usually detuned.
Performance	Relative disturbance gain (RDG)	Pairings are selected to reduce unfavorable interaction ($\lvert\text{RDG}\rvert\lvert K_d\rvert$ small) and provide fast feedback dynamics for important loops.
Enhancements		Designs, such as cascade and feedforward, that reduce the effects of disturbances are always beneficial. Decoupling can be used to reduce the effects of unfavorable interaction ($\lvert\text{RDG}\rvert > 1$) when the transmission interaction (RGA) is not too large

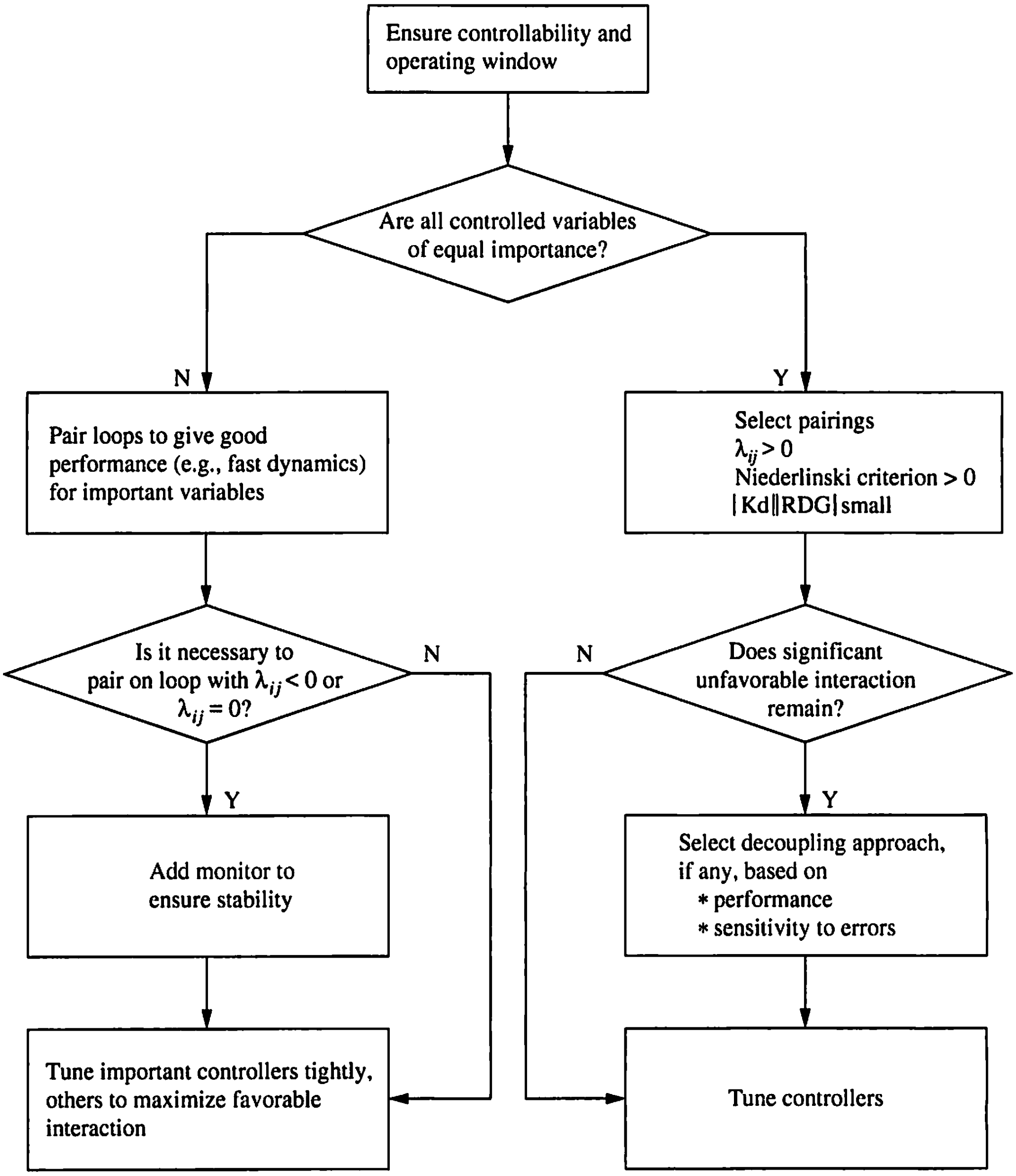


FIGURE 21.15

Flowchart for selecting 2 × 2 pairing and tuning.

is whether both controlled variables are of equal importance. If one is of much greater importance, the left branch is taken. The important controlled variable is paired with the manipulated variable that provides the fastest feedback dynamics (along with satisfactory range) if a significant difference exists. A check is made to determine whether the controlled variable can be improved (through faster dynamics) by pairing it with a manipulated variable giving a zero relative gain; this step would be taken only in unusual situations in which the controlled variable is extremely important. After pairing has been selected, the control loops are tuned. Since the left-hand path is for unequal control priorities, the more important loops should be tuned to retain favorable interaction and reduce unfavorable interaction, and the less important loops should be tuned in a manner consistent with improving the overall performance and maintaining stability. Decoupling would probably not be considered, because detuning alone would reduce the effects of unfavorable interaction.

If the controlled variables are of equal importance, the pairings should be selected according to the analysis of the relative disturbance gain. If substantial unfavorable interaction remains, consideration would be given to decoupling, especially one-way decoupling to prevent the sensitivity problems encountered with two-way decoupling when the process has a large relative gain. Finally, the controllers would be tuned using methods described in Chapter 20. This procedure can lead to a good multiloop control strategy for the given process.

The concepts and methods presented in this chapter can be applied to a multiloop system of any order. However, the equations for the relative disturbance gain in this chapter are limited to a 2×2 system; they have been extended for higher-order systems by Skogestad and Morari (1987*a*), who also introduce an alternative measure of multiloop performance.

Finally, this approach often, but not always, provides satisfactory performance. However, depending on factors such as the feedback dynamics and the disturbance type, magnitude, and frequency, situations exist in which no multiloop feedback design provides acceptable dynamic performance. Other steps for improving control performance include multivariable control, which is covered in Chapter 23, and process alterations.

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ADDITIONAL RESOURCES

The "valve position" controllers introduced in the next chapter involve pairing on zero relative gain. In addition, a few examples of control designs with pairings on zero steady-state relative gains are given in McAvoy (1983) and in

Finco, M., W. Luyben, and R. Pollack, "Control of Distillation Columns with Low Relative Volatility," *IEC Res., 28*, 76–83 (1989).

In addition to Shinskey (1988), example multiloop control designs are presented in

Balchen, J., and K. Mumme, *Process Control: Structures and Applications*, Van Nostrand Reinhold, New York, 1988.

Additional results for systems with complex interactive dynamics (e.g., inverse responses), are given in

Holt, B., and M. Morari, "Design of Resilient Process Plants: VI The Effect of Right Half Plane Zeros on Dynamic Resilience," *CES, 40*, 59–74 (1985*a*).
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The methods in Chapters 20 and 21 can be applied in sequence, as shown in Figure 21.15, to eliminate poor alternatives, rank likely performance of feasible designs, and evaluate the appropriateness and sensitivity of decoupling. This analysis is based on quantitative analysis of the linearized system.

QUESTIONS

21.1. The following transfer functions were provided by Wood and Berry (1973) for a methanol-water separation in a distillation column similar to Figure 20.3. The products are expressed as mole % light key, and the reflux F_R, the reboiler steam F_S, and the disturbance feed flow F are in lb/min; time is in min.

$$\begin{bmatrix} X_D(s) \\ X_B(s) \end{bmatrix} = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \begin{bmatrix} F_R(s) \\ F_S(s) \end{bmatrix} + \begin{bmatrix} \dfrac{3.8e^{-8.1s}}{14.9s+1} \\ \dfrac{4.9e^{-3.4s}}{13.2s+1} \end{bmatrix} F(s)$$

Answer the following questions for the feed flow disturbance.

(*a*) Determine whether the input-output combination is controllable.

(*b*) Determine whether either loop pairing can be eliminated based on the sign of the relative gains ($\lambda_{ij} > 0$).

(*c*) Select the loop pairing based on an estimate of the control performance.

(*d*) Determine the initial tunings for PI controllers for the best loop pairing. Answer this question for (1) the two product compositions of equal importance and (2) the top product quality more important.

(*e*) Discuss whether decoupling is recommended and if so, design the decoupler.

(*f*) Discuss whether feedforward compensation would improve the control performance and if so, design the feedforward controller.

(*g*) The model was determined from empirical identification experiments. Discuss the likely errors in the model and the effects of these errors on the design conclusions.

For (*c*) through (*f*), compare the multiloop control performance for each controlled variable with its single-loop performance.

21.2. (*a*) Derive the expressions for the relative disturbance gain (RDG_1) and the integral error ($\int E_1 \, dt$) for the following inputs (1) ΔSP_1, (2) ΔSP_2, (3) a disturbance that has the same transfer function as MV_1, and (4) a disturbance that has the same transfer function as MV_2.
(*b*) Relate the value of the relative disturbance gain, RDG_1, to the ratio of changes in the manipulated variable for single-loop and multiloop control, $(\Delta MV_1)_{ML}/(\Delta MV_1)_{SL}$, to the same disturbance.
(*c*) Why is the magnitude, not the value, of the RDG used in evaluating performance?
(*d*) Is the RDG scale-dependent?

21.3. For a 2 × 2 control system with PID controllers and decoupling, write the equations for digital implementation of all control equations, or provide a sample computer program.

21.4. A linear transfer function model of a chemical reactor was determined by Foss et al. (1980) and simplified by Marino-Galarraga et al. (1987*a*). The reaction of oxygen and hydrogen over a catalyst occurs in two beds, with cold hydrogen quench added between the beds. The reactor is shown in Figure Q21.4, and the model is given below. The units are composition in mole%, temperatures in °C/167.4, flow in L/min/13.5, and time in sec/87.5. Assume that both controlled variables are of equal importance. Answer the following questions for two cases: (1) the input perturbation is a set point change to the composition controller and (2) the input perturbation is a change to the cooling medium temperature, so that the disturbance transfer function is the second column of the following matrix (the same effect as a change in the manipulated quench temperature).

$$\begin{bmatrix} T(s) \\ C(s) \end{bmatrix} = \begin{bmatrix} \dfrac{-2.265e^{-1.326s}}{0.786s+1} & \dfrac{0.746e^{-2.538s}}{0.092s+1} \\ \dfrac{1.841e^{-0.445s}}{0.917s+1} & \dfrac{-0.654e^{-0.786s}}{0.870s+1} \end{bmatrix} \begin{bmatrix} F_Q(s) \\ T_Q(s) \end{bmatrix}$$

(*a*) Determine whether the input-output combination is controllable.
(*b*) Determine if either loop pairing can be eliminated based on the sign of the relative gains.
(*c*) Select the loop pairing based on an estimate of the control performance.
(*d*) Determine the initial tunings for PI controllers for the best loop pairing. Answer this question for (1) the temperature and product composition of equal importance and (2) the temperature more important.
(*e*) Discuss whether decoupling is recommended and if so, design the decoupler.
(*f*) Discuss whether feedforward compensation would improve the control performance and if so, design the feedforward controller.

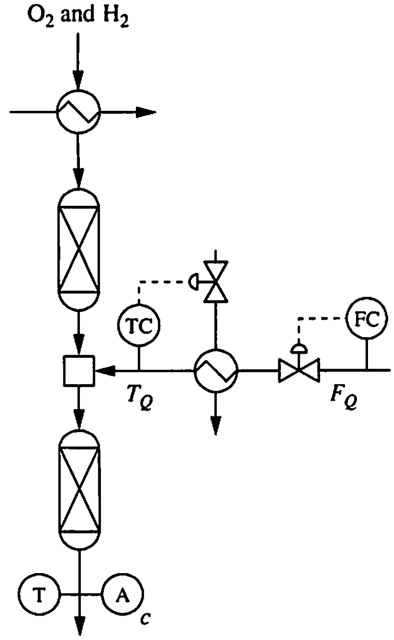

FIGURE Q21.4

For (*c*) through (*f*), compare the multiloop control performance for each controlled variable with the single-loop performance.

21.5. Two physical systems with exactly the same equipment structure, pressures, and flow rates in Figure Q21.5 are considered in this question. The only difference is that in system (*a*) phase I is a liquid (this is a decanter), whereas in system (*b*) phase I is a vapor (this is a flash drum). You may assume that the flows are proportional to the square root of the pressure drop and the valve % open; the valves are all 50% open at the base-case conditions. The three valves are available for manipulation, and three controlled variables are shown as sensors. The following additional information is provided about the variability of the process operation: the feed flow is 1400 to 2600 L/min, the percent overhead material in feed is 1 to 5%, and the external pressures are essentially constant. Select the best control loop pairing and discuss the differences, if any, between the results for systems (*a*) and (*b*).

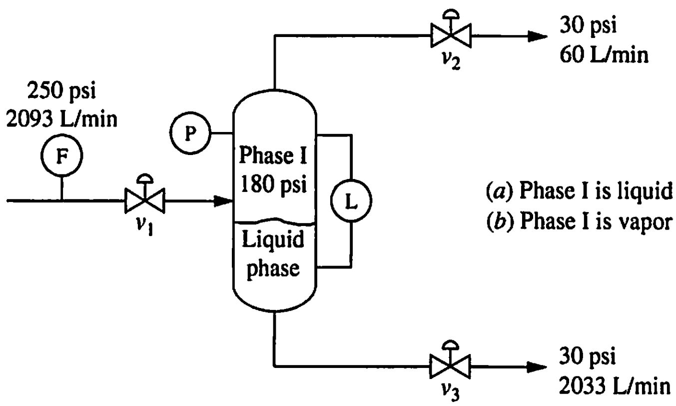

FIGURE Q21.5

21.6. Answer the following questions.

(*a*) Is there a feedback control system for system B2 in equation (21.6) that will prevent the inverse response?

(*b*) For system B1 in equation (21.5), can the multiloop feedback system experience an inverse response with two PID controllers?

(*c*) Values of the relative disturbance gain (RDG) can be related to the change in the manipulated variables under multiloop control. Determine the value of ΔMV_1 for a disturbance and relate this to RDG_1.

(*d*) Is it possible to have a relative gain ≈ 1.0 and a large RDG?

(*e*) Is it possible to have no interaction of any type (e.g., $K_{12} = K_{21} = 0$) and have a large RDG?

(*f*) Feedforward control can be applied on a multiloop system. Modify the calculation of the relative disturbance gain (RDG_1) and the integral error ($\int E_1\, dt$) for various feedforward control designs (feedforward to MV_1 only, to MV_2 only, and to both) using the same disturbance.

(*g*) The relative disturbance gain provides the ratio of multiloop to single-loop performance. Discuss how to use this information when comparing the performance of two designs with different single-loop performances.

21.7. The outlet temperature of the process fluid and the oxygen in the flue gas can be controlled in the fired heater in Figure Q20.10 by adjusting the fuel pressure (flow) and the stack damper % open. A dynamic model for the fired heater in Figure Q20.10 was reported by Zhuang et al. (1987) and is repeated here.

$$\begin{bmatrix} T(s) \\ A(s) \end{bmatrix} = \begin{bmatrix} \dfrac{0.6}{2400s^2 + 85s + 1} & \dfrac{-0.04}{3000s^2 + 90s + 1} \\ \dfrac{-1.1}{70s + 1} & \dfrac{0.30}{70s + 1} \end{bmatrix} \begin{bmatrix} P_{sp}(s) \\ V_2(s) \end{bmatrix}$$

The inputs and outputs are in percent of the range of each instrument, and the time is in sec.

(*a*) Determine whether the input-output combination is controllable.
(*b*) Determine whether either loop pairing can be eliminated based on the sign of the relative gains.
(*c*) Determine whether decoupling will improve the control performance.
(*d*) Determine the PI controller tuning for the best multiloop control, with or without decoupling.

21.8. The following transfer functions were provided by Waller et al. (1987) for a distillation column. System I was similar to Figure 20.3 except that the controlled product compositions were not measured directly; they were inferred from tray temperatures (°C) near the top, T_4, and near the bottom, T_{14}, trays. System II had the distillate/(distillate + reflux) as a manipulated variable rather than the reflux; this is designated as R. The flows are in kg/h; time is in min. Answer the following questions for both systems (the same process with different regulatory control designs) and compare the results.

System I: Energy balance regulatory control

$$\begin{bmatrix} T_4(s) \\ T_{14}(s) \end{bmatrix} = \begin{bmatrix} \dfrac{-0.045e^{-0.5s}}{8.1s + 1} & \dfrac{0.048e^{-0.5s}}{11s + 1} \\ \dfrac{-0.23e^{-1.5s}}{8.1s + 1} & \dfrac{0.55e^{-0.5s}}{10s + 1} \end{bmatrix} \begin{bmatrix} F_R(s) \\ F_S(s) \end{bmatrix} + \begin{bmatrix} \dfrac{0.004e^{-s}}{8.5s + 1} \\ \dfrac{-0.65e^{-s}}{9.2s + 1} \end{bmatrix} X_F(s)$$

System II: Modified regulatory control; $R = F_D/(F_D + F_R)$

$$\begin{bmatrix} T_4(s) \\ T_{14}(s) \end{bmatrix} = \begin{bmatrix} \dfrac{6.7e^{-0.5s}}{11s + 1} & \dfrac{0.01e^{-0.5s}}{13s + 1} \\ \dfrac{34e^{-1.3s}}{12s + 1} & \dfrac{0.35e^{-0.5s}}{10s + 1} \end{bmatrix} \begin{bmatrix} R(s) \\ F_S(s) \end{bmatrix} + \begin{bmatrix} \dfrac{-0.026e^{-2.5s}}{23s + 1} \\ \dfrac{-0.81e^{-s}}{13s + 1} \end{bmatrix} X_F(s)$$

(*a*) Determine whether the input-output combination is controllable.
(*b*) Determine whether either loop pairing can be eliminated based on the signs of the relative gains.
(*c*) Select the loop pairing based on an estimate of the control performance.
(*d*) Determine the initial tunings for PI controllers for the best loop pairing. Answer this question for (1) the two product compositions of equal importance and (2) the top product quality more important.

(*e*) Discuss whether decoupling is recommended and if so, design the decoupler.
(*f*) Discuss whether feedforward compensation would improve the control performance and if so, design the feedforward controller.

For (*c*) through (*f*), compare the multiloop control performance for each controlled variable with the single-loop performance.

21.9. (*a*) The limit for the integral error of a decoupled system in equation (21.20) as the gain errors approach zero is $\lambda_{11} K_{d1} T_{I1} / K_{c1} K_{11}$. Explain why this differs from equation (21.9).
(*b*) Explain why the gain decoupler errors in Example 21.10 lead to an unstable system. (Hint: Consider the relative gain or Niederlinski criterion for the system with decouplers.)
(*c*) Derive the expression in equation (21.20) for the integral error for a 2×2 multiloop system with PI controllers and decouplers, with gain errors in the decouplers.

21.10. The process with two series chemical reactors in Example 3.3 is considered in this question. The process flexibility is increased by allowing the temperatures of the two reactors to be manipulated independently. The two controlled variables are the concentrations of reactant A in the two reactors. The rate constant can be expressed as $5.87 \times 10^5 e^{-5000/T}$ (with temperature in K), and the disturbance is feed composition, C_{A0}.
(*a*) Determine whether the input-output combination is controllable.
(*b*) Determine whether either loop pairing can be eliminated based on the signs of the relative gains.
(*c*) Determine whether decoupling could improve the dynamic performance, especially if the most important controlled variable is the concentration in the second reactor.

21.11. Doukas and Luyben (1978) reported the transfer function model for the distillation column with a side stream product, shown in Figure Q21.11. The feed contains benzene (B), toluene (T), and xylene (X). The controlled and manipulated variables are given in the figure, with the benzene in the side stream of much less importance than the other controlled variables. The linearized transfer function model is

$$\begin{bmatrix} XD_T(s) \\ XS_B(s) \\ XS_X(s) \\ XB_T(s) \end{bmatrix} = \begin{bmatrix} \dfrac{-1.986e^{-0.7s}}{66.7s+1} & \dfrac{5.24e^{-60s}}{400s+1} & \dfrac{5.984e^{-2.24s}}{14.3s+1} \\ \dfrac{0.002e^{-0.6s}}{(7.14s+1)^2} & \dfrac{-0.33e^{-0.7s}}{(2.4s+1)^2} & \dfrac{2.38e^{-0.42s}}{(1.43s+1)^2} \\ \dfrac{-0.176e^{-0.5s}}{(6.9s+1)^2} & \dfrac{4.48e^{-0.5s}}{11.1s+1} & \dfrac{-11.7e^{-1.9s}}{12.2s+1} \\ \dfrac{0.374e^{-7.75s}}{22.2s+1} & \dfrac{-11.3e^{-3.8s}}{(21.7s+1)^2} & \dfrac{-9.81e^{-1.6s}}{11.4s+1} \end{bmatrix} \begin{bmatrix} RR(s) \\ LS(s) \\ QB(s) \end{bmatrix}$$

For this system, determine the best loop pairing by following the method in Figure 21.15.

21.12. Design an improved control system to improve the dynamic performance of the composition in the fuel system in Figure 21.7 when

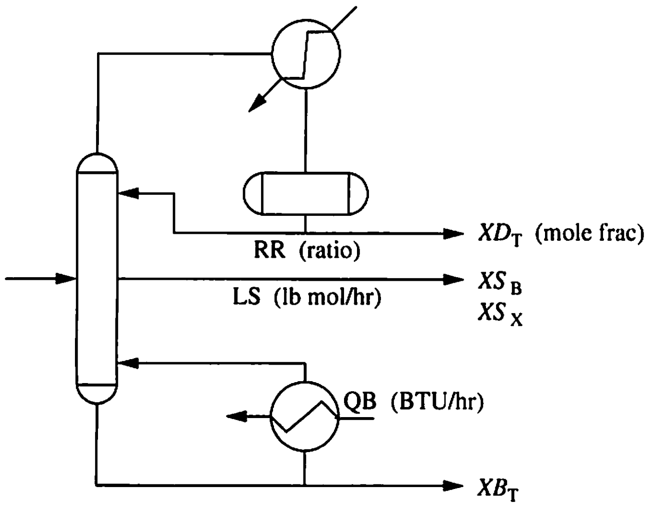

FIGURE Q21.11

(*a*) A measurement of the total fuel flow to the consumers is available.
(*b*) A measurement of the gas fuel (L) is available.

21.13. Calculate the controller tuning for the blending system in Table 21.1 with $A_1 = 0.95$. Discuss which loop pairing would be preferred.

21.14. (*a*) Derive the closed-loop transfer function for a 2×2 system with decoupling.
(*b*) From the result in (*a*), determine whether one-way decoupling influences the stability of the closed-loop system.

21.15. The series of well-stirred chemical reactors with equal volumes shown in Figure Q21.15 is to be controlled. The controlled variables are the temperature and reactant concentration in the third reactor, and the manipulated variables are the inlet concentration set point and the cooling valve v_2. The chemical reaction is first-order, the rate constant has an Arrhenius relationship with temperature, and the heat of reaction is negligible. The heat exchanger dynamics are negligible. For this example, the concentration is much more important than the temperature, but both should have zero steady-state offset for a steplike disturbance. Design the loop pairings and tuning and discuss the rationale for the design.

21.16. Answer the following questions for two physical processes: (1) the chemical reactor described in Section C.2, and (2) the same chemical reactor with no heat of reaction, $\Delta H_{rxn} = 0$. Both processes have two feedback PI controllers: $T \rightarrow F_c$ and $C_A \rightarrow C_{A0}$ (with the feed flow unchanged).
(*a*) Does process interaction influence the stability of the closed-loop system? Provide quantitative analysis to support your conclusion.
(*b*) Does process interaction influence the dynamic performance (behavior) of the closed-loop system? Explain your answer briefly.

21.17. Design feedforward controllers for the distillation column under energy balance control, described by equation (21.1), for a measured disturbance in feed composition. Design the feedforward controller for the two following

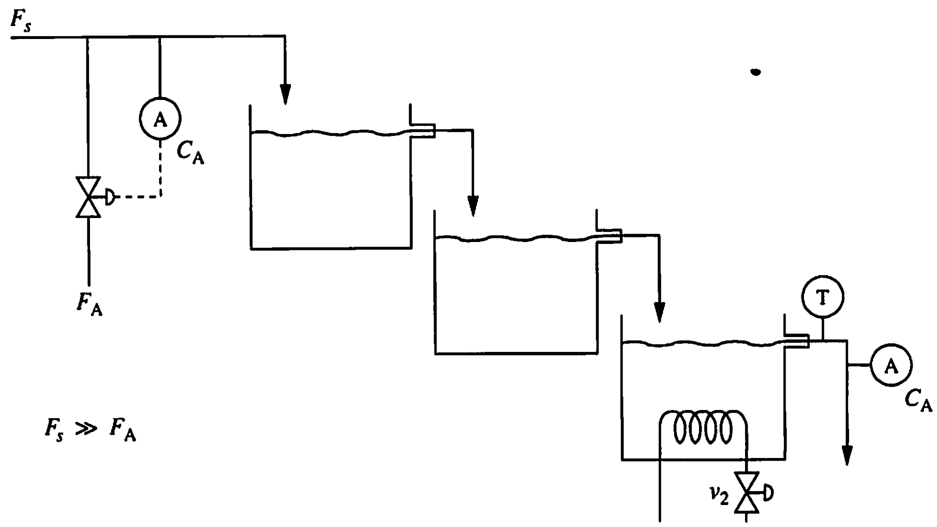

FIGURE Q21.15

situations, discuss the differences in the results, and discuss the implications for application of each.

(*a*) The distillate composition X_D is to be maintained constant, and the bottoms composition X_B is not controlled and may vary.

(*b*) The distillate composition X_D and the bottoms composition X_B are both to be maintained constant via the feedword controller.