# Multiloop Control: Effects of Interaction

CHAPTER 20

## 20.1 ◻ INTRODUCTION

Multivariable control occurs in nearly all processes, because production rate (flow), inventory (level and pressure), process environment (temperature), and product quality are normally controlled simultaneously. The *multiloop* approach, using multiple single-loop controllers, was the first approach used for multivariable control in the process industries. Through decades of research and experience, many successful multiloop strategies have been developed and continue to be used.

One advantage of multiloop control is the use of simple algorithms, which is especially important when the control calculations are implemented with analog computing equipment. A second advantage is the ease of understanding by plant operating personnel, which results from the simplicity of the control structure. Since each controller uses only one measured controlled variable and adjusts only one manipulated variable, the actions of the controllers are relatively easy to monitor. A third advantage is that standard control designs have been developed for the common unit operations, such as furnaces, boilers, compressors, and simple distillation towers. This does *not* mean that a single control design functions well for all unit operations of the same type. However, several general structures are in common use, and selection among alternatives can be based on analysis and experience. Considering these advantages, one could conclude that multiloop designs will continue to be used extensively, although not exclusively.

An example of multiloop control of a flash process is given in Figure 20.1. Let us consider the behavior of the system when the feed flow rate increases. An initial effect is an increase in the amount of vapor entering the drum, although the

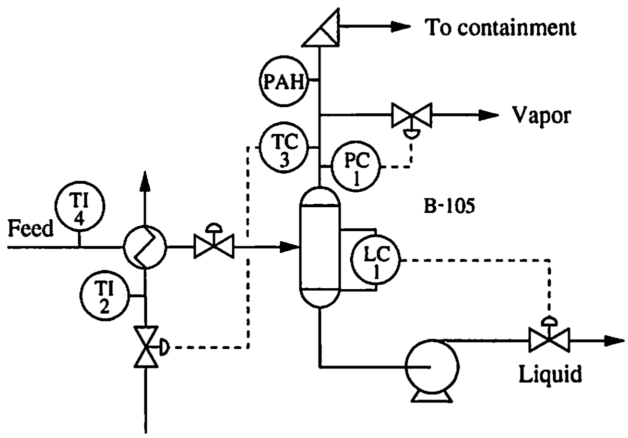



**FIGURE 20.1**

**Example of a multiloop control system.**

percentage feed vaporized decreases because of a slight decrease in inlet temperature. The pressure in the drum increases because of the additional vapor; therefore, the pressure controller PC-1 takes action by increasing the percent opening of the valve in the vapor line. Another effect is a decrease in the temperature after the heat exchanger, which is sensed by TC-3. This feedback controller increases the steam flow to the exchanger, which returns the temperature to its set point and causes even more feed to be vaporized. This additional vapor causes the pressure to increase, and the pressure controller has to respond to this change as well. The increase in feed rate and changes in percent vaporized introduce changes in the liquid rate into the liquid inventory in the drum. The level controller increases the opening of the valve in the liquid product line to maintain the level near its set point.

Two important features of this system become clear when observing its dynamic behavior:

1. The single-loop controllers are completely independent algorithms that do not communicate directly among themselves.
2. The manipulations made by one controller can influence other controlled variables; that is, there can be *interaction through the process* among the individual control loops.

The interaction is the key effect addressed in this chapter, where we will demonstrate that several single-loop controllers on a process should not generally be analyzed as though each were a single-loop system.

We shall use the following definition of interaction.

> A multivariable process is said to have **interaction** when process input (manipulated) variables affect more than one process output (controlled) variable.

This definition is consistent with the use of the word in the vernacular and will serve us in the study of multivariable systems. However, the definition does not distinguish between various important properties that will be introduced in this chapter. Thus, careful attention must be paid to the effects of various types of interaction on control stability and performance.

In this chapter the basic principles of multiloop control are presented, with the goal of understanding multivariable systems. As with single-loop control, we start with the process by reviewing modelling approaches for multivariable processes and developing models for two sample systems, which will be used in later examples. Then the concept of interaction is discussed to highlight its effects on system behavior, and a quantitative measure of interaction is introduced. Finally, some approaches for tuning multiloop controllers are presented. All of the concepts developed in this chapter are employed in the next chapter, which addresses the performance of multiloop control systems.

## 20.2 ▫ MODELLING AND TRANSFER FUNCTIONS

Process models for multivariable control can be derived from fundamental principles or can be estimated based on empirical data. Regardless of the modelling method used, the analysis, design, and tuning of multiloop controllers will be based on linear input-output models employing block diagram manipulation, stability analysis, and frequency response. The following two examples demonstrate the modelling approaches applied to blending and distillation, and the resulting models will be employed in several subsequent examples.

**EXAMPLE 20.1.**

Blending is an important unit operation and is employed in a wide variety of industries, as in the production of gasoline (Stadnicki and Lawler, 1985) and cement (Sakr et al., 1988). Typically, the controlled variables in a blending process are production rate and blended product composition. The blending process in Figure 20.2 is modelled with the following assumptions:

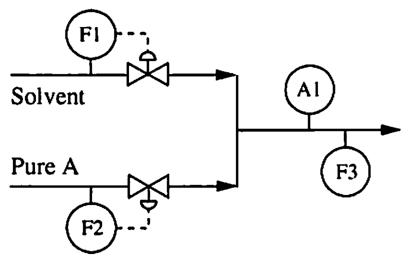



**FIGURE 20.2**

**Example blending process.**

1. The inlet concentrations are constant.
2. Mixing where the flows merge is perfect.
3. The densities of the solvent and component A are equal.

The overall and component A material balances at the point of mixing are

$$F_m = F_A + F_S \tag{20.1}$$

$$F_m X_m = F_A X_A + F_S X_S \tag{20.2}$$

where
$F_m$ = flow rate of mixed liquid (mass/time)
$X_A$ = mass fraction of component A in pure A = 1.0
$X_S$ = mass fraction of component A in solvent = 0.0
$X_m$ = mass fraction of component A in the mixed liquid

Equation (20.2) can be linearized about the steady state to give

$$X'_m(t) = \left[\frac{F_S}{(F_S + F_A)^2}\right]_s F'_A(t) + \left[\frac{-F_A}{(F_S + F_A)^2}\right]_s F'_S(t) \tag{20.3}$$

with the prime indicating deviation variables. The system is liquid-filled; thus, there is essentially no delay between a change in a component flow rate and a change in the mixed-product flow rate. It is also assumed that the concentration at the location of the analyzer is essentially the same as at the mixing point; that is, there is no transportation dead time. Also, the inlet flow measurements are assumed to be exactly and instantaneously equal to the actual flows, $F_1 = F_S$ and $F_2 = F_A$. The dynamics of the mixed stream flow and concentration sensors are not instantaneous and are characterized by a first-order-with-dead-time model with gains of 1.0 and the following dynamic parameters:

| | Dead time | Time constant |
|---|---|---|
| Flow | $\theta_F$ | $\tau_F$ |
| Concentration | $\theta_A$ | $\tau_A$ |

Thus, the measured controlled variables are related to the instantaneous process variables in equations (20.1) and (20.3) by

$$\tau_A \frac{dA_1'(t)}{dt} = X_m'(t - \theta_A) - A_1'(t) \tag{20.4}$$

$$\tau_F \frac{dF_3'(t)}{dt} = F_m'(t - \theta_F) - F_3'(t) \tag{20.5}$$

Equations (20.1) and (20.3) to (20.5) can be combined to give the following linearized dynamic model:

$$A_1(s) = \frac{\left[\frac{-F_A}{(F_S + F_A)^2}\right]_s e^{-\theta_A s}}{1 + \tau_A s} F_1(s) + \frac{\left[\frac{F_S}{(F_S + F_A)^2}\right]_s e^{-\theta_A s}}{1 + \tau_A s} F_2(s) \tag{20.6}$$

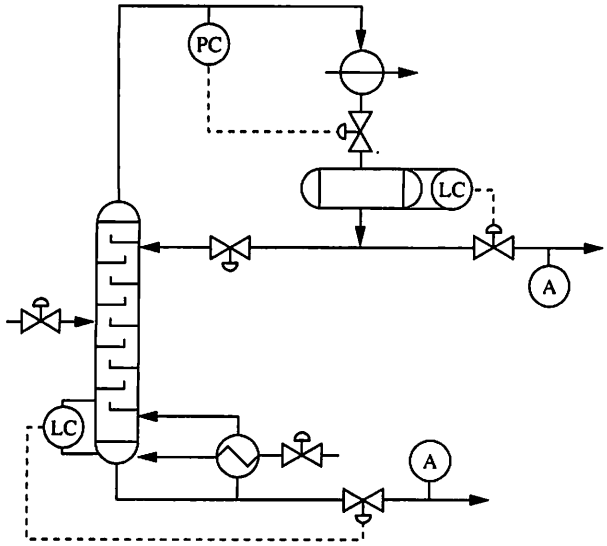
$$F_3(s) = \frac{1.0 e^{-\theta_F s}}{1 + \tau_F s} F_1(s) + \frac{1.0 e^{-\theta_F s}}{1 + \tau_F s} F_2(s) \tag{20.7}$$

Clearly, interaction is present in this process, because each output is affected by both inputs. Numerical values will be determined for different operating conditions later in this chapter.

**EXAMPLE 20.2.**

The empirical identification procedures described in Chapter 6 can be applied to the distillation process shown in Figure 20.3. (This design was originally suggested by McAvoy and Weischedel (1981) and was approximated for constant relative volatility by Sampath (1991).) The manipulated variables are reflux and reboiler flow rates, and the controlled variables are distillate and bottoms composition. Other important variables, such as pressure and levels, are controlled tightly as shown.

One experiment must be performed for each input variable, and the responses of all output variables (after 2 min analyzer dead time) are recorded. Either the process reaction curve or statistical methods can be used to fit parameters in the transfer functions. The models derived by this empirical procedure are as follows




| | |
|---|---|
| Relative volatility | 2.4 |
| Number of trays | 17 |
| Feed tray | 9 |
| Analyzer dead times | 2 min |
| Feed light key | $X_F$ = 0.50 |
| Distillate light key | $X_D$ = 0.98 mole fraction |
| Bottoms light key | $X_B$ = 0.02 mole fraction |
| Feed flow | $F_F$ = 10.0 kgmole/min |
| Reflux flow | $F_R$ = 8.53 kgmole/min |
| Distillate flow | $F_D$ = 5.0 kgmole/min |
| Reboiler flow | $F_V$ = 13.53 kgmole/min |
| Tray holdup | $H$ = 1.0 kgmole |
| Holdup in drums | $HD$ = 10.0 kg mole |

**FIGURE 20.3**

**Example distillation tower.**

with time in minutes:

$$X_D(s) = \frac{0.0747e^{-3s}}{12s+1}F_R(s) - \frac{0.0667e^{-2s}}{15s+1}F_V(s) + \frac{0.70e^{-5s}}{14.4s+1}X_F(s) \qquad (20.8)$$

$$X_B(s) = \frac{0.1173e^{-3.3s}}{11.7s+1}F_R(s) - \frac{0.1253e^{-2s}}{10.2s+1}F_V(s) + \frac{1.3e^{-3s}}{12s+1}X_F(s) \qquad (20.9)$$

Note that the reflux flow ($F_R$) and amount vaporized in the reboiler ($F_V$) are potential manipulated variables, and the feed composition ($X_F$) is a disturbance, because it depends on upstream operations and is assumed not free to be adjusted.

Finally, the linearized models in Examples 20.1 and 20.2 will be used in subsequent system analysis examples. When the dynamic responses are determined via simulation, the linearized distillation model will be used, but the nonlinear blending model will be used because of the large range of operating conditions considered in the blending examples.

Linearized models, whether derived from fundamental balances or from experiments, can be used to analyze the system with and without control. To understand the entire system, it is helpful to present the process in a block diagram. The block diagram of a general 2 × 2 system, recalling that each process transfer function relates one input to one output, is shown in Figure 20.4. Each term $G_{ij}(s)$ relates manipulated input $j$ to output $i$, and the terms $G_{di}(s)$ relate the effects of a disturbance on each process output. If more than one important disturbance is to be considered, additional disturbance transfer functions can be included. Note that if both $G_{12}(s)$ and $G_{21}(s)$ [or alternatively $G_{11}(s)$ and $G_{22}(s)$] are zero, the process has no interaction, because one input affects only one output. In such a case, the system behaves like two independent processes, and the behavior of each control loop is independent.

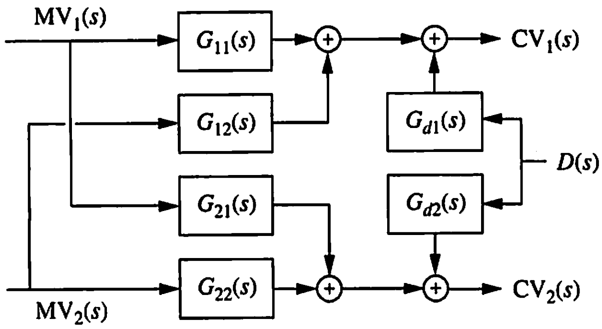



**FIGURE 20.4**

**Block diagram of 2 × 2 open-loop system.**

The set of simultaneous equations relating inputs to outputs in Figure 20.4 are often presented in matrix form as follows:

$$\begin{bmatrix} CV_1(s) \\ CV_2(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} MV_1(s) \\ MV_2(s) \end{bmatrix} + \begin{bmatrix} G_{d1}(s) \\ G_{d2}(s) \end{bmatrix} D(s) \tag{20.10}$$

Each element of the matrix is a transfer function relating one input to one output. Thus:

> Linear models for multivariable systems can be developed using the same analytical and empirical procedures as for single-variable systems.

## 20.3 ▪ INFLUENCE OF INTERACTION ON THE POSSIBILITY OF FEEDBACK CONTROL

Previously, some basic requirements were stated for the variables involved in a single-loop feedback control system. Briefly, the controlled variable should be closely related to process performance; the manipulated variable should be independently adjustable; there should be a causal relationship between the manipulated and controlled variables; and the dynamics should be favorable. These guidelines are still useful, but a somewhat more thorough analysis is required for multivariable systems, because range and controllability are influenced by process interactions.

### Operating Window

The first issue is the control system's range of attainable variable values. The term *operating window* will be used for the range of possible (or feasible) steady-state values of process variables that can be achieved with the equipment available. The operating window can be sketched using different variables as coordinates; in one approach, the controlled variables are used to characterize the range of possible set points, with all disturbances constant. Another common approach is to use the disturbance variables as coordinates to characterize the range of disturbance values that can be compensated by the control system (i.e., for which the con-

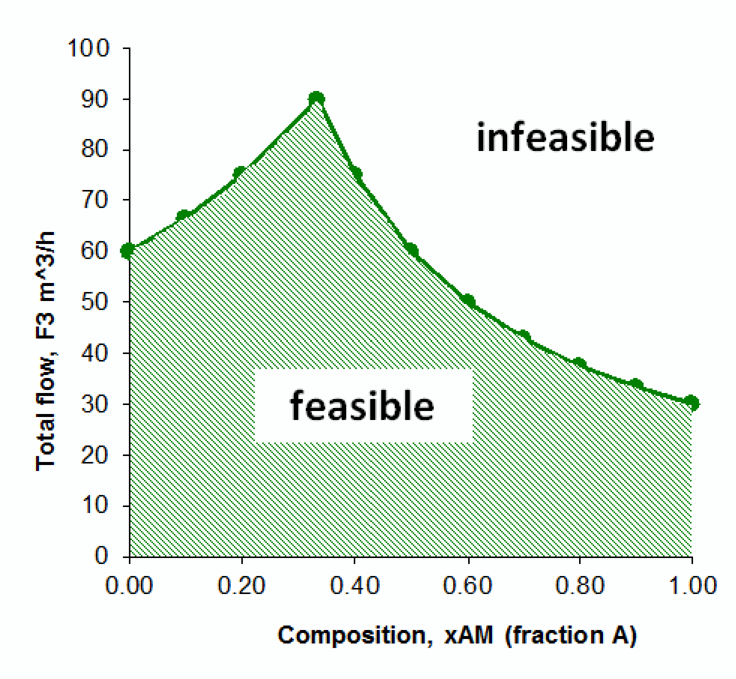
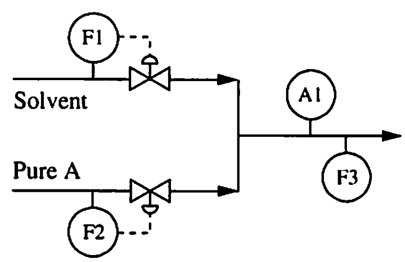



**FIGURE 20.5**

**Operating window for blending with controlled variables as coordinates.**

trolled variables can be maintained at constant set points). The two approaches are demonstrated in the following examples.

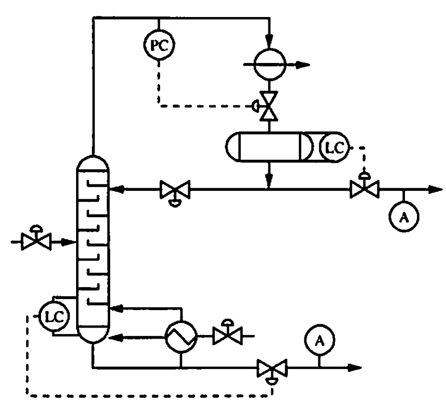
**EXAMPLE 20.3.**




The component flow rates in the blending example can be adjusted continuously from zero to maximum rates, $F_{A\max}$ and $F_{S\max}$. Draw the operating window of attainable total flow rate and composition, assuming that the component compositions remain unchanged.

The attainable total flow $F_3$ and composition $A_1$ are shown in Figure 20.5. The limiting values are easily determined by solving equations (20.1) and (20.2) for various values of one flow, with the other flow at its maximum value. The interaction between variables is clear, because the value of one variable influences the range of the other variable. If the variables were independent and no interaction occurred, the operating window would be rectangular, which it clearly is not.

**EXAMPLE 20.4.**




The feed flow rate and composition to the distillation tower in Example 20.2 change over ranges of 8 to 12 kmole/min and mole fraction 0.4 to 0.6, respectively. Also, the vapor condensed in the condenser cannot be greater than 15.0 kmole/min. Determine the range of disturbances for which the product qualities can be maintained at 0.98 and 0.02 mole fraction.

The method for calculating the operating window for this example depends on the equation-solving methods available. A trial-and-error method could be used to specify the disturbances and simulate the tower with $X_D$ and $X_B$ at their set points. This trial-and-error procedure, involving many simulations, would be executed until the disturbance value that resulted in the maximum overhead vapor flow was found. A direct method of solving this problem would be to specify $X_D$ and $X_B$ and calculate the feed composition $X_F$ that resulted in the overhead vapor flow meeting

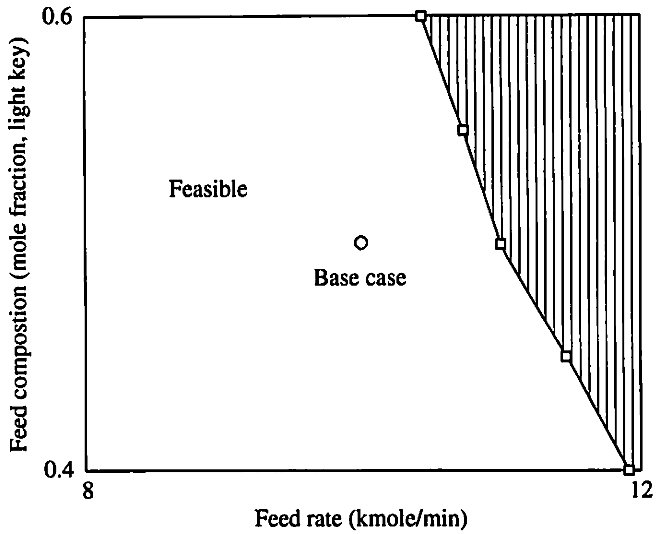



**FIGURE 20.6**

**Operating window for distillation with disturbance variables as coordinates.**

its maximum limit; this approach is possible with a steady-state model solved using an equation-based approach (Perkins, 1984). The results of the analysis, performed by either method, are the feasible values of feed rate and composition, with $X_D$ and $X_B$ maintained at their desired values; the operating window is given in Figure 20.6. Again, the interaction is apparent by the shape of the operating window. The maximum feed rate is attainable with a feed containing the least light key, because the least amount of distillate product is generated by this feed and the least distillate requires the minimum overhead vapor.

## Controllability

Another important issue in multivariable control is the independence of the input-output process relationships between selected manipulated variables ($MV_j$'s) and controlled variables ($CV_i$'s); a process in which the relationships are independent is termed *controllable*. Many definitions for the term *controllability* are used in automatic control (e.g., Franklin et al., 1990); for the purposes of this book we will use the following definition, which is appropriate for continuously operating plants that should attain steady-state conditions (a somewhat less restrictive version of Rosenbrock's (1974) "functionally controllable (f)"):

> A system is **controllable** if the controlled variables can be maintained at their set points, in the steady state, in spite of disturbances entering the system.

Controllability is defined for a selected set of manipulated and controlled variables, and a system may be controllable for one selection and uncontrollable for another selection. A system's controllability is not always easy to determine by observation; thus, a quantitative method for determining controllability is presented in this section. There is no general method for nonlinear systems; therefore, the controllability of the locally linearized system will be analyzed to evaluate the system. As a result, the results of the controllability test are strictly valid only at the operating point at which the linear model is evaluated.

The multivariable dynamic system can be described by a model of the form given in equation (20.10); only a 2 × 2 system is given, but the extension to higher orders is straightforward. We will assume that the system begins at steady state. The definition of controllability will be met if the controlled variables can be maintained at their set points, so that their deviation variables are zero, by adjusting the specified manipulated variables in the presence of steplike disturbances, which achieve a constant value, at least asymptotically. The behavior of the system at steady state can be determined through the final value theorem. As noted in Chapter 4, the final value theorem can be applied if the output is bounded, which excludes bounded input–bounded output unstable systems. Applying the final value theorem to equation (20.10), with $\mathrm{CV}_i(s) = 0$ for all $i$, gives

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} \mathrm{MV}'_1 \\ \mathrm{MV}'_2 \end{bmatrix} + \begin{bmatrix} K_{d1} \\ K_{d2} \end{bmatrix} D' \tag{20.11}$$

with $K_{ij} = \lim_{s \to 0} G_{ij}(s)$ being the steady-state gains.

The system is controllable if there is a solution for this set of linear algebraic equations for arbitrary nonzero values of $K_{d1}$, $K_{d2}$, and $D'$ (i.e., all possible disturbances).

**A solution exists for a square system of linear equations (20.11) when an inverse to the matrix of feedback process gains (K) exists; thus, the system is controllable if the determinant of the gain matrix is nonzero.**

A square physical system (numbers of manipulated and controlled variables are constant) is not controllable if any of the following conditions occurs:

1. Any two process inputs are linearly dependent (giving dependent columns).
2. Any two process outputs are linearly dependent (giving dependent rows).
3. A process output is not influenced by any input (giving a column of zeros).
4. A process input does not influence any output (giving a row of zeros).

The controllability test is applied to the two processes in the following example to ensure that they are controllable.

**EXAMPLE 20.5.**

Evaluate the controllability of the blending and distillation processes.

The gain matrices and their determinants are

Blending $\begin{bmatrix} \dfrac{-F_A}{(F_S + F_A)^2} & \dfrac{F_S}{(F_S + F_A)^2} \\ 1.0 & 1.0 \end{bmatrix}$ Determinant: $\dfrac{-F_S + F_A}{(F_S + F_A)^2} \neq 0.0$

Distillation $\begin{bmatrix} 0.0747 & -0.0667 \\ 0.1173 & -0.1253 \end{bmatrix}$ Determinant: $-0.001536 \neq 0.0$

Since each determinant is nonzero, each process is controllable for the selected manipulated and controlled variables.

Note that a controllable system indicates that the manipulated variables can compensate for effects of disturbances on selected controlled variables for some small region over which the linearization is valid and constraints are not encountered in the manipulated variables. In contrast, the operating window, which is evaluated using the nonlinear steady-state models including constraints, defines the entire possible region of operation. Both analyses should be performed to ensure the possibility of multivariable control.

Finally, the controllability and range of the system are affected by the process design and operating conditions, along with the selected controlled variables. Therefore, deficiencies in controllability and range must be compensated through changes to the equipment or process operating point, not control algorithms.

## 20.4 ▨ PROCESS INTERACTION: IMPORTANT EFFECTS ON MULTIVARIABLE SYSTEM BEHAVIOR

We now continue investigating the effects of interaction on multivariable system behavior, assuming that the process has a controllable input-output selection. The goal of this section is to demonstrate how the responses of a control system are influenced by interaction. To simplify the analysis, only relationships for two-input, two-output systems are considered, but the results obtained can be extended to control systems of higher order. Insights will be provided in this section through analyzing several examples and are formalized in the next section.

The first step is to derive the transfer function for the multiloop feedback control system and determine the main differences from single-loop control. We begin this procedure by considering the same system (1) without control, (2) with one controller, and finally (3) with two controllers. First, suppose that a single controller were to be implemented on the system in Figure 20.4, with the goal of controlling $CV_1(s)$ by adjusting $MV_1(s)$. The transfer function $G_{11}(s)$ would have to be considered when tuning the controller, as demonstrated by the transfer function:

$$\frac{CV_1(s)}{MV_1(s)} = G_{11}(s) \qquad \text{no control} \tag{20.12}$$

In this case, the control loop could be considered a single-loop system; however, changes in $MV_1(s)$ caused by the controller would affect $CV_2(s)$ because of interaction.

Next, we consider a more complex structure to determine whether it affects the first loop. The block diagram for a multivariable process with one single-loop controller is given in Figure 20.7. This example is considered to demonstrate the effects of interaction on closed-loop systems. The transfer function relating $MV_1(s)$ with $CV_1(s)$ would have to be considered when tuning the controller using these measured and manipulated variables. This transfer function follows

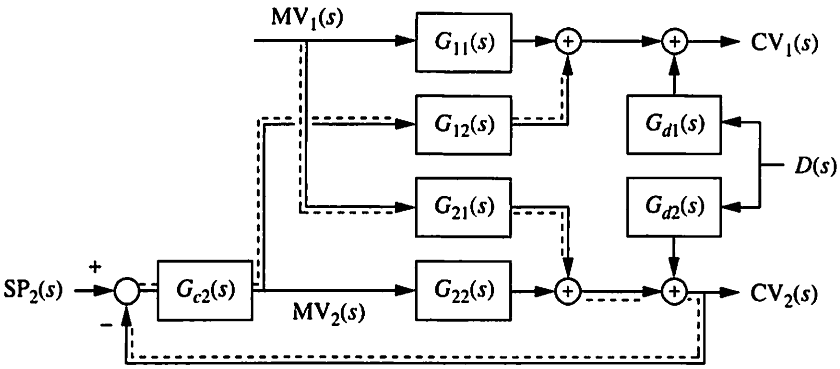





**FIGURE 20.7**

**Block diagram of 2 × 2 system with one single-loop controller.**

for the case with $G_{c2}(s)$ implemented:

$$\frac{CV_1(s)}{MV_1(s)} = G_{11}(s) - \frac{G_{12}(s)G_{21}(s)G_{c2}(s)}{1 + G_{c2}(s)G_{22}(s)} \qquad G_{c2}(s) \text{ implemented} \qquad (20.13)$$

This equation differs from the transfer function with no control of $CV_2(s)$, equation (20.12), by the second term, and the path represented by the second term is shown as a dashed line in Figure 20.7. Clearly, this path results from the process interaction and the second controller. The second term on the right-hand side in equation (20.13) would be zero if either or both $G_{12}(s)$ and $G_{21}(s)$ were zero, in which case the controller $G_{c2}(s)$ would have no effect on the transfer function for $CV_1(s)/MV_1(s)$. The path shown with the dashed line will be referred to as *transmission* interaction and will be seen to have an important influence on stability.

> **Transmission interaction** exists when a change in the set point of a controller affects its controlled variable through a path that includes another controlled variable and controller.

Note that it is possible to have process interaction [i.e., only $G_{12}(s)$ *or* $G_{21}(s)$ nonzero] without having transmission interaction, which requires *both* to be nonzero.

The control design can be completed by applying two single-loop controllers to the process, as shown in Figure 20.8. The following closed-loop transfer functions can be determined from block diagram manipulation. (The results for the other controlled variable, $CV_2(s)$, can be obtained by transposing the subscripts.)

$$\frac{CV_1(s)}{SP_1(s)} = \frac{G_{c1}(s)G_{11}(s) + G_{c1}(s)G_{c2}\left[G_{11}(s)G_{22}(s) - G_{12}(s)G_{12}(s)\right]}{CE(s)} \qquad (20.14)$$

$$\frac{CV_1(s)}{SP_2(s)} = \frac{G_{c2}(s)G_{12}(s)}{CE(s)} \qquad (20.15)$$

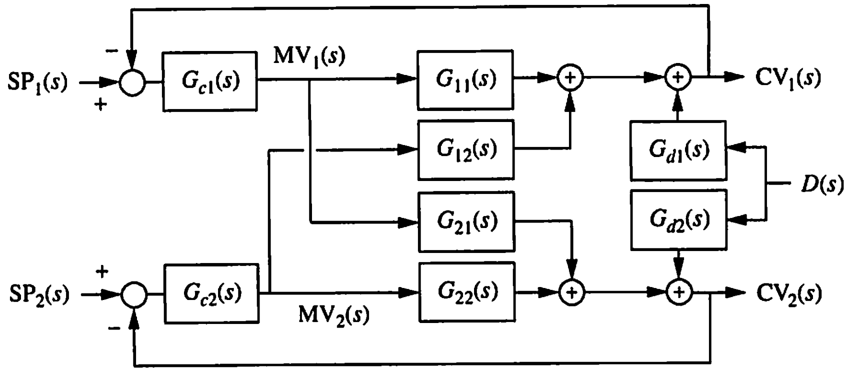



**FIGURE 20.8**

**Block diagram of 2 x 2 system with two single-loop controller.**

$$\frac{\mathrm{CV}_1(s)}{D(s)} = \frac{\left\{G_{d1}(s) - \dfrac{G_{d2}G_{12}(s)G_{c2}(s)}{[1+G_{c2}(s)G_{22}(s)]}\right\}[1+G_{c2}(s)G_{22}(s)]}{\mathrm{CE}(s)} \tag{20.16}$$

with the characteristic expression CE($s$), which is the same for equations (20.14) through (20.16),

$$\begin{aligned}\mathrm{CE}(s) = {} & 1 + G_{c1}(s)G_{11}(s) + G_{c2}(s)G_{22}(s) \\ & + G_{c1}(s)G_{c2}(s)[G_{11}(s)G_{22}(s) - G_{12}(s)G_{21}(s)]\end{aligned} \tag{20.17}$$

When both interaction terms $G_{12}(s)$ and $G_{21}(s)$ are nonzero, the dynamic response of a single-loop controller between $\mathrm{CV}_1(s)$ and $\mathrm{MV}_1(s)$ depends on all terms in the closed-loop transfer function. As a result, the stability and performance of loop 1 depend on the tuning of loop 2. By a similar argument, the stability and performance of loop 2 depend on the tuning of loop 1. Therefore,

> **The two controllers must be tuned *simultaneously* to achieved desired stability and performance.**

Further insight can be obtained by considering the steady-state behavior of the multivariable system. In particular, the necessary adjustments in the manipulated variables can be used as an indication of how interaction changes the system's behavior. The general steady-state relationship for a 2 × 2 system is expressed here in deviation variables:

$$\mathrm{CV}_1' = K_{11}\mathrm{MV}_1' + K_{12}\mathrm{MV}_2' \tag{20.18}$$

$$\mathrm{CV}_2' = K_{21}\mathrm{MV}_1' + K_{22}\mathrm{MV}_2' \tag{20.19}$$

These equations are often written in matrix form as

$$\begin{bmatrix}\mathrm{CV}_1' \\ \mathrm{CV}_2'\end{bmatrix} = \mathbf{K}\begin{bmatrix}\mathrm{MV}_1' \\ \mathrm{MV}_2'\end{bmatrix} \quad \text{with } \mathbf{K} = \begin{bmatrix}K_{11} K_{12} \\ K_{21} K_{22}\end{bmatrix} \tag{20.20}$$

Equation (20.20) can be rearranged to give

$$\begin{bmatrix} MV'_1 \\ MV'_2 \end{bmatrix} = \mathbf{K}^{-1} \begin{bmatrix} CV'_1 \\ CV'_2 \end{bmatrix} \tag{20.21}$$

where $\mathbf{K}^{-1}$ is the inverse of the steady-state gain matrix and exists for a controllable system. Note that equation (20.21) represents the calculation performed by the controller with zero steady-state offset. For example, equation (20.21) could be used to determine the steady-state changes in $MV'_1$ and $MV'_2$ for any specified changes in $CV'_1$ and $CV'_2$ (i.e., set point changes). Several hypothetical systems are considered first so that the extent of interaction can be changed incrementally from the base model; then some realistic processes are considered.

The process gain matrices in Table 20.1 represent hypothetical systems with various extents of interaction: A has no interaction ($K_{12} = K_{21} = 0$); B has moderate transmission interaction; C has strong transmission interaction; D is not controllable (the determinant of the gain matrix is zero), and E has one-way interaction ($K_{21} = 0$). Thus, their behaviors are expected to vary. In particular, multivariable control is not possible with system D, because it is not possible to

**TABLE 20.1**

**Summary of manipulated-variable changes for example systems with differing amounts of interaction**

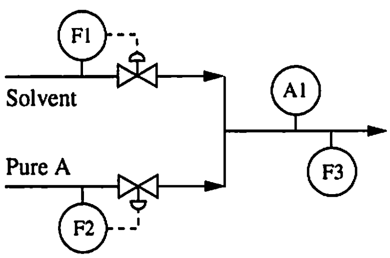
| System | Process gain matrix K | Inverse gain matrix $\mathbf{K}^{-1}$ | $CV'_1 = 1.0$, $CV'_2 = 0.0$ |
|---|---|---|---|
| A<br>No interaction | $\begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}$ | $\begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}$ | $MV'_1 = 1.0$<br>$MV'_2 = 0.0$<br>Same as single-loop |
| B<br>Moderate transmission interaction | $\begin{bmatrix} 1.0 & 0.75 \\ 0.75 & 1.0 \end{bmatrix}$ | $\begin{bmatrix} 2.29 & -1.71 \\ -1.71 & 2.29 \end{bmatrix}$ | $MV'_1 = 2.29$<br>$MV'_2 = -1.71$<br>Larger than single-loop |
| C<br>Strong transmission interaction | $\begin{bmatrix} 1.0 & 0.90 \\ 0.90 & 1.0 \end{bmatrix}$ | $\begin{bmatrix} 5.26 & -4.74 \\ -4.74 & 5.26 \end{bmatrix}$ | $MV'_1 = 5.26$<br>$MV'_2 = -4.74$<br>Much larger than single-loop |
| D<br>Not controllable | $\begin{bmatrix} 1.0 & 1.0 \\ 1.0 & 1.0 \end{bmatrix}$ | Singular; inverse does not exist | |
| E<br>One-way interaction | $\begin{bmatrix} 1.0 & 1.0 \\ 0.0 & 1.0 \end{bmatrix}$ | $\begin{bmatrix} 1.0 & -1.0 \\ 0.0 & 1.0 \end{bmatrix}$ | $MV'_1 = 1.0$<br>$MV'_2 = 0.0$<br>Same as single-loop |

control $CV'_1$ and $CV'_2$ independently. This system is not considered further, because a process design change would be required to control the selected controlled variables.

The changes in the manipulated variables required for the specified CV changes are given in Table 20.1. The manipulated variable changes for systems B and C with transmission (two-way) interaction differ from the single-loop values reported as system A. Also, the differences in manipulated variable behavior from system A increase with increases in the interaction terms. For cases reported in the table, the manipulations for systems B and C are greater than those for system A, but for other specified CV changes, systems B and C could be smaller than system A. The following points summarize the major differences in steady-state behavior between single-loop and multivariable systems.

**1. The values of the manipulated variables that satisfy the desired controlled variables must be determined simultaneously.**
**2. Differences between single-variable and multivariable behavior increase as the transmission (two-way) interaction increases.**

Before we conclude this section, two examples of process gain matrices are considered. These examples demonstrate that the behavior shown in Table 20.1 occurs in realistic chemical processes.




**EXAMPLE 20.6.**

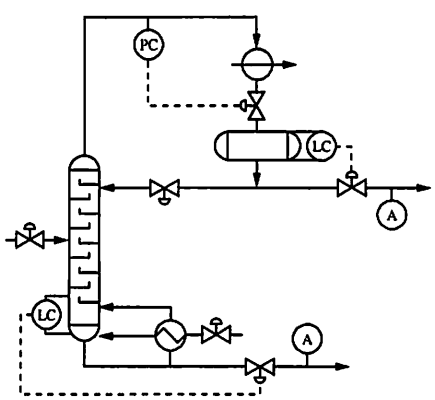
The first is the blending system shown in Figure 20.2, where the product flow and composition are controlled by adjusting the flows of the two component streams. The gains are determined from the linearized model in equations (20.6) and (20.7). The base conditions are taken to be

$$F_1 = 95.0 \text{ kg/min} \qquad F_2 = 5.0 \text{ kg/min}$$
$$A_1 = 0.05 \text{ wt fraction A} \qquad F_3 = 100 \text{ kg/min} \tag{20.22}$$

The gain matrix and its inverse for these conditions are

$$\begin{bmatrix} A_1 \\ F_3 \end{bmatrix} = \begin{bmatrix} -0.0005 & 0.0095 \\ 1.0 & 1.0 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \qquad \mathbf{K}^{-1} = \begin{bmatrix} -100 & 0.95 \\ 100 & 0.05 \end{bmatrix} \tag{20.23}$$

The gain and inverse matrices have one element that is nearly zero. Thus, the system is likely to behave similar to system E in Table 20.1. As a result, this system is not expected to experience very strong departures from the single-variable behavior in manipulated-variable adjustment magnitudes.




**EXAMPLE 20.7.**

The second example is the binary distillation tower in Figure 20.3, where the product compositions are controlled by adjusting the reflux and reboiler flows. The steady-state gains can be taken from the transfer function matrix in equations (20.8) and (20.9).

$$\mathbf{K} = \begin{bmatrix} 0.0747 & -0.0667 \\ 0.1173 & -0.1253 \end{bmatrix} \qquad \mathbf{K}^{-1} = \begin{bmatrix} 81.58 & -43.42 \\ 76.36 & -48.63 \end{bmatrix} \tag{20.24}$$

The distillation tower appears highly interactive in the two-way manner similar to systems B and C. To complete this distillation example, steady-state changes in manipulated variables are calculated for single-loop and multivariable control. In both cases, the bottoms mole fraction of light key is to be decreased by 0.01. In the first case, only the bottoms mole fraction is specified and the distillate mole fraction is not controlled. This is single-loop control, and the necessary change in vaporization is

$$\text{Single-loop:} \qquad \Delta F_V = \frac{\Delta X_B}{K_{XB,V}} = \frac{-0.01}{-0.1253} = 0.0798 \text{ kmole/min}$$

Since the bottoms composition is not controlled, $\Delta F_R = 0$ and $\Delta X_D \neq 0$. In the alternative multivariable case, the distillate mole fraction is maintained unchanged ($\Delta X_D = 0$), while the bottoms composition is changed by −0.01.

$$\text{Multivariable:} \qquad \begin{bmatrix} \Delta F_R \\ \Delta F_V \end{bmatrix} = \begin{bmatrix} 81.53 & -43.42 \\ 76.36 & -48.63 \end{bmatrix} \begin{bmatrix} 0 \\ -0.01 \end{bmatrix} = \begin{bmatrix} 0.4343 \\ 0.4863 \end{bmatrix}$$

The results demonstrate that the change in the vaporization in the reboiler was much larger in magnitude for the multivariable system (0.4863 compared with 0.0798 kmole/min), and in addition, a large change in reflux was required. Clearly, the interaction has strongly affected the steady-state behavior of the system.

In conclusion, interaction can strongly influence the steady-state and dynamic behavior of multivariable systems. There exists a range of interaction from completely independent through nearly dependent (i.e., nearly singular), with this interaction dependent on the *process characteristics*, not on control. In general, the closer the system approaches singularity (system D in Table 20.1), the more its behavior differs from that of independent loops. The final two process examples demonstrated that real processes can have interaction similar to the range of examples in Table 20.1. In the next section, a quantitative measure of interaction is introduced.

## 20.5 □ PROCESS INTERACTION: THE RELATIVE GAIN ARRAY (RGA)

As shown in the previous section, process interaction is an important factor influencing the behavior of multivariable systems. A quantitative measure of interaction is needed to proceed with a multiloop analysis method, and the relative gain array, which has proved useful in control system analysis, is introduced to meet this need. The relative gain array was developed by Bristol (1966) and extended by many engineers, most notably Shinskey (1988) and McAvoy (1983*b*). In this section, the relative gain is defined, special properties and methods for calculation are given, and interpretations for control analysis are presented. The relative gain array (RGA) is a matrix composed of elements defined as ratios of open-loop to closed-loop gains as expressed by the following equation, which relates the *j*th input and the *i*th output.

$$\lambda_{ij} = \frac{\left(\dfrac{\partial \text{CV}_i}{\partial \text{MV}_j}\right)_{\text{MV}_k=\text{const},k\neq j}}{\left(\dfrac{\partial \text{CV}_i}{\partial \text{MV}_j}\right)_{\text{CV}_k=\text{const},k\neq i}} = \frac{\left(\dfrac{\partial \text{CV}_i}{\partial \text{MV}_j}\right)_{\text{other loops open}}}{\left(\dfrac{\partial \text{CV}_i}{\partial \text{MV}_j}\right)_{\text{other loops closed}}} \qquad (20.25)$$

Consistent with prior terminology, the open-loop gain ($K_{ij}$) is the change in output $i$ for a change in input $j$ with all other inputs constant (for stable processes). By *closed-loop gain* we mean the steady-state relationship between $MV'_j$ and $CV'_i$ with all other control loops closed (i.e., in automatic). In this definition, it is assumed that the controllers have an integral mode so that the steady-state values of the controlled variables are maintained constant (i.e., $CV'_k = 0$ for those under feedback control). If the relative gain is 1.0, the process gain is unaffected by the other control loops and no (transmission) interaction exists. Thus, the amount that the relative gain deviates from 1.0 indicates, in some sense, the "extent" of transmission interaction in a quantitative manner.

Before control-relevant interpretations of the relative gain are developed, some important properties must be noted:

1. The relative gain is scale-independent. This is important because rules for interpretation do not change when the units of a variable change (e.g., from percent to parts per million).
2. The expression in equation (20.25) suggests that both open- and closed-loop data is required to determine the relative gain. However, the relative gain can be calculated from the open-loop data alone, which can be demonstrated by rearranging equation (20.25) to give

$$\lambda_{ij} = \left(\frac{\partial \mathrm{CV}_i}{\partial \mathrm{MV}_j}\right)_{\mathrm{MV}_k=\mathrm{const}, k\neq j} \left(\frac{\partial \mathrm{MV}_j}{\partial \mathrm{CV}_i}\right)_{\mathrm{CV}_k=\mathrm{const}, k\neq i} \tag{20.26}$$

   The procedure for calculating the relative gain array is to evaluate the open-loop gain matrix $\mathbf{K}$; calculate its inverse transposed $(\mathbf{K}^{-1})^{\mathrm{T}}$; and multiply them in an *element-by-element manner.* This type of matrix multiplication is referred to as the *Hadamard product* (McAvoy, 1983*b*). The following expression gives the result for each element in the relative gain array, with $K_{ij}$ being the elements in the gain matrix and $\mathrm{KI}_{ij}$ being the elements of the inverse of the gain matrix,

$$\lambda_{ij} = K_{ij}\mathrm{KI}_{ji} \tag{20.27}$$

   For a $2 \times 2$ system, the (1,1) element of the relative gain array can be shown to be

$$\lambda_{11} = \frac{1}{1.0 - \dfrac{K_{12}K_{21}}{K_{11}K_{22}}} \tag{20.28}$$

3. The rows and columns of the relative gain array sum to 1.0. This property enables $2 \times 2$ systems to be characterized by the $\lambda_{11}$ element, as follows:

$$\begin{array}{c|cc} & \mathrm{MV}_1 & \mathrm{MV}_2 \\ \hline \mathrm{CV}_1 & \lambda_{11} & 1-\lambda_{11} \\ \mathrm{CV}_2 & 1-\lambda_{11} & \lambda_{11} \end{array} \tag{20.29}$$

4. The relative gain calculation can be very sensitive to errors in the gain calculation. As an example, consider the following relative gain for a $2 \times 2$ process, and assume that each process gain can be in error by a factor $\epsilon_{ij}$, which is 1.0

for no error.

$$\lambda_{11} = \frac{1}{1.0 - \dfrac{(K_{12}\epsilon_{12})(K_{21}\epsilon_{21})}{(K_{11}\epsilon_{11})(K_{22}\epsilon_{22})}} \tag{20.30}$$

When the relative gain element has a large magnitude, the relative gain can take widely varying values and can even change sign for small errors in individual process gains, as shown by the following example cases. In this example, the actual values for the gains are $K_{11} = K_{22} = 1.0$ and $K_{12} = K_{21} = 0.949$, and the erroneous relative gain is shown for a few example sets of gain errors.

| True $\lambda_{11}$ | $\epsilon_{11}$ | $\epsilon_{12}$ | $\epsilon_{21}$ | $\epsilon_{22}$ | $\lambda_{11}$ calculated with model errors | |
|---|---|---|---|---|---|---|
| 10 | 1.0 | 1.0 | 1.0 | 1.0 | 10.0 | No error |
| 10 | 1.0 | 1.1 | 1.0 | 1.0 | 100.0 | |
| 10 | 1.0 | 1.2 | 1.0 | 1.0 | −16.6 | |
| 10 | 0.97 | 1.03 | 1.03 | 0.97 | −7.8 | Only 3% errors |

Since the sign of the relative gain is of great importance in control design decisions, the sensitivity to model errors demonstrated as the foregoing property 4 must be considered, to prevent incorrect results. Thus, great accuracy is required in the process gains used for calculating the relative gain. Probably the best method is to derive an analytical model and evaluate the process gains from analytical derivatives. This can be done for the blending example using the linearized model and the foregoing property 2:

$$[\lambda_{ij}] = \begin{bmatrix} \dfrac{F_1}{F_1 + F_2} & \dfrac{F_2}{F_1 + F_2} \\ \dfrac{F_2}{F_1 + F_2} & \dfrac{F_1}{F_1 + F_2} \end{bmatrix} \tag{20.31}$$

However, few complex industrial processes can be accurately modelled by sets of equations small enough to be conveniently manipulated analytically by hand, although advances in algebraic processing by computers could change this situation in the future. Thus, numerical differentiation using steady-state process simulators is a common approach to evaluating process gains. In this procedure, a separate simulation is performed at the base case and at a case with each input $MV_j$ changed a small amount from the base case. The process gains are calculated using the equation below, and the relative gain array is determined from equation (20.27):

$$K_{ij} \approx \frac{CV_i(MV_1, MV_2, \ldots, MV_j + \Delta MV_j, \ldots) - CV_i(MV_1, MV_2, \ldots)}{\Delta MV_j} \tag{20.32}$$

Special care is required when using this method because of the accuracy required for the relative gain. When numerical differentiation is used, two potential causes of errors are introduced: the convergence tolerances in solving the equations and the use of approximate rather than exact derivatives. As demonstrated by McAvoy (1983*b*), the convergence tolerances and ΔMVs used in equation (20.32) in calculating the approximate gain must be reduced until the estimated process gains are not significantly influenced by further reductions in their values. The conclusion of this analysis can be stated as follows:

> The gains $K_{ij}$ used for calculating the relative gains must be accurate; the use of gains from linearized fundamental models is recommended. Given the typical errors in empirical model identification, the use of empirically determined process gains using methods in Chapter 6 is *not recommended* for calculating relative gains.

Some very useful control-related interpretations based on the RGA are summarized as follows and will be used in a hierarchical analysis procedure in the next chapter.

| | |
|---|---|
| $\lambda_{ij} < 0$ | In this case, the open- and closed-loop process gains are of different signs. In a 2 × 2 process, if the single-loop ($CV_i - MV_j$) controller gain were positive for stable feedback control, the same controller gain would have to be negative for stable multiloop feedback control. Thus, the sign of the controller gain to retain stability would depend on the mode of other controllers in the multiloop system—not a desirable situation. |
| $\lambda_{ij} = 0$ | One situation in which the relative gain is zero occurs when the open-loop process gain ($\Delta CV_i/\Delta MV_j$ with the other loops open) is zero, which indicates no steady-state relationship between the input and output variables. Thus, the controller with this pairing can function, if at all, only when other controllers are in automatic. Again, this is not generally a desirable situation but is acceptable in special circumstances, as explained in the next chapter. |
| $0 < \lambda_{ij} < 1$ | From equation (20.25), the steady-state loop process gain with the other loops closed [e.g., equation (20.13)] is larger than the same process gain with the other loops open. |
| $\lambda_{ij} = 1$ | In this situation, there is no transmission interaction, in the sense that the *product* of $K_{12}K_{21}$ is zero, but either one of the terms may be nonzero. Thus, a change in $MV_j(s)$ is transmitted to $CV_i(s)$ only through $G_{ij}(s)$. Note that this does not preclude the possibility that the manipulated variable might affect another controlled variable (i.e., one-way interaction). |
| $\lambda_{ij} > 1$ | From equation (20.25), the steady-state loop process gain with the other loops closed [e.g., equation (20.13)] is smaller than the same process gain with the other loops open. |
| $\lambda_{ij} = \infty$ | When the process gain is zero with the other loops closed, it is not possible to control the variable in a multiloop system. |

As examples, the relative gains for all cases in Table 20.1 and the two process examples are reported here. (Note that the model for the distillation tower was developed from very small perturbations in the nonlinear model without noise; the probability of obtaining an accurate relative gain value from empirical model fitting is quite small.)

| System | Relative gain, $\lambda_{11}$ | |
|---|---|---|
| A | 1.0 | |
| B | 2.29 | |
| C | 5.26 | |
| D | $\infty$ | |
| E | 1.0 | |
| Blending | $\lambda_{A1-F2} = 0.95$ | Operating conditions in equation (20.22) |
| Distillation | $\lambda_{XD-FR} = 6.09$ | Operating conditions in Figure 20.3 |

These values are consistent with the previous, qualitative evaluations of interaction in that systems with relative gain deviating most from 1.0 deviate most from single-loop behavior. Note that system E with only one-way interaction has $\lambda_{11} = 1.0$; in general, the relative gain array is the identity matrix for systems with a steady-state gain matrix that is lower (or upper) diagonal [i.e., with nonzero entries only on and below (or above) the diagonal].

Finally, the relative gain can be related directly to the closed-loop transfer function of a $2 \times 2$ system. To do this, the definition of relative gain has been extended by Witcher and McAvoy (1977) to include frequency-dependent terms by replacing the steady-state gains with the corresponding transfer functions. Thus, the frequency-dependent relative gain is

$$\lambda_{11}(s) = \frac{1}{1 - \dfrac{G_{12}(s)G_{21}(s)}{G_{11}(s)G_{22}(s)}} \tag{20.33}$$

This expression can be used, by setting $s = j\omega$, to evaluate the magnitude of the relative gain elements at various frequencies.

Using the foregoing expression, the characteristic expression (20.17) can be rewritten as

$$\text{CE}(s) = 1 + G_{c1}(s)G_{11}(s) + G_{c2}(s)G_{22}(s) + \frac{G_{c1}(s)G_{c2}(s)G_{11}(s)G_{22}(s)}{\lambda_{11}(s)} \tag{20.34}$$

This analysis demonstrates the fundamental nature of the relative gain and the close relationship between the relative gain and system stability for $2 \times 2$ systems.

A summary of the key results for the relative gain array follows:

1. The deviation from single-loop behavior, specifically the transmission interaction, is related to the difference of the relative gain element from the value of 1.0.
2. The condition of $\lambda_{ij} \leq 0$ results in multiloop systems that, to maintain acceptable performance, must alter the ($\text{CV}_i - \text{MV}_j$) controller gain or automatic status, depending on the status of other controllers.
3. A direct relationship between frequency-dependent relative gain and control system stability has been demonstrated for $2 \times 2$ systems.

## 20.6 ▣ EFFECT OF INTERACTION ON STABILITY AND TUNING OF MULTILOOP CONTROL SYSTEMS

The final major topic in this chapter is controller tuning. Analysis of the closed-loop transfer function demonstrates that interaction influences the characteristic equation and, therefore, stability; thus, controller tuning must consider interaction as well as the single-loop feedback process dynamics. The following example provides further insight into the effect of interaction on stability and tuning.

**EXAMPLE 20.8.**

A dynamic system with the following model is to be controlled by two PI controllers. The input-output pairings are 1–1 and 2–2 as shown in Figure 20.8. Determine the allowable range of tuning constants that yield a stable system.

$$\begin{bmatrix} \mathrm{CV}_1(s) \\ \mathrm{CV}_2(s) \end{bmatrix} = \begin{bmatrix} \dfrac{1.0e^{1.0s}}{1+2s} & \dfrac{0.75e^{-1.0s}}{1+2s} \\ \dfrac{0.75e^{-1.0s}}{1+2s} & \dfrac{1.0e^{-1.0s}}{1+2s} \end{bmatrix} \begin{bmatrix} \mathrm{MV}_1(s) \\ \mathrm{MV}_2(s) \end{bmatrix} \tag{20.35}$$

The example system has transmission interaction, because both off-diagonal elements are nonzero; thus, it would not be correct to tune each controller independently. The stability limit is determined by the characteristic expression, given in equation (20.17). Finding the limiting values of the tuning constants would be an arduous task because all four controller tuning constants ($K_{c1}$, $T_{I1}$, $K_{c2}$, and $T_{I2}$) appear in the characteristic equation and, therefore, all affect stability *simultaneously.* To simplify the calculations and allow graphical presentation of the results, the integral times of the controllers will be held constant at 3.0 min, which are reasonable values, being the sum of the dead time and time constant of each transfer function. Note that this selection will not necessarily yield the best performance, but it is a reasonable choice for this example calculation.

With the integral times fixed, the characteristic equation has two remaining tuning parameters, the controller gains.

$$\mathrm{CE}(s) = 1 + G_{\mathrm{OL}}(s) \tag{20.36}$$

where

$$G_{\mathrm{OL}}(s) = K_{c1}\left(1+\frac{1}{3s}\right)\left(\frac{1.0e^{-1.0s}}{1+2s}\right) + K_{c2}\left(1+\frac{1}{3s}\right)\left(\frac{1.0e^{-1.0s}}{1+2s}\right)$$
$$+ K_{c1}\left(1+\frac{1}{3s}\right)K_{c2}\left(1+\frac{1}{3s}\right)\left(\frac{1.0e^{-1.0s}}{1+2s}\frac{1.0e^{-1.0s}}{1+2s} - \frac{0.75e^{-1.0s}}{1+2s}\frac{0.75e^{-1.0s}}{1+2s}\right)$$

To calculate the stability region, one gain (e.g., $K_{c2}$) was given a value, and the Bode stability analysis was performed to determine the ultimate value of $K_{c1}$ that defines the stability limit. These calculations involve extensive manipulations of complex numbers and were therefore performed using a computer program. The results of the calculations are displayed in Figure 20.9. If there had been no interaction, the stability region would have encompassed the entire box defined by values of the controller gains of (0,0) and (3.76, 3.76) shown in the figure, because the tuning of one controller would not have influenced the tuning of the other. As can be seen, the interactions in this example *reduced* the allowable values for the controller gains.

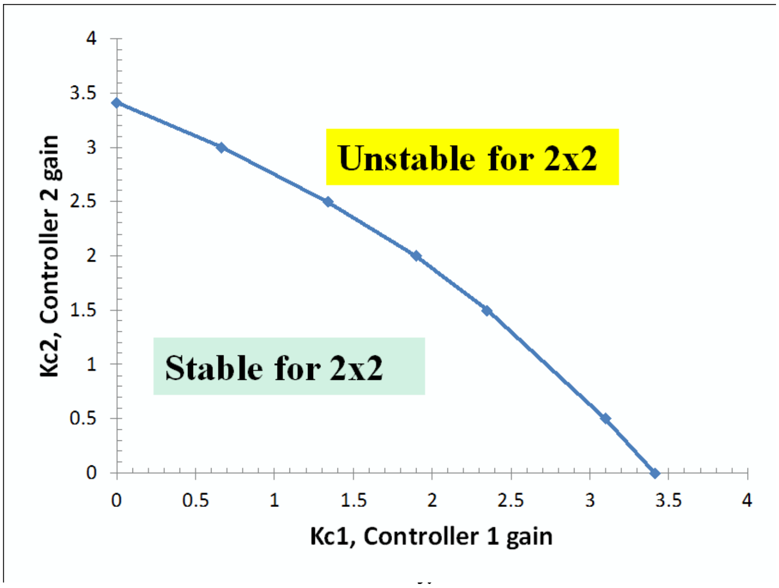



**FIGURE 20.9**

**Map of stable and unstable controller gain regions for Example 20.8 with $T_{I1} = T_{I2} = 3.0$.**

**TABLE 20.2**

**Summary of example tuning for 2 × 2 system**

| Case | $K_{c1}$ | $T_{I1}$ | $K_{c2}$ | $T_{I2}$ | $IAE_1$ | $IAE_2$ | $IAE_1 + IAE_2$ |
|---|---|---|---|---|---|---|---|
| Figure 20.10*a* | 0.95 | 3.0 | 0.95 | 3.0 | 7.22 | 5.41 | 12.63 |
| Figure 20.10*b* | 1.40 | 3.0 | 0.50 | 3.0 | 4.90 | 10.3 | 15.2 |
| Figure 20.10*c* | 0.50 | 3.0 | 1.40 | 3.0 | 13.7 | 3.67 | 17.37 |
| Figure 20.11 | 1.23 | 1.76 | 0.89 | 1.06 | 3.46 | 2.46 | 5.92 |

**EXAMPLE 20.9.**

Although all tuning within the defined region yields a stable system, the control performance is different for various tunings chosen from within the stable region. To investigate by example the effect of tuning on performance, three sets of tuning constants were chosen for the system in Example 20.8 from within the stable area shown in Figure 20.9. The tuning was selected to have a reasonable gain margin (i.e., margin from the stability boundary). The simulation results for multiloop PI controllers responding to a $CV_1$ set point change of 1.0 for three different tuning constants are given in Figure 20.10*a* through *c* and tabulated in Table 20.2. Figure 20.10*a* gives equal weight to both controlled variables. Figure 20.10*b* gives more importance to controlled variable 1, whereas Figure 20.10*c* gives more importance to controlled variable 2. These results demonstrate that controller tuning influences multiloop system performance, so tuning can be used as a method for adapting system performance to conform to specific priorities in the importance of controlled variables. This result will be exploited in the next chapter.

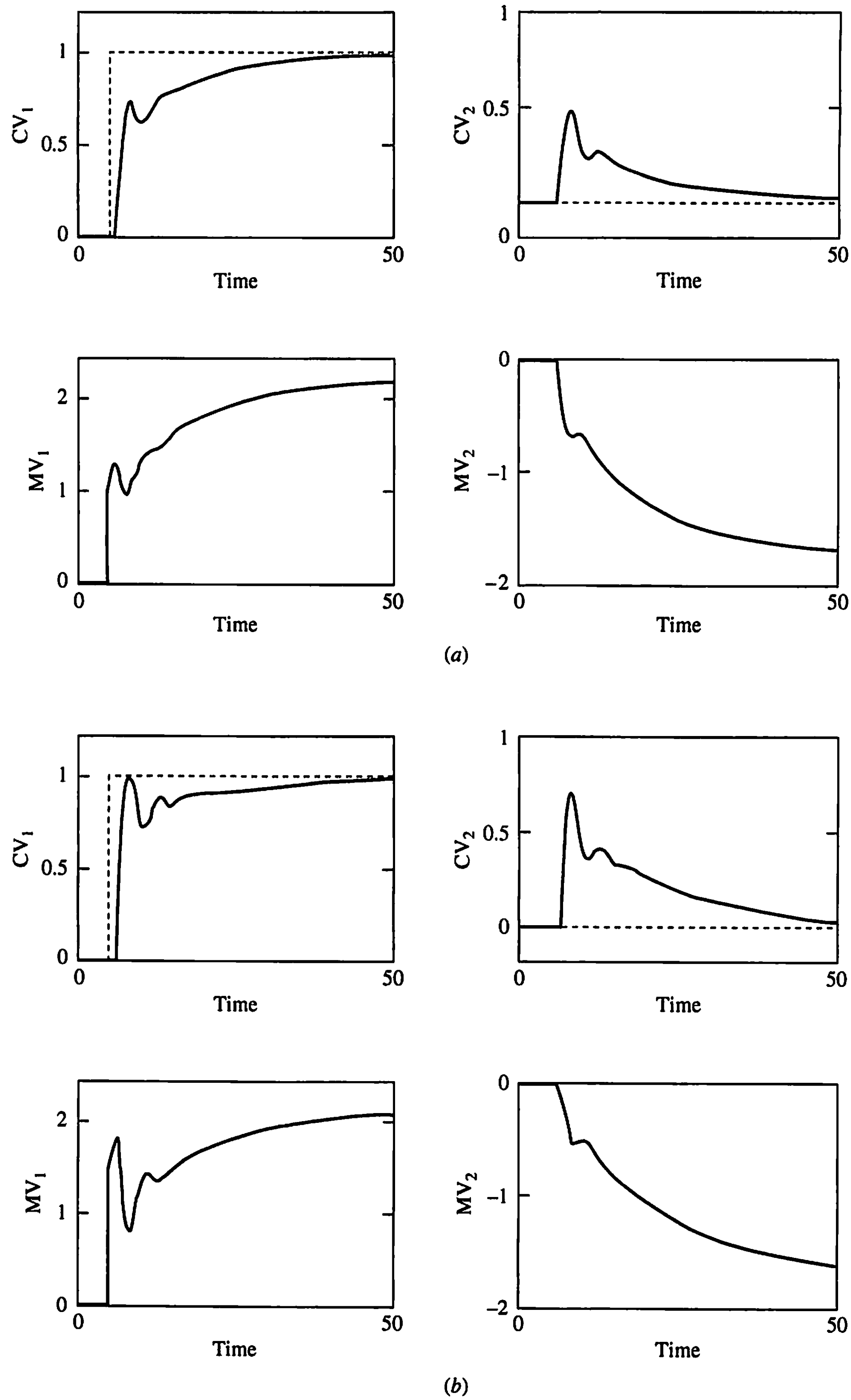
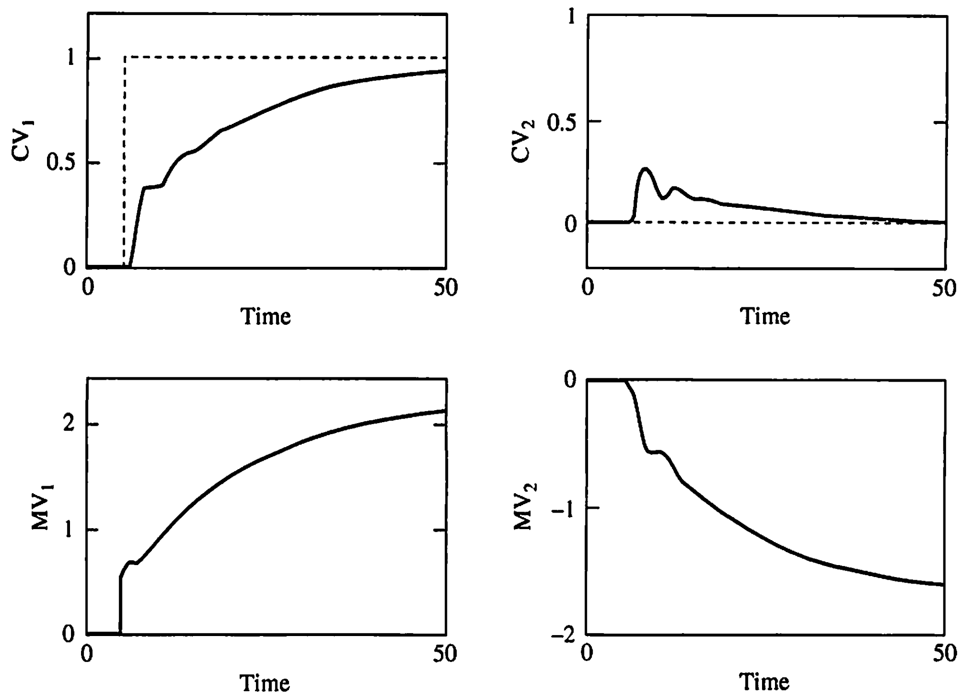



**FIGURE 20.10**

**Multiloop control: (*a*) with the same gains for both controllers; (*b*) with loop 1 gain higher.**




(*c*)

**FIGURE 20.10 Con't.**

**Multiloop control (*c*) with loop 2 gain higher.**

Since tuning influences performance, the engineer should be able to use this flexibility to obtain good control system performance. Three approaches are typically used for tuning multiloop systems, and each is described here.

## Trial and Error

Although potentially tedious, a trial-and-error method is often used in practice. Initial tuning constant values are typically the single-loop values altered for stability, perhaps with the gains reduced by a factor of 2 or more. These initial values are adjusted through fine tuning, as described in Chapter 9, with trials performed on a simulation or directly on the process. The final tuning must be conservative (i.e., not too close to the stability margin) to account for changes in process operating conditions that would occur after the trial-and-error procedure has been completed. Naturally, the success of this approach depends on the expertise of the engineer, but the approach can reach reasonable results quickly when transmission interaction is not too strong.

## Optimization

An optimization approach, similar to the approach described in Chapter 9 that optimized a simulated transient response, can be implemented to automate the trial-and-error procedure. This approach would require a computer optimization of the simulated transient response to obtain good initial values for each control system (Edgar and Himmelblau, 1988). Optimization is justified when process interaction is strong and the trial-and-error method would be time-consuming or result in severe process disturbances.

As an example, the tuning for the control system considered in Example 20.9 was optimized for a unit step change in controlled variable 1, assuming equal importance of the two controlled variables and no other objectives; therefore, the objective was to minimize the total integral of absolute value of errors ($IAE_1$ + $IAE_2$). The tuning and transient response are given in Figure 20.11 and included in Table 20.2. The optimization method yielded initial estimates with little engineering effort and modest computing resources. The reader is cautioned that the results in Figure 20.11 are *not satisfactory*, because of the lack of robustness and the very aggressive manipulated variable adjustments; a more complete definition of control performance, including these factors, should be used. However, it does provide a useful bound for the lowest IAE that can be attained with PI control.

## Approximate, Noniterative Approach

A few methods have been proposed for estimating the tuning for multiloop systems without the time-consuming iterations associated with trial and error or the computer computations associated with the optimization approach. The goal of these methods is to provide initial tuning constants that are much closer than single-loop tuning constants to the "best" multiloop values. Naturally, fine tuning based on

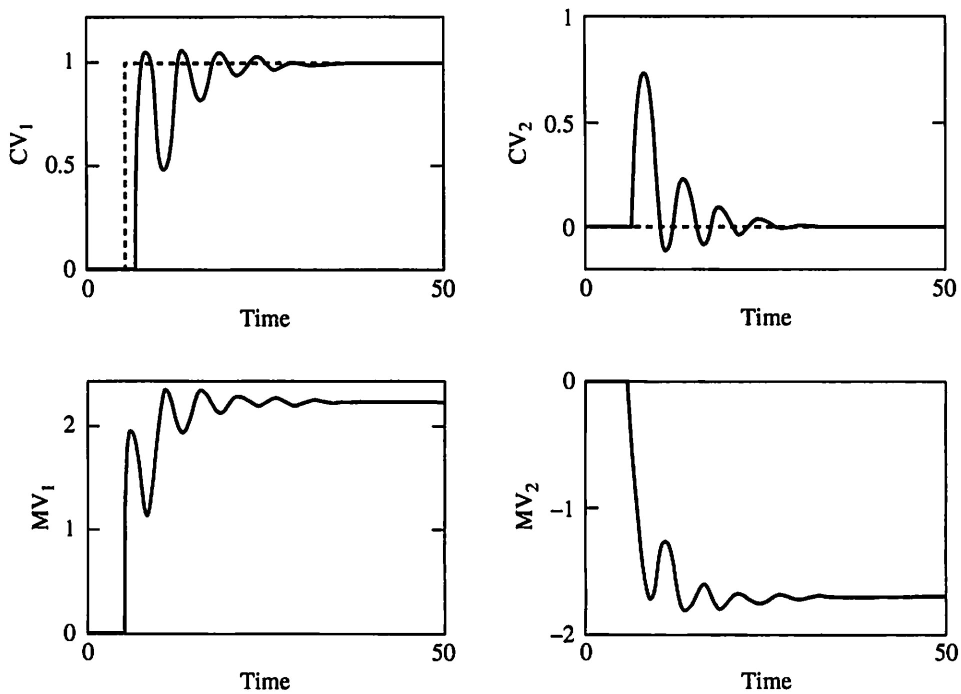



**FIGURE 20.11**

**Multiloop control with PI tuning that minimizes $\sum$ IAE (tuning is too aggressive).**

plant experience is still required. Unfortunately, there is no generally accepted method for quickly estimating multiloop tuning. The method explained here is selected because it provides insight and introduces some key process-related issues. It also provides a useful correlation for many 2 × 2 systems; however, it is not easily extended to higher-order systems.

The method takes advantage of simplifications to determine the tuning for three cases of limiting process dynamics for 2 × 2 systems with PI multiloop controllers (McAvoy, 1983*a*, and Marino-Galarraga et al., 1987). In all of these cases, the relative importances of the controlled variables are considered equal; this is the most demanding case for tuning, but other situations are considered in the next chapter as we tailor the performance to control objectives. The general approach is to establish how much the PI controller tuning must be changed from single-loop values when applied in a multiloop system.

The basis of the analysis is the closed-loop characteristic expression (20.34) divided by $1 + G_{c2}(s)G_{22}(s)$, which does not change the stability limit:

$$\mathrm{CE}(s) = 1 + G_{c1}(s)G_{11}(s)\left[\frac{1 + G_{c2}(s)G_{22}(s)/\,\lambda_{11}(s)}{1 + G_{c2}(s)G_{22}(s)}\right] \tag{20.37}$$

As demonstrated in Chapter 10, the closed-loop characteristic expression given by equation (20.37) determines the stability of the control system. To evaluate potential simplifications, the relative importance of each term must be determined at the critical frequency of the loop. Since the approach is based on stability analysis, which considers only the denominator of the closed-loop transfer function, the same tuning is obtained for all disturbances and set point changes. The method considers three limiting cases for tuning loop 1: loop 1 much faster than loop 2; loop 1 much slower; and both loops having the same dynamics. (The following analysis considers loop 1, but the same results can be obtained for loop 2 by simply transposing the subscripts.)

**LOOP 1 MUCH FASTER THAN LOOP 2.** When the loop 1 process is much faster than loop 2, the term $G_{c2}(j\omega)G_{22}(j\omega)$ is very small at the loop 1 critical frequency because of the tendency of processes to have amplitude ratios that decrease rapidly after the corner frequency (for example, see Figure 10.13*b*). Assuming that $\lambda_{11}$ is not a strong function of frequency, as is most often true,

$$\left|\frac{1 + G_{c2}(j\omega)G_{22}(j\omega)/\lambda_{11}}{1 + G_{c2}(j\omega)G_{22}(j\omega)}\right| \approx 1.0 \tag{20.38}$$

which gives

$$\mathrm{CE}(j\omega) \approx 1 + G_{c1}(j\omega)G_{11}(j\omega) \tag{20.39}$$

Therefore, the very fast loop 1 in this case can be tuned like a single-loop controller without interaction.

This result confirms a qualitative argument in which we would consider the interaction from the slow loop to be a slow disturbance to the very fast loop 1, which could be tuned using single-loop methods.

**LOOP 1 MUCH SLOWER THAN LOOP 2.** When loop 1 is much slower, the term for the fast controller, $G_{c2}(j\omega)$, would have a very large magnitude at the critical frequency of loop 1, because the amplitude ratio of the integral mode in $G_{c2}(j\omega)$ will have a very high value at a frequency much less than the loop 2 critical frequency (see Figure 10.13*f*). Therefore, $|G_{c2}(j\omega)| \gg 1.0$, which leads to the following simplification in the characteristic equation:

$$\mathrm{CE}(j\omega) \approx 1 + G_{c1}(j\omega)G_{11}(j\omega)\left[\frac{G_{c2}(j\omega)G_{22}(j\omega)/\lambda_{11}}{G_{c2}(j\omega)G_{22}(j\omega)}\right] \tag{20.40}$$

$$\mathrm{CE}(j\omega) \approx 1 + G_{c1}(j\omega)\frac{G_{11}(j\omega)}{\lambda_{11}(j\omega)} \approx 1 + G_{c1}(j\omega)\frac{G_{11}(j\omega)}{\lambda_{11}} \tag{20.41}$$

As noted, the steady-state relative gain has been used as an approximation for the frequency-dependent relative gain. In this case, the gain of the process "seen" by the controller 1 in the multiloop system is changed by $1/\lambda_{11}$ from the single-loop gain ($K_{11}$). Therefore,

**The slow controller gain can be modified to be the product of the relative gain and the single-loop tuning, $K_{cML} = (\lambda_{11})(K_{cSL})$, to maintain the desired stability margin. Since the phase lag is not affected, the integral time can retain its single-loop value.**

Again, this result seems consistent with a qualitative argument that a very fast associated loop would "become part of the process" and affect only the closed-loop process gain.

The tuning result for the slow loop has a *potential flaw*. When the relative gain has a value much different from 1.0, the controller gains for the single-loop and multiloop situations have very different values. Thus, the correct value for the controller gain depends whether an interacting controller is in automatic or manual! To ensure that the stability of the slow loop does not depend on the status of the interacting loop, the slow loop's controller gain is often limited by its single-loop value, $K_{cML} \le K_{cSL}$. If this limit is significantly exceeded, a real-time computer program could be implemented to monitor the status of the interacting loop and adjust the controller gain of the slow loop accordingly.

**LOOPS 1 AND 2 HAVE THE SAME DYNAMICS.** The entire closed-loop characteristic equation (20.34), as follows, must be considered.

$$\mathrm{CE}(s) \approx 1 + 2\Lambda(s) + \frac{\Lambda^2(s)}{\lambda_{11}} \tag{20.42}$$

with $\Lambda(s) = G_{c1}(s)G_{11}(s) = G_{c2}(s)G_{22}(s)$, because the loop dynamics are equal.

With the simplification that all transfer functions in the process, $G_{ij}(s)$, have similar dynamics, the effects of interaction on tuning are completely represented by the relative gain, and the results can be condensed into detuning correlations in Figure 20.12*a* and *b* (Marino-Galarraga et al., 1987). These figures show how single-loop tuning must be altered for 2 × 2 multiloop control when all input-output

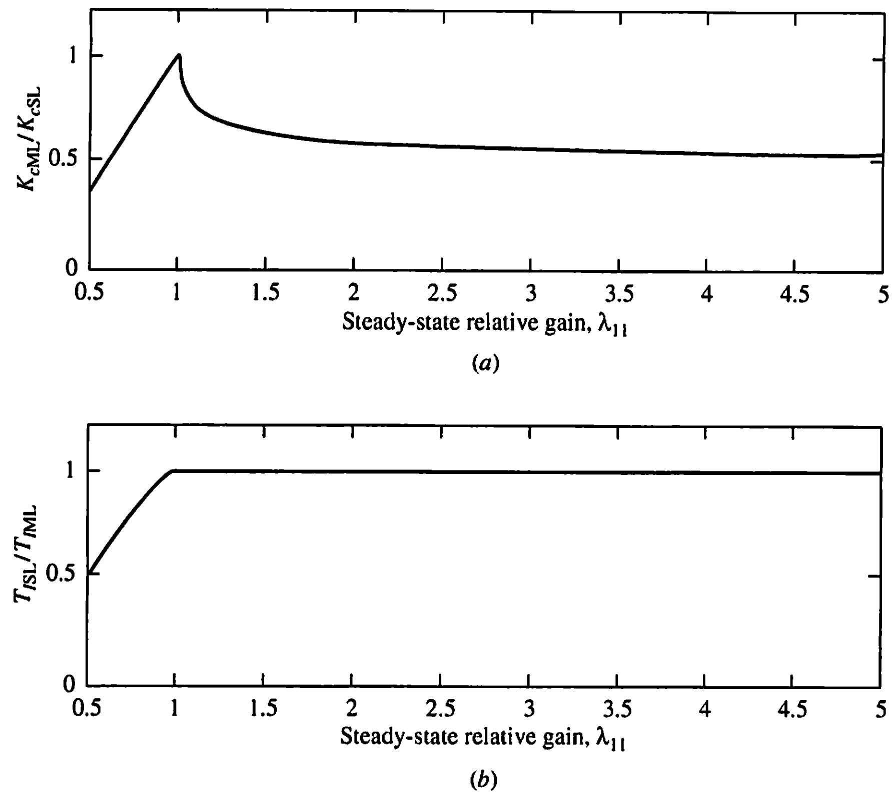



**FIGURE 20.12**

**Relationship between single-loop (SL) and multiloop (ML) PI controller tuning when both loops have similar dynamics.**

dynamics are similar. The controller gain is reduced by about a factor of 2.0 as the relative gain changes from 1.0. Also, the integral time increases by a factor of about 2.0 as the relative gain decreases to 0.5.

Two important conclusions for systems with similar dynamics become apparent from this plot:

1. The multiloop controllers must be detuned from their single-loop tuning over the entire range of relative gain.
2. The change in tuning constants is not very large.

Thus, interaction results in controller detuning, which slows feedback action for most 2 × 2 multiloop systems. The tuning results for 2 × 2 PI control presented in this section are summarized in Table 20.3.

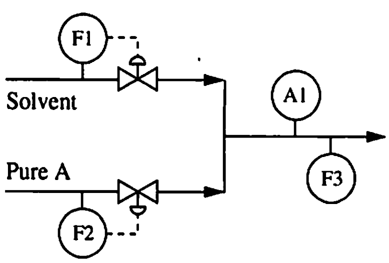
Note that these results are appropriate for systems that satisfy the assumptions employed. At the current time, there is no approximate method for the general case with very different dynamics of all paths, $G_{ij}(s)$. The trial-and-error or optimization-based methods must be used in these cases. Also, the importances of the controlled variables have been assumed to be relatively equal; the case for unequal importances is covered in the next chapter. The next two examples apply the tuning approach to realistic processes.

**TABLE 20.3**

**Summary of example tuning for 2 × 2 system**

| Situation | Characteristic expression | Interaction effect |
|---|---|---|
| General | $1 + G_{c1}(s)G_{11}(s) + G_{c2}(s)G_{22}(s)$ $+ G_{c1}(s)G_{11}(s)G_{c2}(s)G_{22}(s)/\lambda_{11}(s)$ | Transmission interaction affects stability |
| Loop 1 much faster | $1 + G_{c1}(s)G_{11}(s)$ | Loop 1 stability is not strongly affected by interaction; use single-loop tuning |
| Loop 1 much slower* | $1 + G_{c1}(s)G_{11}(s)/\lambda_{11}$ | Loop 1 stablility is affected by the change in close-loop process gain; multiply single-loop controller gain by $\lambda_{11}$ |
| Both loops with equal dynamics | $1 + 2\Lambda(s) + \Lambda(s)^2/\lambda_{11}$ with $\Lambda(s) = G_{c1}(s)G_{11}(s) = G_{c2}(s)G_{22}(s)$ | Loop 1 stability is affected by changes in gain and phase; use Figure 20.12 |

*This approach will lead to a very large controller gain for large $\lambda$. If the interacting controller is switched to manual, loop 1 could become unstable. Thus, the additional limit $(K_c)_{ML} \leq (K_c)_{SL}$ is often applied to ensure stability for both single- and multiloop systems.




**EXAMPLE 20.10.**
Determine initial tuning constants for multiloop PI controllers applied to the blending system operating at the conditions given in equations (20.22), 5% A in the product, and the following sensor dynamics:

| | Dead time | Time constant |
|---|---|---|
| Flow | 1 | 2 sec |
| Concentration | 15 | 30 sec |

Consider first the $A_1$–$F_2$ and $F_3$–$F_1$ controlled-manipulated variable pairing. The basis for the tuning values is the linear transfer function models in equations (20.6) and (20.7) with gain values from equation (20.23), and any single-loop tuning method could be used. The dynamics above indicate that this case fits the situation having one fast and one slow loop. Referring to Table 20.3, and noting that $\lambda_{11} = \lambda_{22} = 0.95 \approx 1.0$, both the fast and slow loops can be tuned very close to their single-loop values. The tuning results using the Ciancone single-loop correlations are summarized in Table 20.4.

A transient response of this system, simulated using the linearized equations, for a set point change of 0.01 in the mixed concentration, is given in Figure 20.13. This is a reasonably well-behaved response, which could be fine-tuned as needed. An important result of this analysis is that the tuning for this loop pairing does not

**TABLE 20.4**

**Tuning for the blending system with dilute product** ($x_m = 0.05$, $\lambda = 0.95$)

| Tuning term | $A_1$–$F_2$ controller (slow loop) Single-loop | Multiloop | $F_3$–$F_1$ controller (fast loop) Single-loop | Multiloop |
|---|---|---|---|---|
| Process gain | $K_{11} = 0.0095$ | $K_{11}/\lambda_{11} = 0.01$ | $K_{22} = 1.0$ | $K_{22} = 1.0$ |
| $\theta/(\theta + \tau)$ | 0.333 | 0.333 | 0.333 | 0.333 |
| $K_c K_p$ | 1.0 | 1.0 | 1.0 | 1.0 |
| $T_I/(\theta + \tau)$ | 0.85 | 0.85 | 0.85 | 0.85 |
| $K_c$ (kg/min/wt fraction) | 105.0 | 100.0 | 1.0 | 1.0 |
| $T_I$ (sec) | 38.0 | 38.0 | 2.6 | 2.6 |

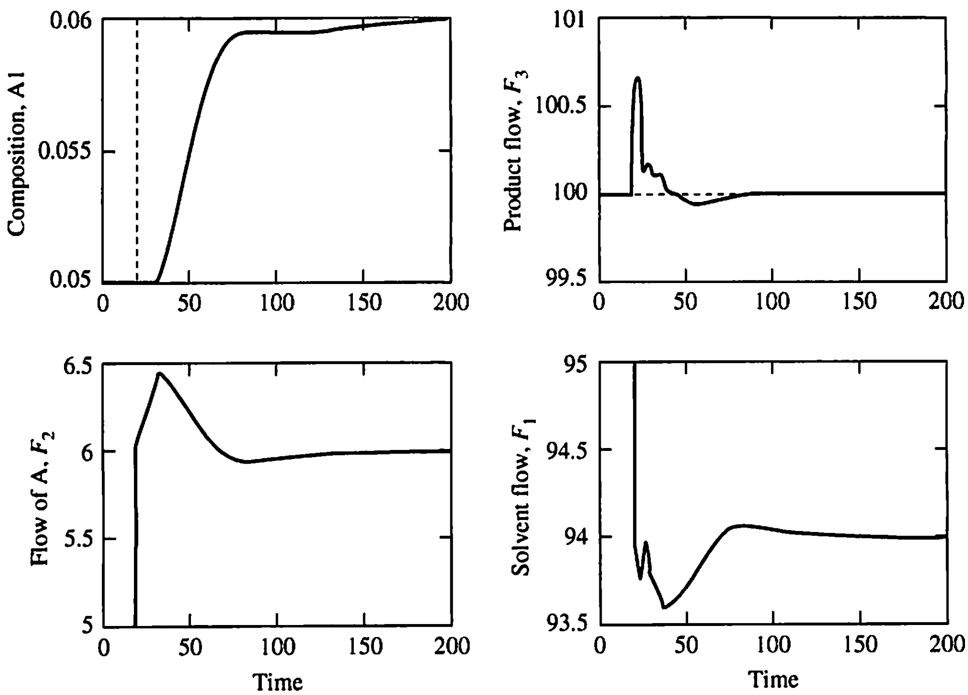



**FIGURE 20.13**

**Set point response for multiloop blending system in Example 20.10.**

change significantly from single-loop to multiloop; in other words, the tuning of the controllers does not depend on the control status (automatic/manual) of the other controller. This is a good situation.

Now consider the alternative loop pairing, $A_1$–$F_1$ and $F_3$–$F_2$. Again, the system consists of a fast and slow loop, so that the same approach can be used. However, in this system, the relative gain has a value far from unity, $\lambda_{11} = \lambda_{22} = 0.05$. Therefore, the response of the slow loop ($A_1$–$F_1$), which has an effective process gain of $K_{11}/\lambda_{11}$, is significantly altered by interaction. The results, using the recommendations in Table 20.3 and the Ciancone single-loop tuning correlations, are summarized in Table 20.5.

**TABLE 20.5**

**Tuning for the blending system with dilute product ($x_m = 0.05$, $\lambda = 0.05$)**

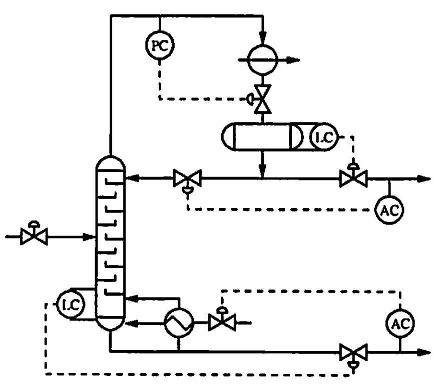
| Tuning term | $A_1$–$F_1$ pairing (slow loop) Single-loop | Multiloop | $F_3$–$F_2$ pairing (fast loop) Single-loop | Multiloop |
|---|---|---|---|---|
| Process gain | $K_{11} = -0.0005$ | $K_{11}/\lambda_{11} = -0.01$ | $K_{22} = 1.0$ | $K_{22} = 1.0$ |
| $\theta/(\theta+\tau)$ | 0.333 | 0.333 | 0.333 | 0.333 |
| $K_c K_p$ | 1.0 | 1.0 | 1.0 | 1.0 |
| $T_I/(\theta+\tau)$ | 0.85 | 0.85 | 0.85 | 0.85 |
| $K_c$ (kg/min/wt fraction) | −2000.0 | −100.0 | 1.0 | 1.0 |
| $T_I$ (sec) | 38.0 | 38.0 | 2.6 | 2.6 |

The transient response of the multiloop system with the multiloop tuning given in Table 20.5 is essentially the same as that for the previous pairing and is not shown. However, the single-loop and multiloop tunings are very different in Table 20.5, because the relative gain is much different from 1.0. If both loops are in automatic, the $A_1$ controller gain must be the (small) multiloop value given in the table. When the $F_3$ controller is in manual, the effective process gain for the $A_1$ controller changes to its single-loop value (which is lower by a factor of about 20).

A summary of the implications of the multiloop system in Table 20.5 follows:

| Tuning of $A_1$ | Single-loop ($A_1$) system | Multiloop system |
|---|---|---|
| Single-loop ($K_c = -2000$) | Good performance | Unstable system |
| Multiloop ($K_c = -100$) | Poor performance (very slow) | Good performance |

Thus, the controller tuning in Table 20.5 must be matched to the status of the controllers—*a situation to be avoided if possible.* This complexity in updating tuning online suggests that the pairing in Table 20.4, which can have the same tuning for any combination of loop statuses (since $\lambda \approx 1.0$), is a *much better choice.*




**EXAMPLE 20.11.**

Determine initial tuning constants for the distillation tower with the pressure and level controller pairings given in Figure 20.3, resulting in the model in equations (20.8) and (20.9). Evaluate the dynamic behavior for a step change in the feed light key of −0.04 mole fraction light key.

This process has similar dynamics for both loops, so that the summary in Table 20.3 recommends the tuning correlations in Figure 20.12. The large value

**TABLE 20.6**

**Tuning analysis for distillation control system**

| Tuning term ($\lambda = 6.09$) | $X_D$–$F_R$ controller Single-loop | $X_D$–$F_R$ controller Multiloop | $X_B$–$F_V$ controller Single-loop | $X_B$–$F_V$ controller Multiloop |
|---|---|---|---|---|
| Process gain | 0.0747 | | $K_{22} = -0.1253$ | |
| $\theta/(\theta+\tau)$ | 0.20 | | 0.16 | |
| $K_c K_p$ | 1.55 | | 1.7 | |
| $T_I/(\theta+\tau)$ | 0.60 | | 0.50 | |
| $K_c$ | 20.75 | $K_{cSL}/2 = 10.4$ | −13.6 | $K_{cSL}/2 = -6.8$ |
| $T_I$ | 9.0 | 9.0 | 6.1 | 6.1 |

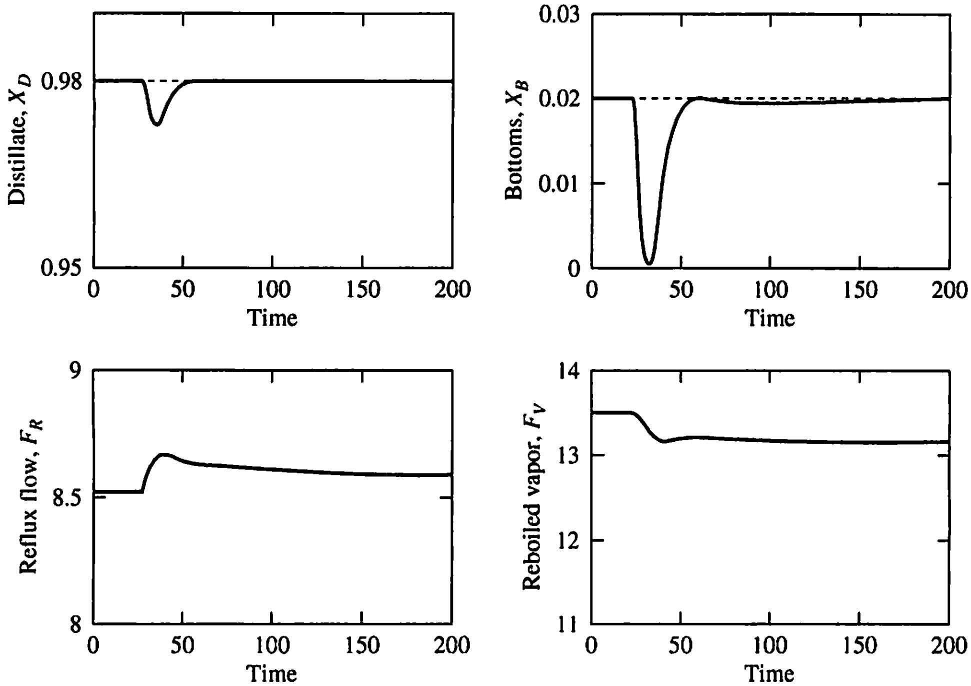



**FIGURE 20.14**

**Example disturbance response for multiloop distillation in Example 20.11.**

of the relative gain (6.09) indicates that the controller gains must be reduced by a factor of 2.0 from their single-loop values, and the integral times can remain unchanged. The results from applying this approach are given in Table 20.6, and a dynamic response of the multiloop system, using the multiloop tuning from the table, is shown in Figure 20.14. The response is well behaved, because the controlled variables return to their set points reasonably quickly and the manipulated variables experience moderate adjustments. Thus, the correlations provide acceptable initial tuning, which can be tailored to specific objectives through fine tuning.

## 20.7 ▧ ADDITIONAL TOPICS IN INTERACTION ANALYSIS

The material on interaction in this chapter is only introductory, and a coverage of much more material would be required for a mastery of the topic. Some of the key additional topics are reviewed briefly in this section.

### Modelling

Models for multivariable control should be developed with their ultimate use in mind. Recent results on model consistency (Skogestad, 1991; Haggblom and Waller, 1988) give useful relationships that can be used to verify that linearized models observe fundamental properties of the nonlinear system. Also, new experimental designs (Kwong and MacGregor, 1994) could be of use in obtaining better empirical estimates of process gains, but even with these careful experimental steps the use of empirical models for calculating relative gains with large magnitudes is problematical.

### Interaction Measures

The important features of systems with transmission interaction discussed in Section 20.3 can be developed through singular-value analysis for systems of arbitrary size and dynamics. The relevant matrix, here the process gain matrix **K**, can be decomposed into three matrices, which can be used to determine the directions in the CVs that cause the manipulated variables to change the "most" and the "least" (as measured by the root sum of squares of the changes in the MVs). Also, the ratio of the largest to the smallest changes in these two directions can be determined and is called the *condition number.* Clearly, the larger the condition number, the more interaction affects the multiloop system. Also, the condition number indicates the sensitivity of the calculation to model errors. The basic mathematics of this analysis is presented in Ortega (1987), and control applications are given in Barton et al. (1991) and Arkun (1984). The relationship between the relative gain and condition number is given by McAvoy (1983*b*) and Grosdidier et al. (1985). An alternative measure of interaction has been proposed by Grosdidier and Morari (1987). Finally, the controllability and relative gain calculations can be extended to systems with pure integration, such as liquid levels, by replacing the derivative of the variable ($dL/dt$) with a surrogate variable $\xi$ and proceeding with the standard method thereafter (McAvoy, 1983*b*).

### Frequency-Dependent Measures

The material in this chapter on controllability and interaction relied principally on steady-state measures. The definition of controllability used here involves steady-state behavior. An alternative frequency-dependent definition involves the ability to influence the dynamic trajectory of the output variables and requires that det $\mathbf{G}(s) \neq 0$ (Rosenbrock, 1974). Since this book deals mainly with continuous processes operated at specified steady-state conditions, the definition of controllability used here involves steady-state ($\omega = 0$) controllability.

In addition, the effects of interaction should be evaluated near the critical frequencies of the control loops. Frequency-dependent interaction is discussed by McAvoy (1981).

### Tuning

Another approach to tuning multiloop PID controllers that seems to have met with success is presented by Monica et al. (1988). This method can be extended to higher-order systems with frequency response calculations. The definition of modelling errors to be considered in tuning multivariable systems is much more difficult, because errors in the individual transfer functions and parameters within an individual transfer function are not independent.

## 20.8 CONCLUSIONS

Multiloop process control systems have been introduced, and the important concept of process interaction defined. Standard modelling methods can be used to represent the input-output behavior of the process without control. Interaction—one input affecting more than one output—is seen to influence the behavior of multivariable systems. Using the convention that the single-loop controllers are paired on the 1–1 and 2–2 elements in a two-variable process, interaction occurs when at least one of the interacting terms, $G_{12}(s)$ or $G_{21}(s)$, is nonzero. The process model can be employed to determine a useful measure of interaction: the relative gain array.

Requirements of controllability and values for relative gain are really extensions of conditions that are required for good single-loop feedback control, as summarized in the following table.

| Required condition | Single-loop system | Multiple-loop system |
|---|---|---|
| Controllability | A causal relationship exists between the manipulated and controlled variables, $K_p \neq 0$ | $n$ independent, causal relationships exist between the manipulated and controlled variables, $\det \mathbf{K} \neq 0$. |

Since the requirements are less obvious in multiloop systems, the rigorous mathematical tests are provided.

Transmission interaction—the additional connection path between an input and output through an interacting controller—occurs when both interacting terms in a $2 \times 2$ system are nonzero. Transmission interaction can strongly affect the behavior of a multivariable system. First, depending on the directions of the desired changes in controlled variables, it can substantially influence the adjustments required in the manipulated variables. Second, it can influence the system's stability and proper controller tuning.

Some of the results introduced in this chapter are general for all multiloop systems of any order ($n \times n$), while some are restricted to two-variable ($2 \times 2$) systems. The following summary is provided to help the reader.

| $n \times n$ **systems** | $2 \times 2$ **systems only** |
|---|---|
| Modelling | Closed-loop transfer function, |
| Controllability | equations (20.14) to (20.17) |
| Relative gain array definition, | Relationship between RGA and |
| equation (20.25) | stability, equation (20.34) |
| Relative gain calculation, | PI tuning, Section 20.5 |
| equation (20.27) | |
| Interpretations of relative | |
| gain in Section 20.5 | |

Finally, an important interpretation concerning control performance can be reached from these tuning results by considering a system having similar dynamics and a relative gain much larger than 1.0. (Many important processes have large relative gains.) In this system, the multiloop process gain is *smaller* than the single-loop process gain by a factor of about $1/\lambda$, as shown in equation (20.25). However, stability and tuning analysis indicated that the controller gain in the multiloop system must be *reduced* from its single-loop value, as shown in Figure 20.12! As a result, the reduction in effective process gain caused by interaction in the multiloop system *cannot* be compensated by an increase in the controller gain; if an attempt is made to increase the controller gain to improve control performance, the system will become unstable! Thus, the product $K_p K_c$ can be small (i.e., very much less than 1.0) in the interactive system, and feedback adjustments in response to some disturbances can be very slow because of this "detuning" effect of interaction. This stability limit for multiloop systems accounts for the very slow return to set point experienced by some processes with large relative gains.

To this point, general interpretations of multiloop system behavior have been developed. The many useful insights and quantitative expressions for the effects of interaction on multivariable behavior and stability will be exploited in the next chapter on multiloop control performance, in which specific methods for tailoring control design to performance goals are presented.

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## ADDITIONAL RESOURCES

The effects of process interaction were investigated earlier by several researchers, including

Rijnsdorp, J., "Interaction for Two-Variable Control Systems in Distillation Columns," I, *Automatica, 1,* 15–29 (1965) and II, *Automatica, 1,* 29–51 (1965).

Further information on the effects of interaction on multiloop systems is available in

Despande, P. (ed.), *Multivariable Process Control,* Instrument Society of America, Research Triangle Park, NC, 1989.

Shinskey, F. G., *Controlling Multivariable Processes,* Instrument Society of America, Research Triangle Park, NC, 1981.

Skogestad, S., and I. Postlethwaite, *Multivariable Feedback Control, Design and Analysis*, Wiley, New York, 1996.

A rigorous criterion for the stability of linear multivariable systems is available in Despande (1989, just cited) and

Luyben, W., *Process Modelling, Simulation, and Control for Chemical Engineers* (2nd ed.), McGraw-Hill, New York, 1990.

**The material in the chapter enables the engineer to evaluate the suitability of candidate processes and variables for multiloop control quantitatively. Specifically, controllability and operating window (or range of operation) can be used to establish the feasibility (or infeasibility) of feedback control for potential process designs. Interpretations of the relative gain suggest that only variable pairings with $\lambda_{ij} > 0$ for $2 \times 2$ systems should normally be considered further (but see Chapter 21 for important exceptions). Also, the effects of interaction on tuning are demonstrated by some preliminary tuning rules for $2 \times 2$ systems. The methods in this chapter enable the engineer to eliminate some candidate designs as infeasible for multiloop control, so that future effort can be directed toward evaluating the remaining feasible candidates.**

## QUESTIONS

**20.1.** For the blending process in Figure 20.2, design a control system to control the following three product variables at independent values: (*a*) the total flow ($F_3$), (*b*) the mass fraction of component A, and (*c*) the mass fraction of component $S$. You may assume that both mass fractions can be measured by the analyzer $A_1$.

**20.2.** Answer the following questions for two physical processes: (1) the chemical reactor described in Section C.2 of Appendix C and (2) the same chemical reactor with no heat of reaction. Both processes have two feedback PI controllers: $T \rightarrow F_c$ and $C_A \rightarrow C_{A0}$ (with $F$ unchanged).

(*a*) Does process interaction influence the stability of the closed-loop system? Provide quantitative analysis to support your conclusion.

(*b*) Does process interaction influence the dynamic behavior of the closed-loop system? Explain your answer briefly.

**20.3.** Prove the statements made in this chapter about the relative gain array: (*a*) The elements are scale-independent. (*b*) The sum of values in a row or column is 1.0. (*c*) the $\lambda_{ij}$ in equation (20.27).

**20.4.** Verify the closed-loop transfer functions in equations (20.12) through (20.17).

**20.5.** Answer the following question about controllability.

(*a*) How must the controllability test be modified when a constraint (bound) is encountered in one or more manipulated variables?

(*b*) Develop an alternative definition of controllability and develop a mathematical test for the situation in which the controlled variables must only achieve specified values at a single point in time. This might be valid for batch control or for intercepting a missile.

(*c*) Relate the definition of controllability used in this chapter to the relative gain array.

(*d*) How would the test for controllability in Section 20.3 be modified if the control algorithms were implemented via digital calculation?

(*e*) How far can one extrapolate the conclusions of the controllability test to other operating conditions?

**20.6.** Determine the controllability and possible loop pairings ($\lambda > 0$) for the process in Figure Q20.6 for the following two situations. The feed consists of only solvent and component A. The manipulated variables are the valves, and the controlled variables are the level and the composition of A, $C_A$.

(*a*) The situation without chemical reaction (i.e., a mixing tank).

(*b*) The situation with a single chemical reaction $A \rightarrow B$, $r_A = -kC_A$.

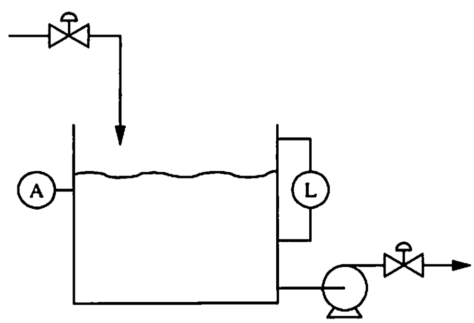


**FIGURE Q20.6**

**20.7.** Consider the CSTR in Figure Q20.7 in which solvent and component A in solvent ($C_{A0}$) are mixed. The two streams can be at different temperatures. A single reaction $A \rightarrow B$ occurs in the reactor. The rate expression is $r_A = -kC_A$, and the heat of reaction can be nonzero. The manipulated variables are the flow rates of the two inlet streams, and the controlled variables are the temperature and concentration of A in the reactor.

(*a*) Determine under what conditions the system is controllable.

(*b*) For the conditions which are controllable, if any, determine allowable loop pairings ($\lambda_{ij} > 0$).

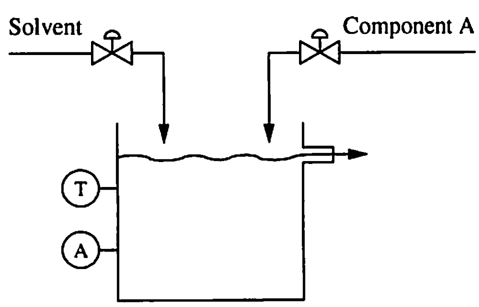


**FIGURE Q20.7**

**20.8.** Answer the following questions for a $2 \times 2$ control system with PI controllers.

(*a*) Is it possible for tuning values to exist that would yield a stable multiloop system and an unstable single-loop system for the same process?
(*b*) Is it possible for tuning values to exist that would yield an unstable multiloop system and a stable single-loop system for the same process?
(*c*) State the criteria for the single-loop system in Figure 20.7 to be stable.
(*d*) Suggest a manner for using the results in Example 20.8 in tailoring the dynamic performance to control system goals.

**20.9.** The following transfer function was provided by Waller et al. (1987) for a distillation column with the levels and pressure controlled with single-loop controllers as in Figure 20.3. The product qualities were not measured directly; they were inferred from tray temperatures (°C) near the top, $T_4$, and near the bottom, $T_{14}$, trays. The manipulated variables are the reflux, $F_R$, and reboiler steam, $F_S$, both in kg/h. Time is in minutes.

$$\begin{bmatrix} T_4(s) \\ T_{14}(s) \end{bmatrix} = \begin{bmatrix} \dfrac{-0.045e^{-0.5s}}{8.1s+1} & \dfrac{0.048e^{-0.5s}}{11s+1} \\ \dfrac{-0.23e^{-1.5s}}{8.1s+1} & \dfrac{0.55e^{-0.5s}}{10s+1} \end{bmatrix} \begin{bmatrix} F_R(s) \\ F_s(s) \end{bmatrix}$$

Answer the following questions for this system.
(*a*) Determine whether the input-output combination is controllable.
(*b*) Determine if either loop pairing can be eliminated based on the relative gains ($\lambda_{ij} > 0$).
(*c*) Determine the initial tunings for PI controllers for all allowable loop pairings.
(*d*) Estimate whether the interaction affects the magnitude of the manipulated variable changes for a set point change between single-loop and multiloop control.

**20.10.** The outlet temperature of the process fluid and the oxygen in the flue gas can be controlled in the fired heater in Figure Q20.10 by adjusting the fuel pressure (flow) and the stack damper % open. A dynamic model for the fired heater in Figure Q20.10 was reported by Zhuang et al. (1987) and is repeated here:

$$\begin{bmatrix} T(s) \\ A(s) \end{bmatrix} = \begin{bmatrix} \dfrac{0.6}{2400s^2+85s+1} & \dfrac{-0.04}{3000s^2+90s+1} \\ \dfrac{-1.1}{70s+1} & \dfrac{0.30}{70s+1} \end{bmatrix} \begin{bmatrix} P_{sp}(s) \\ v_2(s) \end{bmatrix}$$

The inputs and outputs are in percent of the range of each instrument, and the time is in seconds.
(*a*) Determine whether the input-output combination is controllable.
(*b*) Estimate whether the interaction changes the magnitude of the manipulated variable changes for a set point change between single-loop and multiloop control.
(*c*) Determine if either loop pairing can be eliminated based on the relative gains ($\lambda_{ij} > 0$).
(*d*) Determine the initial tunings for PI controllers for all allowable loop pairings.

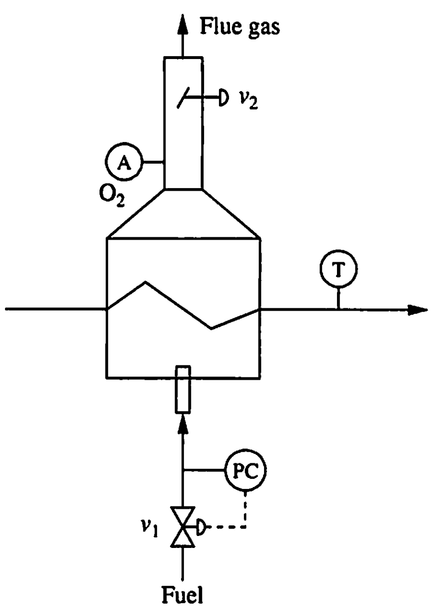

**FIGURE Q20.10**

**20.11.** Three CSTRs with the configuration of Section C.2 and with the following design parameters are considered in this example; the common data is given below, and the unique data and steady states are given in Table Q20.11 for three cases.

$$F = 1\ \text{m}^3/\text{min},\ V = 1\ \text{m}^3,\ C_{A0} = 2.0\ \text{kmole/m}^3,\ C_p = 1\ \text{cal/(gK)},$$
$$\rho = 10^6\ \text{g/m}^3,\ k_0 = 1.0 \times 10^{10}\ \text{min}^{-1},\ E/R = 8330.1\ \text{K}^{-1}$$
$$(F_c)_s = 15\ \text{m}^3/\text{min},\ C_{pc} = 1\ \text{cal/(g K)},\ \rho_c = 10^6\ \text{g/m}^3,\ b = 0.5$$

The controlled variables are $C_A$ and $T$, and the manipulated variables are $C_{A0}$ and $F_c$. Answer the following questions for each chemical reactor and explain the differences among the designs. (Note that this question requires the linearized, steady-state model for each case.)

(*a*) Determine whether the input-output combination is controllable.
(*b*) Estimate whether the interaction changes the magnitude of the manipulated variable changes for a set point change between single-loop and multiloop control.
(*c*) Determine if either loop pairing can be eliminated based on the signs of the relative gains.
(*d*) Determine the initial tunings for PI controllers for all allowable loop pairings.
(*e*) Evaluate the transient responses for a concentration set point change of +0.02 kmole/m$^3$.

**TABLE Q20.11**

| Case | I | II | III |
|---|---|---|---|
| $-\Delta H_{rxn} 10^6$ cal/(kmole) | 130 | 13 | −30 |
| $a$ (cal/min)/K | $1.678 \times 10^6$ | $1.678 \times 10^6$ | $0.7746 \times 10^6$ |
| $T_0$ K | 323 | 370 | 370 |
| $T_{cin}$ K | 365 | 365 | 420 (heating) |
| $T_s$ K | 394 | 368.3 | 392.7 |
| $C_{As}$ kmole/m$^3$ | 0.265 | 0.80 | 0.28 |

**20.12.** Discuss an empirical method for identifying the *inverse* of the process gain matrix directly from experimental data.

**20.13.** Determine whether $\mathbf{K}(\mathbf{K})^{-1}$ would give the same (correct) result as equation (20.27) for the elements of the relative gain array.

**20.14.** The process with two series chemical reactors in Example 3.3 is considered in this question. The process flexibility is increased by allowing the temperatures of the two reactors to be manipulated independently. The two controlled variables are the concentrations of reactant A in the two reactors.

The rate constant can be expressed as $5.87 \times 10^5 e^{-5000/T}$ (with temperature in K).

(*a*) Determine whether the input-output combination is controllable.
(*b*) Determine if either loop pairing can be eliminated based on the signs of the relative gains ($\lambda_{ij} > 0$).

**20.15.** The following transfer functions were provided by Wood and Berry (1973) for a methanol-water separation in a distillation column similar to Figure 20.3. The products are expressed as mole % light key, and the reflux $F_R$ and reboiler steam $F_S$ are in lb/min. Time is in minutes.

$$\begin{bmatrix} X_D(s) \\ X_B(s) \end{bmatrix} = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \begin{bmatrix} F_R(s) \\ F_s(s) \end{bmatrix}$$

(*a*) Determine whether the input-output combination is controllable.
(*b*) Estimate whether the interaction changes the magnitude of the manipulated variable changes for a set point change between single-loop and multiloop control.
(*c*) Determine if either loop pairing can be eliminated based on the sign of the relative gains ($\lambda_{ij} > 0$).
(*d*) Determine the initial tuning for PI controllers for all allowable loop pairings.
(*e*) The model was determined empirically. Discuss the effects of likely model errors on the results in parts (*a*) to (*d*).

**20.16.** A series of nonisothermal CSTRs shown in Figure Q20.16 is analyzed in this question. The heat transfer is adjustable in each reactor, so that each reactor temperature can be considered a manipulated variable. The feed contains only a nonreacting solvent and component A. The potential

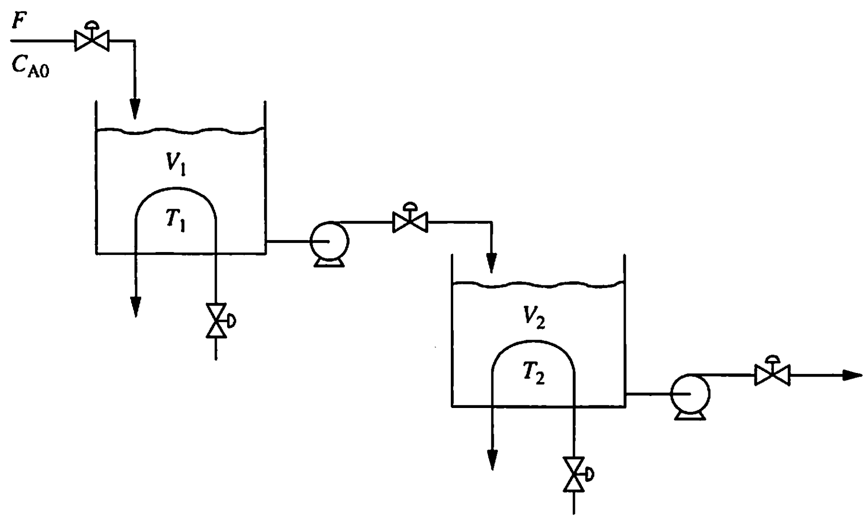


**FIGURE Q20.16**

manipulated variables are $T_1$, $T_2$, $F$, $V_1$, $V_2$, and $C_{A0}$. The variables to be controlled to independent steady-state values are the compositions of B and C in the effluent from the second reactor. For each of the sets of elementary reactions given below, determine (1) for which sets of two manipulated variables the system would be controllable and (2) for the variables selected in (1), whether either pairing of variables could be eliminated based on the relative gain.

(*a*) $\mathrm{A} \xrightarrow{k_1} \mathrm{B} \xrightarrow{k_2} \mathrm{C}$

(*b*) $\mathrm{A} \xrightarrow{k_1} \mathrm{B} + \mathrm{C}$

Assume that the rate constants can be expressed as Arrhenius functions of temperature and the heat of reaction is zero.

**20.17.** The mixing tank in Figure Q20.17 has two independent inlet streams of pure A and B that can be manipulated. The outlet flow cannot be manipulated by the unit; it is set by a unit of higher priority. The composition, the weight percent of B, and the level are to be controlled.

(*a*) Derive a linearized model of the system.
(*b*) Determine whether the system is controllable.
(*c*) Calculate the relative gain array for this process and make conclusions about the possible loop pairings for this system.

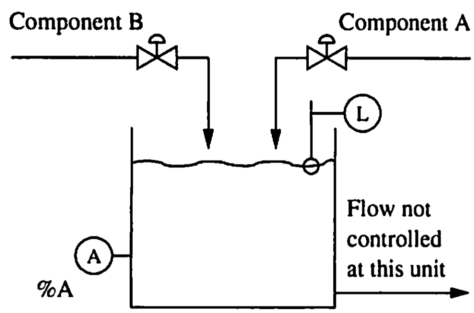


**FIGURE Q20.17**

**20.18.** A proposal is made to control the temperature ($T$) and composition ($C_A$) in the chemical reactor in Figure Q20.18 by manipulating the feed flow and the inlet temperature. The chemical reaction is $\mathrm{A} \rightarrow \mathrm{B}$, with $r_A = -kC_A$ and no heat of reaction. The flow in the pipe is laminar, so that the flow out can be taken to be proportional to level, $F = KL$. The data for this system at the base case operation is the same as for Example 3.2; in addition, the temperature is 323 K and the reaction rate constant is $k = 2.11 \times 10^5 e^{-5000/T}$.

(*a*) Derive the linearized model for this system in deviation variables.
(*b*) Determine whether the system is controllable in the steady state.
(*c*) Derive the four individual single input-output transfer functions.

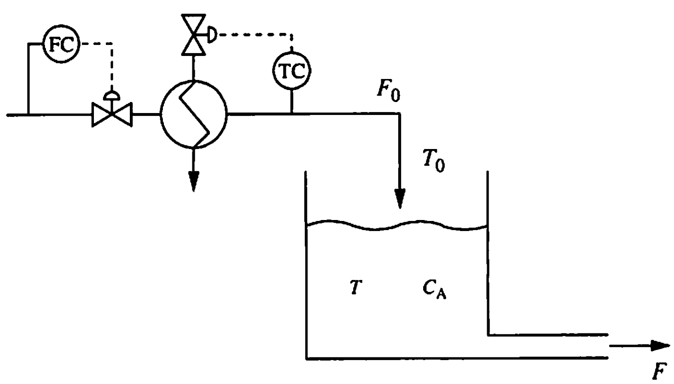


**FIGURE Q20.18**

(*d*) Evaluate the relative gain, both at steady state and as a function of frequency. Explain the differences.
(*e*) Select a feasible loop pairing and design a control system.

**20.19.** Evaluate the controllability and the interaction for the blending and distillation processes modelled in Section 20.2. Discuss the differences, if any, between the steady-state and frequency-dependent results.

**20.20.** The analysis of multiloop tuning summarized in Figure 20.9 considered only positive controller gains. Discuss the control performance when one of the controller gains is allowed to be negative.