Single-Variable Model Predictive Control

CHAPTER 19

19.1 ▪ INTRODUCTION

Most modifications to single-loop feedback control presented in this part of the book have used additional measurements to improve control performance. In contrast, the emphasis in this chapter will be on an alternative to the proportional-integral-derivative (PID) feedback algorithm. The PID controller was introduced in Chapter 8 by explaining the features associated with each mode and by demonstrating that the combined modes could provide reasonable control performance. In subsequent chapters the applications of PID in feedback, cascade, and combined feedforward/feedback have indicated that the adoption of PID as the standard algorithm in the 1940s was an appropriate choice. Perhaps the most remarkable feature of the PID is the success of this single algorithm in so many different applications.

However, the development of the PID lacked a fundamental structure from which the algorithm could be derived, limitations could be identified, and enhancements could be developed. In this chapter a general development is presented that gives great insight into the roles of both the control algorithm and the process in the behavior of feedback systems. This development also provides a method for tailoring the feedback control algorithm to each specific application. Because a model of the process is an integral part of the control algorithm, the controller equation structure depends on the process model, in contrast to the PID controller, which has only one equation structure.

Although the control algorithm is different, the feedback concept is unchanged, and the selection criteria for manipulated and controlled variables are the same as explained in Chapters 1 and 7. In fact, the algorithms presented in this chapter could be used as replacements for the PID controller in nearly all applications so far discussed. Generally, the PID controller is considered the standard algorithm; an alternative algorithm is selected only when the alternative provides better control performance.

The derivation of control algorithms is based on the predictive control structure introduced in the next section. Many methods are possible for deriving practical control algorithms to be implemented within the predictive structure, and two of these—internal model controller (IMC) and Smith predictor—are explained in detail, along with guidance on implementation issues. Finally, some applications are presented in which predictive controllers offer potential improvements over PID. In addition to introducing some very useful single-loop control methods, this chapter offers an opportunity for another perspective on the fundamentals of feedback control and an introduction to the predictive control structure shown to be well suited to multivariable control in Chapter 23.

19.2 ▪ THE MODEL PREDICTIVE CONTROL STRUCTURE

The predictive control structure is based on a very natural manner of interpreting feedback control. Before the general predictive structure is developed, it is worthwhile to consider the typical thought process used by a human operator implementing feedback control manually. Assume that the three-tank mixing process in Figure 19.1*a* is initially at steady state, and the goal is to reduce the outlet concentration by adjusting the flow of component A. First, the operator estimates the amount of change in the valve position (controller output) required to achieve the desired steady-state change in the controlled variable. This estimate requires an estimate of the steady-state model of the process (i.e., K_p). The operator can then estimate the proper adjustment in the valve position to be $\Delta v = (\Delta x_A)_1/K_p$.

Next, the operator would decide whether to implement this entire adjustment in one step or to introduce the change in several smaller steps. If the decision were to introduce the entire adjustment in one step, the dynamic response might look like the initial transient in Figure 19.1*b*. The person waits until steady state is achieved to observe the response and determine whether the estimate was correct. In this example, the concentration change was too large in magnitude, as is shown by a difference between the actual and predicted changes in the concentration. As a result of this error, the operator would have to make another change in the valve position. A clever operator might conclude that the assumed gain is incorrect and modify the estimate of K_p; however, the operator in this example applies a more straightforward approach, in which the next correction is based on the same value of the process gain; that is, $\Delta v = (\Delta x_A)_2/K_p$, where $(\Delta x_A)_2$ is the difference between the *predicted and actual* Δx_A. Several iterations of the procedure result in the transient response achieving steady state, as given in Figure 19.1.

The approach used by the operator has three important characteristics:

1. It uses a model of the process to determine the proper adjustment to the manipulated variable, because the future behavior of the controlled variable can be *predicted* from the values of the manipulated variable.

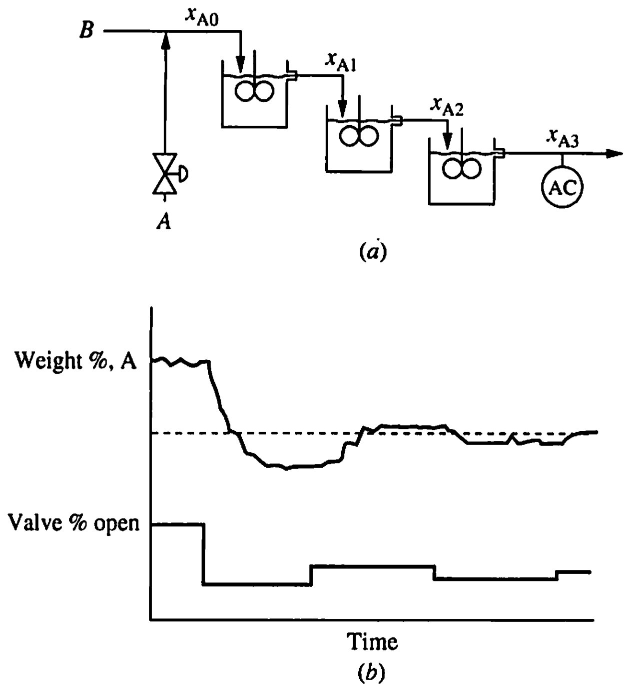


FIGURE 19.1

Example of manual control.

2. The important feedback information is the difference between the predicted model response and the actual process response. If this difference were zero, the control would be perfect, and no further correction would be needed.
3. This feedback approach can result in the controlled variable approaching its set point after several iterations, even with modest model errors.

These characteristics provide the basis for the predictive control structure.

A continuous version of the approach just described can be automated with the general predictive control structure given in Figure 19.2. Three transfer functions represent the true process with the final element and sensor, $G_p(s)$; the controller, $G_{cp}(s)$; and a dynamic model of the process, $G_m(s)$. To avoid confusion, the term *predictive control algorithm* will be used to denote the calculation represented by $G_{cp}(s)$, which is used for the controller in the block diagram in Figure 19.2. The term *predictive control system* will be used to denote all calculations in the control system, which includes the predictive control algorithm, the predictive model, and two differences. All calculations in the predictive control system must be executed every time a value of the final element is determined.

The feedback signal E_m is the difference between the measured and predicted controlled variable values. The variable E_m is equal to the effect of the disturbance, $G_d(s)D(s)$, if the model is perfect [if $G_m(s) = G_p(s)$]; thus, the structure highlights the disturbance for feedback correction. However, the model is essentially never exact, so that the feedback signal includes the effect of the disturbance and the model error, or *mismatch*. The feedback signal can be considered as a model correction; it is used to correct the set point so as to provide a better target value,

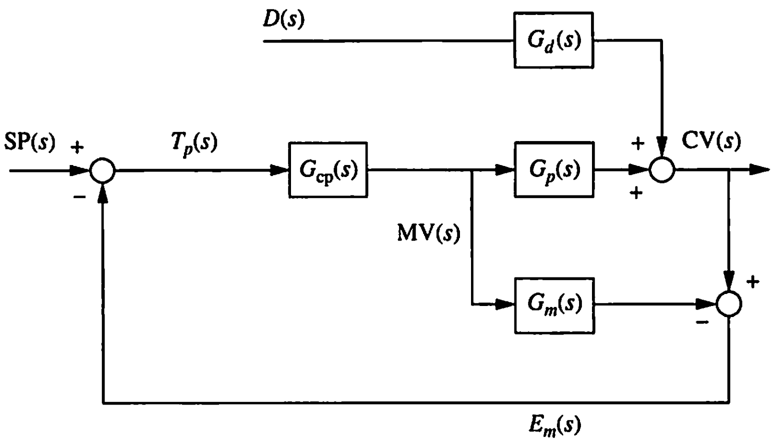


FIGURE 19.2

Predictive control structure.

$T_p(s)$, to the predictive control algorithm. The controller calculates the value of the manipulated variable based on the corrected target.

The following closed-loop transfer functions for responses to set point and disturbances can be derived through standard block diagram algebra.

$$\frac{\text{CV}(s)}{\text{SP}(s)} = \frac{G_{cp}(s)G_v(s)G'_p(s)}{1 + G_{cp}(s)[G_v(s)G'_p(s)G_s(s) - G_m(s)]} \approx \frac{G_{cp}(s)G_p(s)}{1 + G_{cp}(s)[G_p(s) - G_m(s)]} \tag{19.1}$$

$$\frac{\text{CV}(s)}{D(s)} = \frac{[1 - G_{cp}(s)G_m(s)]G_d(s)}{1 + G_{cp}(s)[G_v(s)G'_p(s)G_s(s) - G_m(s)]} \approx \frac{[1 - G_{cp}(s)G_m(s)]G_d(s)}{1 + G_{cp}(s)[G_p(s) - G_m(s)]} \tag{19.2}$$

In all further transfer functions in this chapter, the dynamics of the sensor are considered negligible, and the overall model of the final element and process is taken to be $G_p(s)$. A linear dynamic process model, $G_m(s)$, can be determined using fundamental (Chapters 3 through 5) or empirical (Chapter 6) modelling methods. The controller algorithm, $G_{cp}(s)$, for the predictive structure is as yet unknown and will be determined to give good dynamic performance.

A few properties of the predictive structure are now determined that establish important general features of its performance and give guidance for designing the controller, $G_{cp}(s)$. Normally, a very important control performance objective is to ensure that the controlled variable returns to its set point in steady state. This objective can be evaluated from the closed-loop transfer functions by applying the final value theorem and determining whether the final value of the controlled variable, expressed as a deviation variable from the initial set point, reaches the set point. The application of the final value theorem for this purpose is performed for the following conditions:

1. The input is steplike, in that it reaches a steady state after a transient, $\text{SP}(s) = \Delta\text{SP}/s$ and $D(s) = \Delta D/s$.

2. The process without control reaches a steady state after a steplike input, $G_p(0) = K_p$ and $G_m(0) = K_m$.
3. The closed-loop system is stable, which can be achieved via tuning.

> Note that the use of the steady-state gain of the process, $G_p(0)$, limits the results to stable processes without control.

In fact, the results in this chapter are limited to these stable processes. Under these conditions, application of the final value theorem yields

$$\lim_{t\to\infty} \mathrm{CV}(t) = \lim_{s\to 0} s\mathrm{CV}(s) = s\frac{\Delta \mathrm{SP}}{s}\frac{G_{\mathrm{cp}}(0)G_p(0)}{1 + G_{\mathrm{cp}}(0)[G_p(0) - G_m(0)]} \qquad (19.3)$$
$$= \Delta\mathrm{SP} \quad \text{if and only if } G_{\mathrm{cp}}(0) = G_m^{-1}(0)$$

$$\lim_{t\to\infty} \mathrm{CV}(t) = \lim_{s\to 0} s\mathrm{CV}(s) = s\frac{\Delta D}{s}\frac{[1 - G_{\mathrm{cp}}(0)G_m(0)]G_d(0)}{1 + G_{\mathrm{cp}}(0)[G_p(0) - G_m(0)]} \qquad (19.4)$$
$$= 0 \quad \text{if and only if } G_{\mathrm{cp}}(0) = G_m^{-1}(0)$$

Therefore, the predictive control system will satisfy both of the foregoing equations, thus providing zero steady-state offset for a steplike input, if

$$G_{\mathrm{cp}}(0) = G_m^{-1}(0) \qquad \text{or} \qquad K_{\mathrm{cp}} = 1/K_m \qquad (19.5)$$

Equation (19.5) requires that the steady-state gain of the controller algorithm must be the inverse of the steady-state gain of the *dynamic model* used in the predictive system. This important requirement can be easily achieved, because the engineer has perfect knowledge of the model, although certainly not of the process $G_p(s)$ itself.

> A stable predictive system does not require a perfect model; it must only satisfy equation (19.5) to return the controlled variable to the set point at steady state.

To gain further insight into the predictive structure, the next control performance objective considered is perfect control. Here, the term *perfect control* is taken to mean that the controlled variable never deviates from the set point. As we have seen, this performance is not possible with feedback control and might not generally be desired because of other control performance considerations. However, it is considered here to provide insight into the predictive system and to give further guidance on control algorithm design. The closed-loop transfer functions in equations (19.1) with $\mathrm{CV}(s)/D(s) = 0$ and (19.2) with $\mathrm{CV}(s)/\mathrm{SP}(s) = 1$ provide the basis for the following condition, required for the controlled variable to be equal to the set point at all times during the transient response:

$$G_{\mathrm{cp}}(s) = G_m^{-1}(s) \qquad (19.6)$$

Thus, perfect control performance would be achieved if the controller could be set equal to the inverse of the dynamic model in the predictive system. This might seem to be a simple requirement, since any model, even a constant, could be used for the model, and the controller would be easily evaluated as the inverse. However, block diagram algebra can be applied to derive the following condition for the behavior of the manipulated variable under perfect control:

$$\begin{aligned}\frac{MV(s)}{D(s)} &= \frac{-G_d(s)G_{cp}(s)}{1+G_{cp}(s)[G_p(s)-G_m(s)]}\\ &= \frac{-G_d(s)G_{cp}(s)}{1+G_{cp}(s)G_p(s)-1} = -\frac{G_d(s)}{G_p(s)}\end{aligned} \tag{19.7}$$

This shows that the perfect control system must invert the *true process* in some manner. The following are four reasons why an exact inverse of the process is not possible:

1. *Dead time.* In most physical processes, the feedback transfer function includes dead time in the numerator. The application of equations (19.6) and (19.7) to a typical process model with dead time gives, when the model is factored into two terms with $g_m(s)$ all polynomial terms in s,

$$G_m(s) = g_m(s)e^{-\theta s} \qquad G_{cp}(s) = [G_m(s)]^{-1} = [g_m(s)]^{-1}e^{\theta s} \tag{19.8}$$

   The perfect controller in this situation would have to include the ability to use *future* information in determining the current manipulated-variable value, as indicated by the predictive element $e^{\theta s}$. As discussed in Section 4.3, such noncausal models are not physically realizable—such behavior cannot occur (except in science fiction).
2. *Numerator dynamics.* As demonstrated in Section 5.4 on parallel process structures, some process models have dynamic elements in the numerators of the feedback transfer functions. Application of equation (19.6) to an example gives

$$G_m(s) = K\frac{\tau_2 s+1}{(\tau_1 s+1)^2} \qquad G_{cp}(s) = [G_m(s)]^{-1} = \frac{1}{K}\frac{(\tau_1 s+1)^2}{\tau_2 s+1} = \frac{MV(s)}{T_p(s)} \tag{19.9}$$

   For all values of τ_2 the controlled-variable behavior would be stable, because the product $G_{cp}(s)G_m(s) = 1$. However, the controller algorithm alone would be stable only for $\tau_2 \geq 0$ and would be unstable for $\tau_2 < 0$. (This is termed a right-half-plane zero in $G_m(s)$, leading to a right-half-plane (unstable) pole in $G_{cp}(s)$.) An unstable controller would be expected to cause the manipulated variable to behave in an unstable manner, as is demonstrated in Example 19.2. Thus, the controller in equation (19.9) would not be able to achieve the "perfect" performance when $\tau_2 < 0$.
3. *Constraints.* The manipulated variable must observe constraints. These could be physical constraints, such as a valve, which is limited to 0 to 100% open, or more limiting constraints, such as the fuel to a furnace, which must be above a minimum limit greater than zero to maintain a stable flame. There is no guarantee that the controller defined in equation (19.6), which was derived using linear equations that did not consider constraints, would observe the constraints. Thus, in some cases, values of the manipulated variable that are required to achieve perfect control performance would not be possible. In

such cases, the resulting control performance would not be perfect, and the controlled variable would deviate from its set point.

4. *Model mismatch.* The model used in the predictive system will almost certainly be different from the true process. If this difference is large, the closed-loop system could be unstable, a situation that precludes acceptable control performance. (Recall that the final value theorem assumes stability of the system.)

Thus, the predictive control system clearly shows that dead times, certain numerator process dynamics (right-half-plane zeros), constraints, and model mismatches all prevent perfect feedback control performance.

These results are not new; they were discussed in Part III and summarized in Table 13.3. However, this development reinforces the importance of the process in determining the achievable feedback control performance. It also provides a unified approach to developing these conclusions.

EXAMPLE 19.1.

Feedback control was introduced using the classical (PID) structure. Determine the relationship between the controllers in the classical structure $G_c(s)$ and the predictive system $G_{cp}(s)$.

Solution. Block diagram algebra can be applied to reduce the predictive controller and model into one transfer function, which gives

$$\frac{MV(s)}{SP(s) - CV(s)} = G_c(s) = \frac{G_{cp}(s)}{1 - G_m(s)G_{cp}(s)} \tag{19.10}$$

Therefore, there is an equivalence between the classical and predictive structures, and a control system can be represented by either block diagram, as long as the proper controller transfer function is used. It is important to note that the conversion of a predictive system into a classical system does not necessarily result in a PID controller in the classical system; thus, the behavior of the two closed-loop systems could, and in general would, differ. In this chapter, the predictive controllers will be represented by the block diagram in Figure 19.2 to show the use of an explicit model in the control system clearly; also, there are advantages in performing the calculations in this manner, as will become clear in later sections.

EXAMPLE 19.2.

When the predictive model is perfect [i.e., $G_m(s) = G_p(s)$], what else is required for the closed-loop system to be stable?

Solution. We would like both the controlled *and* manipulated variables to be stable (referred to as *internal stability* by Morari and Zafiriou, 1989), which requires that the following transfer functions be stable:

$$\frac{CV(s)}{SP(s)} = G_{cp}(s)G_p(s) \tag{19.11}$$

$$\frac{CV(s)}{D(s)} = G_d(s) \tag{19.12}$$

$$\frac{\text{MV}(s)}{\text{SP}(s)} = G_{cp}(s) \tag{19.13}$$

Thus, the product of the controller and the process, the disturbance, and the controller itself must be stable for the entire control system to behave in a stable manner. Clearly, the manipulated variable would be stable only if the controller is stable. [Recall that the controller could be unstable, while the product $G_{cp}(s)G_p(s)$ is stable.] Also, the final value theorem in equations (19.3) and (19.4) involves the terms $G_p(0)$ and $G_{cp}(0)$, the transfer functions evaluated at $s = 0$, which were taken to be constant values. This result is valid only when the transfer functions are stable; if they are unstable, the final value theorem is not applicable.

So far, the predictive concept has been introduced, the block diagram structure presented, and the closed-loop transfer function derived. The starting points for the predictive control algorithm design are the requirement for zero steady-state offset in equation (19.5) and the definition of the perfect controller in equation (19.6). Since the perfect controller is not possible even if it were desirable, a manner for deriving an *approximate inverse* of the model is required, with an approximate inverse being a $G_{cp}(s)$ that does not exactly satisfy equation (19.6) but contains the important features for control performance. Many methods exist for developing an approximate inverse, and each would result in a different controller algorithm giving different control performance. In the next sections, two methods for designing single-loop predictive control algorithms are presented. They have been selected because they involve straightforward mathematics, are simple to implement in a digital computer, yield good control performance in many cases, and have been applied industrially.

19.3 ■ THE IMC CONTROLLER

The system in Figure 19.2 has been described by several investigators, who have used different terminology for what is now generally referred to as the predictive structure. The publications by Brosilow (1979) and Garcia and Morari (1982), in which they introduced the terms *inferential control* and *internal model control,* respectively, sparked considerable interest in the chemical engineering community. The controller design approach presented in this section follows the developments of these publications, which is generally referred to as the IMC method.

Since an exact inverse is not possible, the IMC approach segregates and eliminates the aspects of the model transfer function that make calculation of a realizable inverse impossible. The first step is to separate the model into the product of the two factors

$$G_m(s) = G_m^+(s)G_m^-(s) \tag{19.14}$$

where $G_m^+(s)$ = the *noninvertible* part has an inverse that is not causal or is unstable. The inverse of this term includes predictions ($e^{\theta s}$) and unstable poles ($1/(1+\tau s)$, with $\tau < 0$) appearing in $G_{cp}(s)$. The steady-state gain of this term must be 1.0.

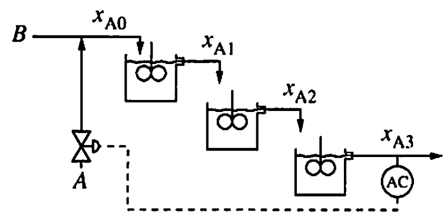
$G_m^-(s)$ = the *invertible* part has an inverse that is causal and stable, leading to a realizable, stable controller. The steady-state gain of this term is the gain of the process model K_m.

> The IMC controller eliminates all elements in the process model $G_m(s)$ that lead to an unrealizable controller by taking the inverse of only the invertible factor to give
>
> $$G_{cp}(s) = [G_m^-(s)]^{-1} \qquad (19.15)$$

This design equation ensures that the controller is realizable and that the system is internally stable (at least with a perfect model), but it does not explicitly guarantee that the behavior of the control system is acceptable. However, the performance of such controllers, as modified shortly, will be seen to be acceptable in many cases. Before we proceed, this procedure is applied to two examples.

EXAMPLE 19.3.

Apply the IMC procedure to design a controller for the three-tank mixing process.



Solution. The IMC controller design requires a transfer function model of the process. The linearized third-order model derived in Example 7.2 will be used. In this case,

$$G_m(s) = \frac{K_m}{(\tau s + 1)^3} = \frac{0.039}{(5s+1)^3} = G_m^-(s) \qquad G_m^+(s) = 1.0$$

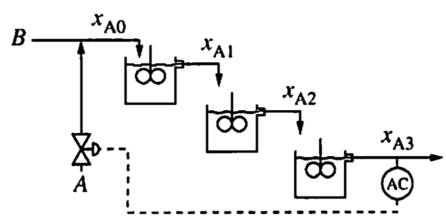
Thus, the model can be inverted directly to give

$$G_{cp}(s) = [G_m^-(s)]^{-1} = \frac{(\tau_m s + 1)^3}{K_m} = \frac{(5s+1)^3}{0.039}$$

This controller in the predictive structure in Figure 9.2 could theoretically provide good control of the controlled variable. However, there are several drawbacks with this design. First, the controller involves first, second, and third derivatives of the feedback signal. These derivatives cannot be calculated exactly, although they can be estimated numerically. Second, the appearance of high-order derivatives of a noisy signal could lead to unacceptably high variation and large overshoot in the manipulated variable. Finally, these high derivatives could lead to extreme sensitivity to model errors. Therefore, this controller would not be used without modification.

EXAMPLE 19.4.

Design an IMC controller for the process in Example 19.3, using the alternative first-order-with-dead-time approximate model for the process that was determined using the process reaction curve in Example 6.4 and as repeated here.



$$G_m(s) = \frac{K_m e^{-\theta_m s}}{\tau_m s + 1} = \frac{0.039 e^{-5.5s}}{10.5s + 1}$$

This model must be factored into invertible and noninvertible parts:

$$G_m^-(s) = \frac{K_m}{\tau_m s + 1} = \frac{0.039}{10.5s + 1}$$

$$G_m^+(s) = e^{-\theta_m s} = e^{-5.5s}$$

The invertible part is then employed in deriving the controller:

$$G_{cp}(s) = [G_m^-(s)]^{-1} = \frac{\tau_m s + 1}{K_m} = \frac{10.5s + 1}{0.039}$$

This controller is a proportional-derivative algorithm, which still might be too aggressive but will be modified to give acceptable performance in Example 19.6.

As discussed in Section 4.3, all realistic processes are modelled by transfer functions having a denominator order greater than the numerator order. Thus, the controller according to equation (19.15), which is the inverse of the process model, will have a numerator order greater than the denominator order. This results in first- or higher-order derivatives in the controller, which generally lead to unacceptable manipulated-variable behavior and, thus, poor performance and poor robustness when model errors occur.

Achieving good control performance requires modifications that modulate the manipulated-variable behavior and increase the robustness of the system. The IMC design method provides one feature to account for both of these concerns: filtering the feedback signal. The filter can be placed before the controller, as shown in Figure 19.3, so that the closed-loop transfer functions for the controlled and manipulated variables become

$$\frac{\mathrm{CV}(s)}{\mathrm{SP}(s)} = \frac{G_f(s)G_{\mathrm{cp}}(s)G_p(s)}{1 + G_f(s)G_{\mathrm{cp}}(s)[G_p(s) - G_m(s)]} \tag{19.16}$$

$$\frac{\mathrm{MV}(s)}{\mathrm{SP}(s)} = \frac{G_f(s)G_{\mathrm{cp}}(s)}{1 + G_f(s)G_{\mathrm{cp}}(s)[G_p(s) - G_m(s)]} \tag{19.17}$$

$$\frac{\mathrm{CV}(s)}{D(s)} = \frac{G_d(s)[1 - G_f(s)G_{\mathrm{cp}}(s)G_m(s)]}{1 + G_f(s)G_{\mathrm{cp}}(s)[G_p(s) - G_m(s)]} \tag{19.18}$$

$$\frac{\mathrm{MV}(s)}{D(s)} = \frac{-G_d(s)G_{\mathrm{cp}}(s)G_f(s)}{1 + G_f(s)G_{\mathrm{cp}}(s)[G_p(s) - G_m(s)]} \tag{19.19}$$

Now, four desirable properties of the filter are determined as a basis for selecting the filter algorithm. First, the steady-state value of the filter needs to be

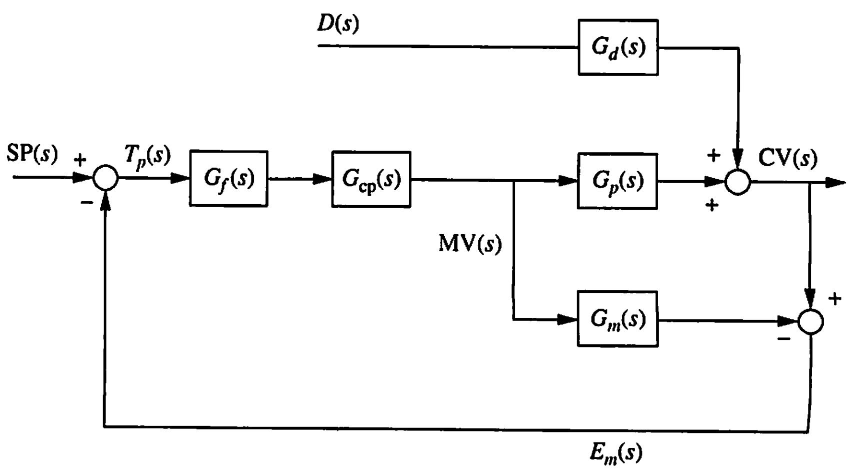


FIGURE 19.3

Predictive structure with single filter.

determined. Application of the final value theorem to the closed-loop transfer function in equation (19.16) with the requirement of zero steady-state offset yields

$$\lim_{t\to\infty} \mathrm{CV}(t) = \lim_{s\to 0} s\frac{\Delta\mathrm{SP}}{s}\left\{\frac{G_f(0)G_{cp}(0)G_p(0)}{1+G_f(0)G_{cp}(0)[G_p(0)-G_m(0)]}\right\} \tag{19.20}$$
$$= \Delta\mathrm{SP} \quad \text{only if } G_{cp}(0) = [G_f(0)G_m(0)]^{-1}$$

By convention, the controller gain is required to be the inverse of the process model; therefore, the steady-state gain of the filter must be unity; that is, $G_f(0) = K_f = 1.0$.

Second, a desired effect of the filter on the manipulated-variable behavior must be decided. Generally, the filter should reduce unnecessary high-frequency fluctuations due to noise. Since $G_f(s)$ appears in the numerator of equations (19.17) and (19.19), the magnitude of the filter magnitude should decrease with increasing frequency. The filter with the proper amplitude ratio attenuates the effects of high-frequency variation in the controlled variable (and set point) on the variation in the manipulated variable while it transmits the lower-frequency variation essentially unchanged. The term introduced in Chapter 12 for this behavior was *low-pass filter.*

Third, the filter influences the controlled-variable performance. Its appearance in the numerators of equations (19.16) and (19.18) indicates that filters with monotonically decreasing amplitude with increasing frequency degrade the performance of the controlled variable: filters lead to larger deviations from set point during transients. Thus, too much damping through the filter is not desirable.

Fourth, the effect of the filter on stability can be interpreted by analyzing the closed-loop transfer function, which has $G_{OL}(s) = G_f(s)G_{cp}(s)[G_p(s) - G_m(s)]$ for the predictive system. Clearly, the system is always stable if the model is perfect (and the controller is stable). However, the model is essentially never perfect, and the filter is required to ensure stability for a reasonable range of model error. Recalling that stability is improved as the magnitude of $G_{OL}(j\omega_c)$ is decreased, a filter that has decreasing magnitude as frequency increases will reduce the effects of model mismatch on $|G_{OL}(j\omega_c)|$ and stabilize the closed-loop system.

In summary, filters with a steady-state gain of 1.0 and decreasing magnitudes as frequencies increase satisfy the general requirements of increased robustness and noise attenuation. Many potential filter transfer functions satisfy the requirements just developed.

In the single-loop IMC design, it is conventional to use the following filter equation to improve robustness and manipulated-variable behavior.

$$G_f(s) = \left[\frac{1}{\tau_f s + 1}\right]^N \tag{19.21}$$

In this equation, the exponent N is selected to be large enough that the product $G_f(s)G_{cp}(s)$ has a denominator polynomial in s of order at least as high as its numerator polynomial. For further examples in this chapter, the model $G_m(s)$ will be first-order with dead time and the filter will be a first-order system ($N = 1$), but this is not always the case for other process models. The filter time constant can be adjusted to satisfy the performance specifications. Increasing the filter time constant modulates the manipulated-variable fluctuations and increases robustness at the expense of larger deviations of the controlled variable from its set point during the transient response.

EXAMPLE 19.5.

The filter location in Figure 19.3 influences the behavior of the control system for both disturbance and set point responses. Develop an alternative structure to separate these effects, so that the disturbance and set point responses can be influenced independently.

To achieve robustness, one filter must be located within the feedback loop. A design is shown in Figure 19.4, which has one filter, $G_{fF}(s)$, in the feedback path and a second filter, $G_{fS}(s)$, for set point. The advantage of this design is the ability to modify the set point and disturbance responses independently. This design is sometimes referred to as a *two-degree-of-freedom controller.*

The predictive control system is difficult to implement in analog computing equipment because of the dead time in the model $G_m(s)$, but it is straightforward with digital computers, regardless of the model structure. The simple models considered in this chapter can be expressed in discrete form by methods already introduced in Chapters 6 and 15 and in Appendix F. The IMC system in Figure 19.3 with a single filter will be considered, and the dynamic model will be assumed to be first-order with dead time. Thus, the predictive control system equations in continuous form are

$$\frac{CV_m(s)}{MV(s)} = G_m(s) = \frac{K_m e^{-\theta_m s}}{\tau_m s + 1} \tag{19.22}$$

$$G_m^-(s) = \frac{K_m}{\tau_m s + 1} \tag{19.23}$$

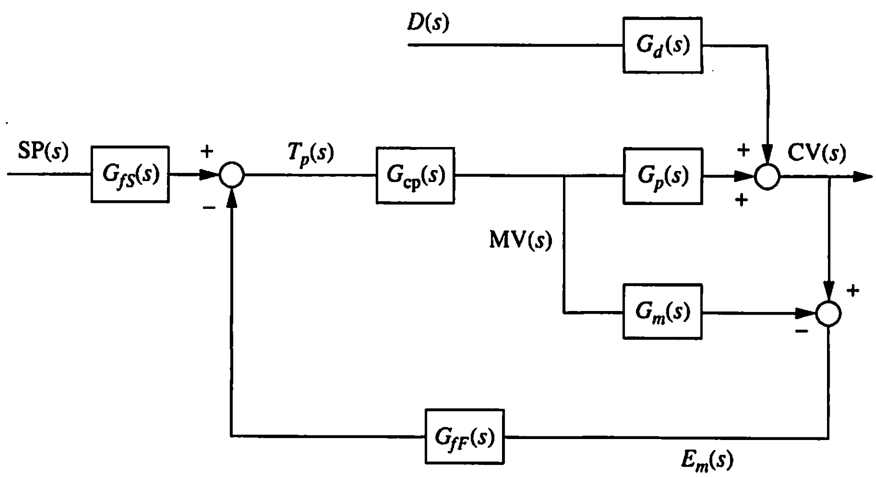


FIGURE 19.4

Two-degree-of-freedom predictive controller.

$$\frac{MV(s)}{T_p(s)} = G_f(s)G_{cp}(s) = \frac{1}{K_m}\frac{\tau_m s + 1}{\tau_f s + 1} \tag{19.24}$$

with $CV_m(s)$ the predicted value of the controlled variable, that is, the output from the model $G_m(s)$. The dynamic model can be simulated in discrete form, as explained in Appendix F.

$$(CV_m)_n = [e^{-\Delta t/\tau_m}](CV_m)_{n-1} + K_m[1 - e^{-\Delta t/\tau_m}]MV_{n-\Gamma-1} \tag{19.25}$$

with Δt the digital controller execution period and the dead time modelled as $\Gamma = \theta_m/\Delta t$, an integer value.

Note that the product of $G_f(s)G_{cp}(s)$ can be implemented as one algorithm in this case: a lead-lag transfer function, which was expressed in discrete form in Section 15.5.

$$MV_n = \left[\frac{\frac{\tau_f}{\Delta t}}{\frac{\tau_f}{\Delta t}+1}\right]MV_{n-1} + \frac{1}{K_m}\left[\frac{\frac{\tau_m}{\Delta t}+1}{\frac{\tau_f}{\Delta t}+1}\right](T_p)_n - \frac{1}{K_m}\left[\frac{\frac{\tau_m}{\Delta t}}{\frac{\tau_f}{\Delta t}+1}\right](T_p)_{n-1} \tag{19.26}$$

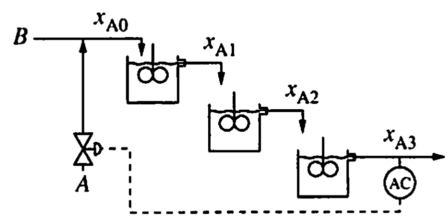
with T_p the *target*—that is, the set point as corrected by the feedback signal; the difference between the measured and predicted values of the controlled variable.

In summary, the predictive control system execution at step n involves the following:

1. Calculate the predicted controlled variable, equation (19.25).
2. Calculate the difference between the measured and model-predicted controlled variables, $(E_m)_n = CV_n - (CV_m)_n$.
3. Correct the set point with the feedback signal, $(T_p)_n = SP_n - (E_m)_n$.
4. Calculate the manipulated-variable value, equation (19.26).

EXAMPLE 19.6.

Simulate the dynamic response of the linearized three-tank mixing process in Example 19.4, operating at the base-case inlet flow rate, under IMC feedback control.



The true process $G_p(s)$ is taken as the linear, third-order system, and the controller and dynamic model $G_m(s)$ will be based on the approximate first-order-with-dead-time model. This structural mismatch, which is typical of realistic applications, precludes perfect control; thus, the results of this exercise give a realistic evaluation of the performance of IMC controllers.

The controller with filter and model transfer functions are

$$G_p(s) = \frac{0.039}{(5s+1)^3} \qquad G_m(s) = \frac{0.039e^{-5.5s}}{10.5s+1} \qquad G_f(s)G_{cp}(s) = \frac{1}{0.039}\frac{10.5s+1}{\tau_f s+1}$$

The controller calculations can be converted to discrete form with $\Delta t = 0.10$ to give

$$\begin{aligned}(CV_m)_n &= [e^{-0.1/10.5}](CV_m)_{n-1} + 0.039[1 - e^{-0.1/10.5}]MV_{n-55-1}\\ &= 0.9905(CV_m)_{n-1} + 0.000388\,MV_{n-56}\end{aligned}$$

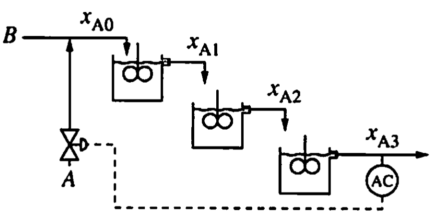
$$MV_n = \left[\frac{\frac{\tau_f}{0.1}}{\frac{\tau_f}{0.1}+1}\right]MV_{n-1} + \frac{1}{0.039}\left[\frac{\frac{10.5}{0.1}+1}{\frac{\tau_f}{0.1}+1}\right](T_p)_n - \frac{1}{0.039}\left[\frac{\frac{10.5}{0.1}}{\frac{\tau_f}{0.1}+1}\right](T_p)_{n-1}$$

In this example, the closed-loop simulation is performed using the foregoing equations for the controller (based on an approximate model) and the linearized third-order model for the plant; thus, significant model mismatch exists between the process and the model. The results are given in Figure 19.5*a* for a feed composition disturbance of magnitude of 0.80%A and in Figure 19.5*b* for a set point change, each for three values of the filter time constant. As the filter time constant increases, the aggressiveness of the controller decreases, as indicated by the slower response of the manipulated variable and slower return to the set point. It is noteworthy that the disturbance response appears acceptable, albeit slow, for all values of the filter tuning, while the set point response experiences extreme manipulated-variable variability for the lowest filter value. This comparison demonstrates the disadvantage for a single filter and the potential for improvement by using separate filters, as shown in Figure 19.4, to influence the disturbance and set point responses separately.

Based on the results in Example 19.6 and our previous experience with the PID controller, we would expect that the performance of predictive control depends on a proper choice of all parameters in the system. In general, all parameters appearing in the IMC model and the control algorithm could be tuned, but it is common practice to use the best estimates for the dynamic model. Thus, only the filter time constant, τ_f, is considered available for tuning.

The considerations for controller tuning were thoroughly discussed in Chapter 9. Clearly, no one value or correlation will suit all situations, but a few studies have been performed to provide initial tuning values, which are applicable to many situations and can be fine-tuned based on empirical experience. One tuning guideline, due to Brosilow (1979), suggests that the filter time constant be related to the likely model error, $\tau_f = 0.25(\delta\theta)$, with $\delta\theta$ the maximum likely error in the estimated dead time; Morari and Zafiriou (1989) recommend that a thorough robust tuning analysis be performed.

The method for tuning discussed in Chapter 9 and summarized in Appendix E for PID controllers, which minimized the IAE of the noisy controlled variable subject to limitations on variations in the manipulated variable over a range of model mismatch, has been applied to the IMC design as well (Ciancone et al., 1993). The results for a first-order-with-dead-time process model are given in Figure 19.6 for good performance for a step disturbance. The filter tuning constant has a large value for small fraction dead times, although one might initially expect the opposite correlation because systems that are easier to control require more filtering. The reason for these results is the need to moderate the high-frequency variation in the manipulated variable. Thus, the ratio of process time constant to filter time constant in the lead-lag element in the controller should not be too large; these results indicate that a reasonable ratio is around 2. A smaller filter time constant would be allowable for stability and give good controlled-variable performance, but the variability in the manipulated variable would be unacceptably large for many applications.



EXAMPLE 19.7.

Calculate the filter tuning constant for the IMC controller applied to the three-tank mixing process.

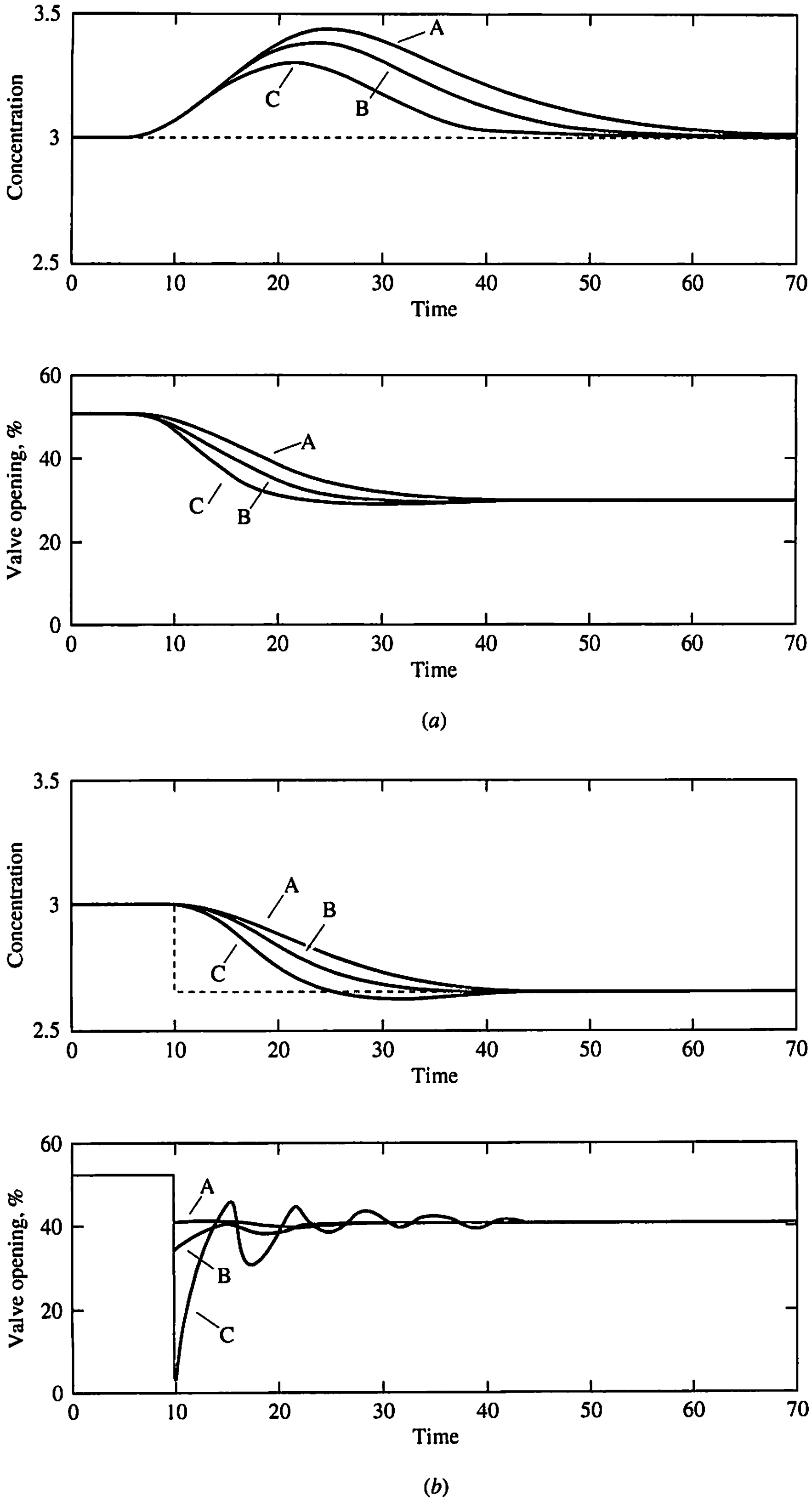


FIGURE 19.5

(*a*) Disturbance responses and (*b*) set point responses for Example 19.6. The values of the filter time constant τ_f are 10 min (case A), 6.1 min (case B), and 2.0 min (case C).

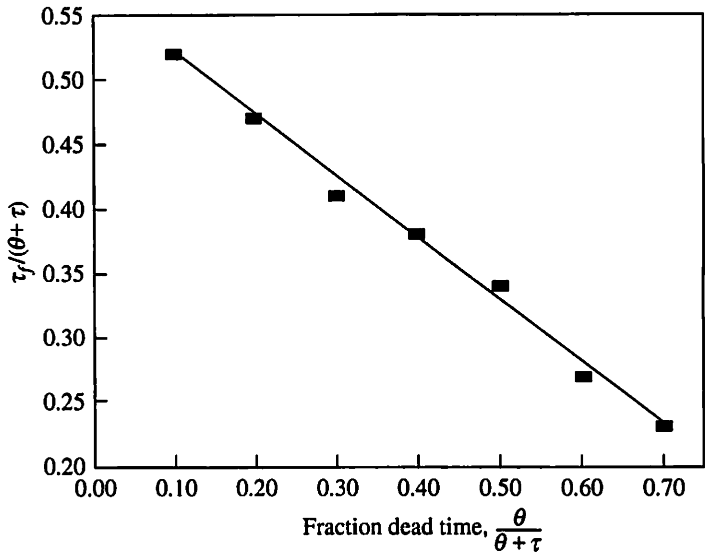


FIGURE 19.6

Tuning correlation for single-loop IMC disturbance response on a first-order-with-dead-time process.

Applying the correlations noted for Brosilow's approach, and assuming that the likely dead time error is 35%, gives

$$\tau_f = (0.35)(5.5)(.25) = 0.48 \text{ min}$$

The Ciancone correlation in Figure 19.6 gives

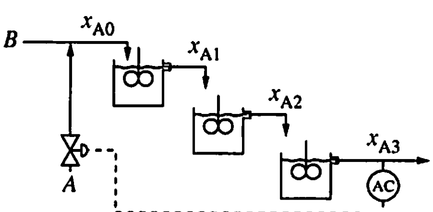
$$\theta/(\theta+\tau) = 5.5/(5.5+10.5) = 0.34$$
$$\tau_f/(\theta+\tau) = 0.38$$
$$\tau_f = (0.38)(16) = 6.1 \text{ min}$$

The dynamic response using tuning values based on Figure 19.6 is case B in Figure 19.5*a* and *b*. The controlled-variable IAE for the IMC disturbance response is 9.1, which is slightly larger than the value obtained under PID control for the same disturbance in Example 9.2.

EXAMPLE 19.8.

Evaluate the robustness for the IMC controllers implemented in Example 19.7.

Assuming that the system closely approximates a continuous system, the analysis could be performed using methods introduced in Chapter 10. However, root locus is not applicable, because the characteristic equation involves exponentials in s. Also, the Bode method is *not generally applicable* because the Bode plots for predictive systems do not always conform to the requirements noted in Table 10.1 (i.e., monotonically decreasing amplitude and phase behavior after the critical frequency). The stability could be determined using the Nyquist *method* applied to $G_{OL}(s) = G_f(s)G_{cp}(s)[(G_p(s) - G_m(s)]$; however, this method has not been stressed in this book.



Therefore, the robustness of this example will be evaluated by simulating cases with a fixed value of the filter time constant and different operating conditions. The range of process operations for the three-tank process is the same as considered in Section 16.2, where the flow rate has its base-case value in case E; is decreased by about 30% in case C; and is decreased by about 55% in case A. The parameters for the process model of the true process are given below.

Case	F_B	K_p (%A/% open)	τ_i(min) ($i = 1, 3$ for third-order system)
A	3.0	0.087	11.4
C	5.0	0.052	6.9
E (basis for tuning)	6.9	0.039	5.0

In this example, the controller parameters are fixed at values appropriate for the base-case approximate first-order-with-dead-time empirical model, as determined in Example 19.6; thus, the model gain was 0.039, the model time constant was 10.5, and the model dead time was 5.5. The filter time constant was determined to be 6.1 min in Example 19.7.

The results are presented in Figure 19.7. The performance is acceptable for the base case and +30% change in process behavior. For the largest model error, the system has very poor performance and appears on the limit of stability. These results are similar to the behavior of PID controllers; for reasonable model errors (previously estimated to be around ±25%), the PID performance usually

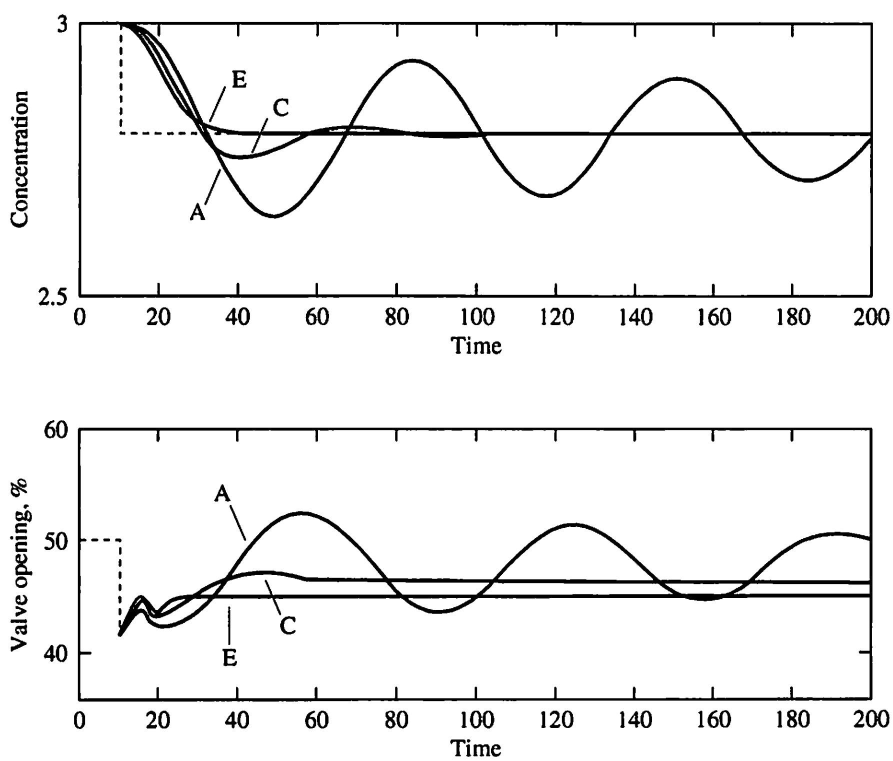


FIGURE 19.7

Dynamic response of three-tank system with IMC control at various operating conditions for Example 19.8 (τ_F = 6.1 min).

does not degrade significantly. However, as the errors become large, the PID performance degrades substantially and can become unstable. As we see, some characteristics of closed-loop feedback systems do not depend strongly on the control calculation.

In conclusion, the IMC controller is based on the general predictive control structure. The controller design method adheres to criteria that ensure zero offset for steplike disturbances, and it employs a factorization approach to obtain a realizable approximate inverse that gives good feedback control performance. An adjustable filter (tuning parameter) was introduced to enable the engineer to moderate the feedback action to maintain good performance of the controller and manipulated variables in the presence of measurement noise and model error.

19.4 THE SMITH PREDICTOR

The control design by O. Smith (1957) preceded much of the general analysis of predictive systems; in fact, it predated the application of digital computers to process control, so that widespread implementation of Smith's results was delayed until real-time digital control computers became commercially available. Smith's approach, shown in Figure 19.8, relies on the general predictive structure in which the controller is calculated by the elements in the dashed box; these elements perform the function of the predictive control algorithm, $G_{cp}(s)$, in Figure 19.2.

Smith reasoned that "eliminating the dead time" from the control loop would be beneficial, which is certainly true but not possible via a feedback controller; only physical changes in the process can affect the feedback dead time. Therefore, Smith suggested that controlling a model of the process, without the dead time (or other noninvertible element), would provide a better calculation of the manipu-

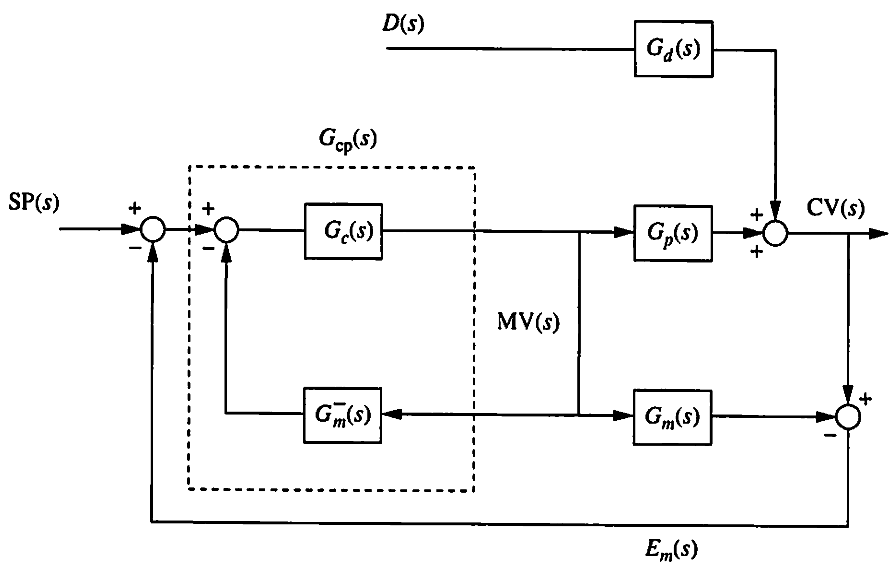


FIGURE 19.8

Block diagram of Smith predictor. G_c is a proportional-integral controller.

lated variable to be implemented in the true process. He retained the conventional PI control algorithm; thus, the system in Figure 19.8 consists of a feedback PI algorithm $G_c(s)$ that controls a simulated process, $G_m^-(s)$, which is easier to control than the real process. $G_m^-(s)$ has the same meaning here as for IMC control in equation (19.14), and the absence of dead time or inverse response (right-half-plane zero) in the model $G_m^-(s)$ allows much more aggressive control of the model than of the true plant.

The calculated manipulated variable resulting from controlling the model is implemented in the true process, which could yield good control as long as the model were perfect. Naturally, the model will not be perfect, and some form of feedback is required to achieve zero steady-state offset. Smith recognized the value of the predictive structure and, as shown in Figure 19.8, proposed correcting the model with the difference between the measured and the predicted controlled variables. Note that the prediction is determined using the complete linear dynamic model $G_m(s)$, including any noninvertible dynamics. The feedback signal $E_m(s)$ can be interpreted as a correction to the model $G_m^-(s)$.

The closed-loop transfer function of the system in Figure 19.8 is

$$\frac{\text{CV}(s)}{\text{SP}(s)} = \frac{G_c(s)G_p(s)}{1 + G_c G_m^-(s) + G_c(s)[G_p(s) - G_m(s)]} \tag{19.27}$$

If the model were perfect, the characteristic equation would not contain a dead time, because $G_m(s)$ and $G_p(s)$ would cancel. Thus, for the case with a perfect model, the characteristic equation involves only the expression $1 + G_c(s)G_m^-(s)$, which is easier to control and allows a more aggressive adjustment of the manipulated variable. Naturally, the true process is never known exactly, and the actual behavior and stability depend on all terms without cancellation. Application of the final value theorem to equation (19.27), for a step change in the set point and a PI algorithm for the controller, gives

$$\lim_{t\to\infty} \text{CV}(t) = \lim_{s\to 0} s \frac{\Delta\text{SP}}{s} \frac{G_c(s)G_p(s)}{1 + G_c G_m^-(s) + G_c(s)[G_p(s) - G_m(s)]} \tag{19.28}$$

For a stable process, $G_p(0) = K_p$ and $G_m(0) = K_m = G_m^-(0) = K_m^-$,

$$\begin{aligned}\lim_{t\to\infty} \text{CV}(t) &= \lim_{s\to 0} \Delta\text{SP} \frac{K_p K_c \left(1 + \frac{1}{T_I s}\right)}{1 + K_c \left(1 + \frac{1}{T_I s}\right) K_m^- + K_c \left(1 + \frac{1}{T_I s}\right)(K_p - K_m)} \\ &= \Delta\text{SP}\end{aligned} \tag{19.29}$$

> Thus, zero steady-state offset for a step input with Smith predictor control does not require a perfect model; it requires only that the steady-state gains for the two models be identical ($K_m = K_m^-$) and that the controller algorithm $G_c(s)$ have an integral mode.

Again, the performance and robustness of the Smith predictor control system depend on the controller tuning. The reader is cautioned that the PI controller in the

Smith predictor should not be tuned using correlations from Part III, which were developed for the conventional control structure, using $G_m^-(s)$ for the feedback dynamics. The purpose of the PI controller is to calculate an approximate inverse rapidly, as demonstrated by the following:

$$G_{cp} = \frac{\text{MV}(s)}{\text{SP}(s) - E_m(s)} = \frac{G_c(s)}{1 + G_c(s)G_m^-(s)} \approx \frac{1}{G_m^-(s)} \quad \text{for "large" } G_c(s) \tag{19.30}$$

Thus, the inverse would be approximated by a tightly tuned controller. A proper tuning procedure should consider the behavior of the controlled and manipulated variables as well as robustness for the model mismatch expected to be encountered. The proper tuning can be related to the IMC tuning by recognizing the equivalence of the IMC and Smith predictor for application to a process with first-order-with-dead-time feedback dynamics:

$$\text{Smith predictor:} \quad \frac{\text{MV}(s)}{T_p(s)} = \frac{G_c(s)}{1 + G_m^-(s)G_c(s)} = \frac{K_c\left(1 + \frac{1}{T_I s}\right)}{1 + K_c\left(1 + \frac{1}{T_I s}\right)\frac{K_m}{1 + \tau_m s}} \tag{19.31}$$

$$\text{IMC controller:} \quad \frac{\text{MV}(s)}{T_p(s)} = \frac{1}{K_m}\frac{\tau_m s + 1}{\tau_f s + 1} \tag{19.32}$$

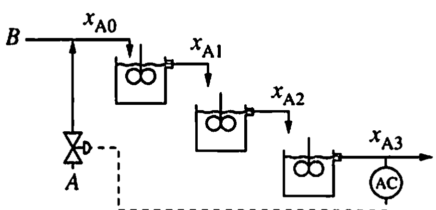
These two expressions can be shown to be equal when

$$K_c = \frac{\tau_m}{\tau_f K_m} \qquad T_I = \tau_m \tag{19.33}$$

Thus, the tuning correlations in Figure 19.6 along with equations (19.33) can be used to estimate initial tuning for the Smith predictor with a first-order-with-dead-time process model. Alternative guidelines are provided by Laughlin and Morari (1987).

The Smith predictor is easily programmed in a digital system. The digital form of the PI controller was presented in Chapter 11, and for a first-order-with-dead-time model, the digital models are programmed using equation (19.25) for $G_m(s)$ and the same equation with no dead time, $\Gamma = 0$, for $G_m^-(s)$.

EXAMPLE 19.9.



Apply the Smith predictor to the same process as considered in Example 19.7, the three-tank mixing process.

Again, the approximate first-order-with-dead-time model will be used as given in Example 19.7. The PI tuning can be estimated using Figure 19.6 and equations (19.33) to give

$$\theta/(\theta + \tau) = 0.34 \qquad \tau_f/(\theta + \tau) = 0.38 \qquad \tau_f = 6.1 \text{ min}$$

$$K_c = \tau/(\tau_f K_m) = 10.5/[(6.1)(.039)] = 44.1(\% \text{ open})/(\%\text{A}) \qquad T_I = \tau = 10.5 \text{ min}$$

The models $G_m(s)$ and $G_m^-(s)$ can be converted to digital approximations, as demonstrated in Example 19.8, and the PI controller can be programmed digitally, as shown in Chapter 11. The dynamic response of the control system, with the controller implemented as a digital algorithm, is essentially identical to the response

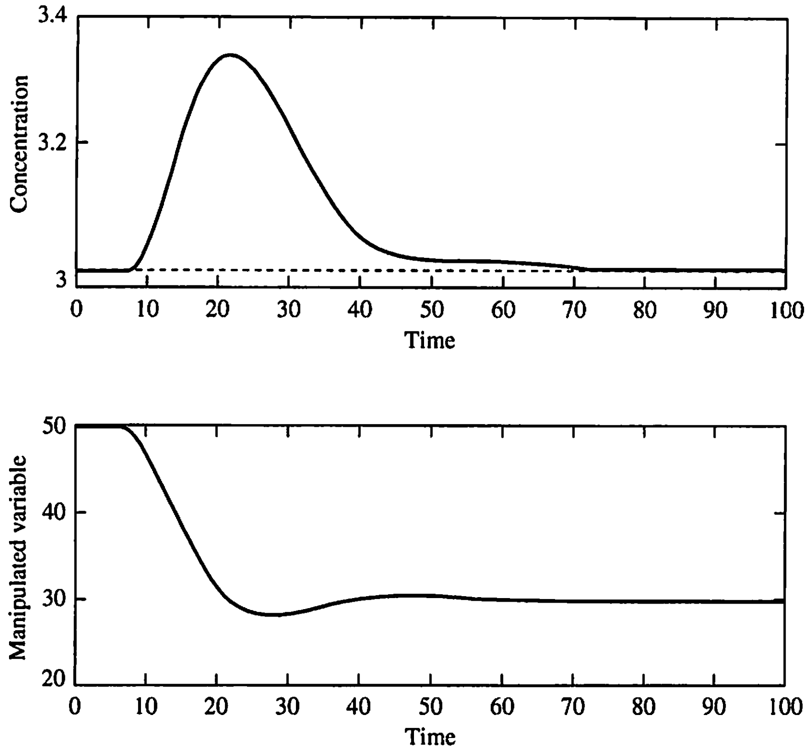


FIGURE 19.9

Dynamic response for three-tank system under Smith predictor control for Example 19.9 with $K_c = 88\%$ open/% A.

for the IMC controller shown in Figure 19.5, case B, so that plot is not repeated. The controller can be fine-tuned using the same approach as described in Section 9.5. For example, the controlled-variable performance for the base-case model can be improved by increasing the controller gain to 88% open/%A, as shown in Figure 19.9, which gives an IAE of 6.8. Since this tuning is more aggressive than the correlations in Figure 19.6, it is less robust and would not normally be used initially, but it could be reached through fine tuning if empirical experience indicated that the actual model errors and noise were smaller than anticipated in deriving the initial tuning correlation.

In conclusion, the Smith predictor conforms to the general principles of the predictive control structure. It employs a unique method for calculating an approximate model inverse: by controlling a model consisting of the invertible part of the model. This structure can achieve zero steady-state offset for steplike disturbances by conforming to easily achieved criteria. Again, the Smith predictor system is simple to implement in digital control and generally yields good control performance. The tuning of the PI controller must be appropriate for the predictive structure and can be adjusted to make the Smith predictor control more or less aggressive to provide the desired controlled- and manipulated-variable performance for the expected range of model mismatch.

19.5 ▣ IMPLEMENTATION GUIDELINES

It is important to remember that predictive controllers employ the same feedback principles as classical structures and involve basically the same tasks to design, implement, and operate. The engineering tasks include selecting the feedback measurement, selecting the manipulated variable, determining an appropriate model structure with parameters, selecting an algorithm, and establishing the tuning constants. In operating the system, process personnel must decide on the status of the controller—automatic or manual—and enter the set point value. Thus, predictive controllers can be presented to plant operating personnel in exactly the same manner as classical PID controllers, so that displays and faceplates need not be altered.

One programming detail is important for proper implementation: the variable used as the input to the model $G_m(s)$. This variable should have the value of the actual process input variable and must observe any limitations that exist in the plant, such as the valve being limited to 0 to 100% open. If this guideline is not observed, the control system will be subject to the undesirable integral (reset) windup, described in Chapter 12. When a limitation is reached in the manipulated variable, the controlled variable cannot be returned to its set point, regardless of the control algorithm used. In this situation, the magnitude of the controller output must not increase without limit (the symptom of integral windup). The behavior of the controller output can be determined by applying the final value theorem to the Laplace transform of the controller output, $MV(s)$, for the IMC control system in Figure 19.10. This is done in the following paragraphs for incorrect and correct implementations for a step disturbance; in both, $MV(s)$ is the output of the controller.

Incorrect (windup occurs): The IMC model input is the signal before any limitation, $MV(s)$, which can differ from the true value of the input variable to the process, $MV^*(s)$. The closed-loop transfer function for this system, when the

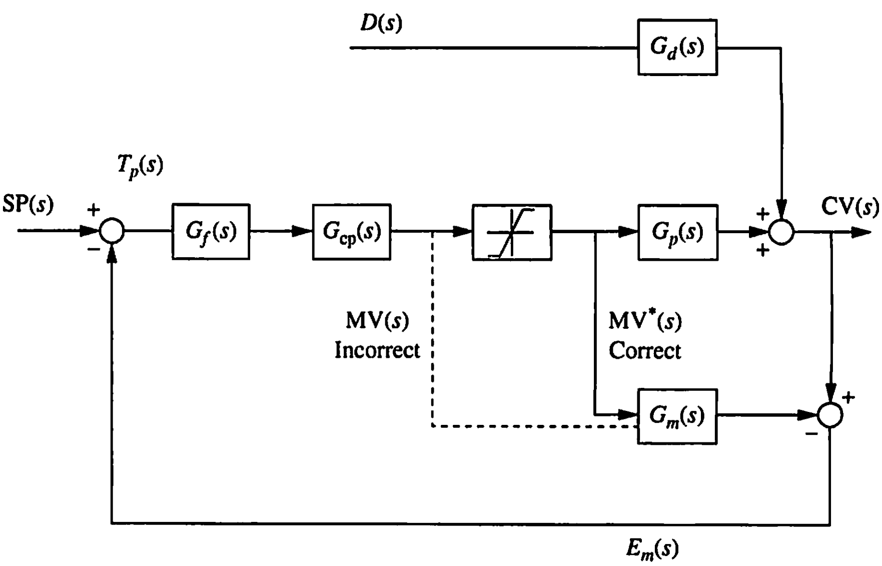


FIGURE 19.10

Predictive control structure with correct (solid) and incorrect (dashed) inputs to the model.

manipulated variable is at an upper or lower limit, is

$$\lim_{t\to\infty} \text{MV}(t) = \lim_{s\to 0} s\text{MV}(s) = \lim_{s\to 0} s \frac{-G_d(s)G_f(s)G_{cp}(s)\frac{\Delta D}{s}}{1 - G_f(s)G_{cp}(s)G_m(s)}$$

$$= \frac{-K_d K_f \frac{1}{K_m}\Delta D}{1 - K_f\left(\frac{1}{K_m}\right)K_m} \to \infty \tag{19.34}$$

Clearly, this implementation suffers from integral windup. This undesirable behavior results because of the requirements that $K_f = 1.0$ and $K_{cp} = K_m^{-1}$ for zero steady-state offset in the normal, unconstrained situation. The cause of reset windup could also be interpreted as the model $G_m(s)$ ceasing to represent the causal relationship between the calculated controller output and the measured controlled variable, which is properly zero when the limitation occurs.

Correct (windup prevented): The IMC model input is the true value of the variable in the process, $\text{MV}^*(s)$, which is affected by the process limitations. When the manipulated variable reaches a limit, it is constant, and the system behaves like an open-loop system.

$$\lim_{t\to\infty} \text{MV}(t) = \lim_{s\to 0} s\text{MV}(s)$$

$$= \lim_{s\to 0} s\left(-G_f(s)G_{cp}(s)G_d(s)\frac{\Delta D}{s}\right) \tag{19.35}$$

$$= -K_f\left(\frac{1}{K_m}\right)K_d\Delta D$$

This approach achieves the proper behavior because the model $G_m(s)$ represents a causal relationship that is valid for all situations. When the values of the predicted and the measured controlled variables reach essentially constant values, the feedback signal is constant except for disturbances, which vary over a limited range of values. The feedback signal in the correct implementation results in a value of the controller output that is limited to proportional responses to disturbances, as is proper to prevent integral windup.

With proper design and care for implementation details, digital implementation of predictive controllers is straightforward; in fact, the algorithms can be preprogrammed so that engineers need only select from a set of possible model structures and enter values for the model parameters and tuning constants. Thus, a predictive controller should not require more effort to implement than a standard PID algorithm.

19.6 ▣ ALGORITHM SELECTION GUIDELINES

To this point, single-loop predictive control has been introduced, the IMC and Smith predictor control algorithms have been presented, and tuning and programming guidelines have been provided. These controllers can be used in place of any PID controller; however, since the PID is the standard algorithm selected, a predictive control system is normally selected only when it performs better than a PID algorithm. In this section four applications are discussed in which predictive control offers potential advantages; the IMC controller will be used in all examples, but similar results can be obtained with a Smith predictor.

Long Dead Times

The Smith predictor is often referred to as a *dead time compensator* because Smith's original goal was to improve the performance of feedback control systems with long dead times in the feedback processes. When the model is perfect in Figure 19.3, the predictive system behaves like a feedforward controller; thus, the control action in this ideal situation can be as aggressive as required for the desired performance, without concern for stability. Even with modest model errors, the predictive system has the potential for improved performance when applied to processes with large fraction dead times. Selection of the proper algorithm (PID or predictive) depends on the particular situation; the advantages of predictive control are greater as the model is more accurate, the noise is small, and the feedback fraction dead time is large, usually greater than about 0.70.

Inverse Response

As explained in Section 13.5, an inverse response in the feedback process degrades the performance of a feedback controller in a manner similar to dead time. The PID algorithm has particular difficulty, because its error signal—the difference between the set point and the measured variable—initially increases in magnitude in spite of a proper initial feedback adjustment to the manipulated variable. The predictive controller has been reported to perform well, because its feedback signal—the difference between the predicted and the measured values—does not experience an inverse response (Iionya and Altpeter, 1962; Shunta, 1984).

Cascade Control

One of the design criteria presented in Chapter 14 for cascade control is that the secondary control loop must be much faster than the primary loop. There are situations in which a cascade is desirable for disturbance response, but the dynamic response of the secondary is not substantially faster than the primary. An example is a distillation composition controller that acts as a primary by resetting a tray temperature controller set point (e.g., Fuentes and Luyben, 1983). If the appropriate cascade criterion is not satisfied, significant fluctuation of the relatively slow secondary controlled variable causes a transient disturbance in the primary. If a PID control algorithm is used as the primary controller, unacceptable oscillations can occur.

A predictive control system for the primary in a cascade offers a distinct advantage, because the feedback signal is the sum of the model error in the primary, along with primary disturbances (Bartman, 1981). Secondary disturbances, which cause deviations in the secondary measurement, appear in both the measured and predicted primary variable at about the same time and magnitude (*if the model* is reasonably accurate). As a result, the secondary disturbances have little or no effect on the feedback signal, $E_m(s)$. This behavior is achieved when the model input is the measured secondary variable, as shown in Figure 19.11. Therefore, the

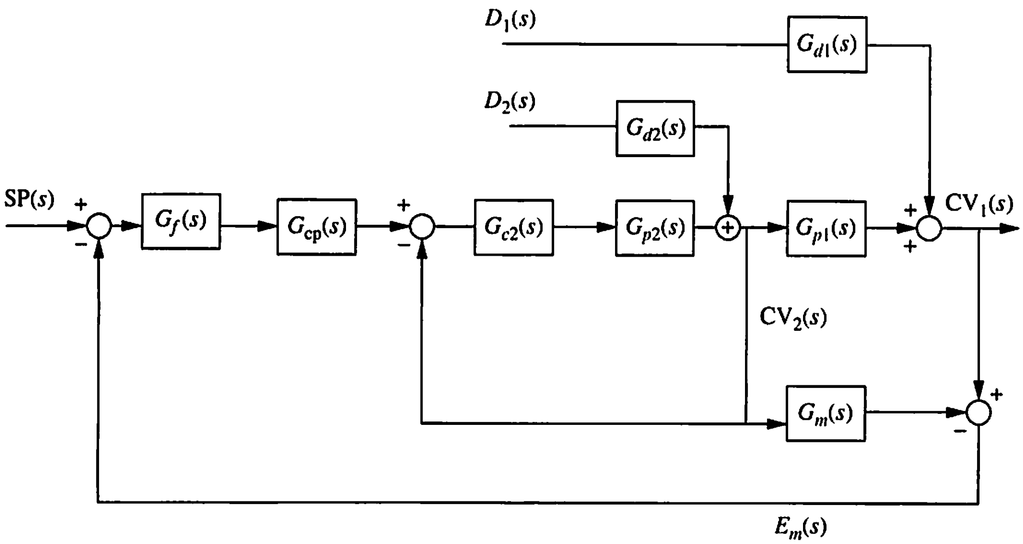


FIGURE 19.11

Cascade control with a primary predictive controller.

dynamic behavior of some cascade systems can potentially be better (i.e., much less oscillatory) when the primary is a predictive control system.

Feedforward

As described in Chapter 15, a feedforward-feedback control system can perform quite well when the process dynamics allow a complete compensation of the disturbance by the feedforward controller. This criterion is satisfied when the disturbance dead time is longer than the feedback process dead time (i.e., $\theta_d \geq \theta_p$). When this criterion is not satisfied, the feedforward controller changes the manipulated variable enough to compensate fully for the disturbance when the steady state is achieved; however, the initial effect of the disturbance appears in the measured controlled variable prior to the effect of the manipulated variable. A PID feedback controller cannot recognize that the feedforward compensation has been introduced and makes an unnecessary *additional* change to the manipulated variable.

The essential deficiency in the conventional feedforward-feedback design is the feedback PID controller, which cannot determine that the proper manipulation has been entered by the feedforward controller. This deficiency can be overcome through the use of a predictive control system for feedback, as shown in Figure 19.12. In this design, the predictive model includes relationships for the manipulated and measured disturbance variables, and the value of the manipulated variable used in the model includes the changes from both feedforward and feedback controllers. Again, the feedback signal is the difference between the measured and predicted controlled variables. As long as the models $G_{dm}(s)$ and $G_m(s)$ are reasonably accurate, the feedback signal, $E_m(s)$, does not change during the transient resulting from imperfect feedforward dynamics. In the situation of "slow" feedforward control ($\theta_d < \theta_p$), the model predictive feedback will not introduce additional adjustments to the manipulated variable.

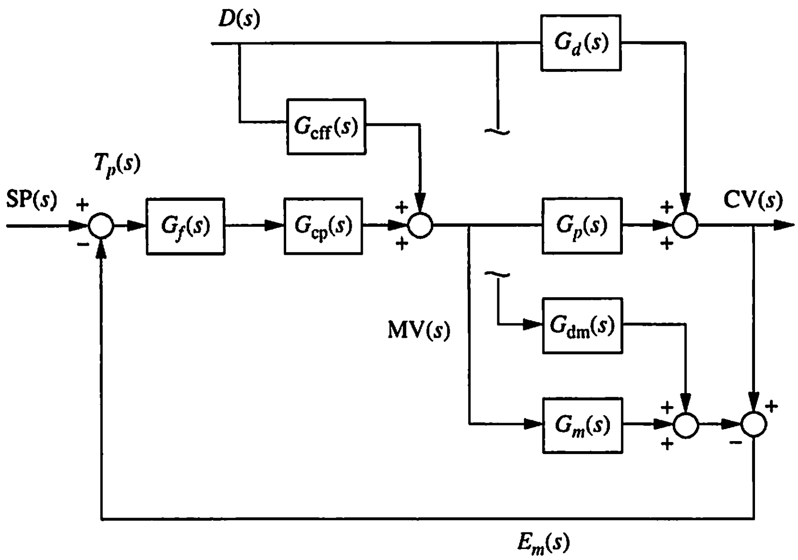


FIGURE 19.12

Feedforward with predictive feedback control.

The guidelines in this section indicate applications in which predictive control is likely to perform better than single-loop PID controllers. In many other applications, PID and predictive control systems give equivalent performance, and the usual selection is PID.

19.7 ◻ ADDITIONAL TOPICS IN SINGLE-LOOP MODEL PREDICTIVE CONTROL

The presentation in this chapter provides only an overview of predictive control, which is still a developing topic. A few important additional issues are introduced briefly in this section with reference to the IMC control system.

Digital Implementation

The procedure used here was to perform the design assuming that all variables were continuous, allowing Laplace transform methods, and subsequently, to convert the resulting model and controller to discrete form. A more general approach is to convert the models to the discrete form before performing the controller design, which enables more general model structures to be used. This approach requires the use of z-transforms (Appendix L; Ogata, 1987) and is presented in Appendix L and in Morari and Zafiriou (1989).

Controller Design

The method for calculating the controller was based on the goal of perfect control (zero deviation of the controlled variable from its set point). Since this goal is not possible, the controller design method factored the model and used only the invertible part, $G_{cp}(s) = [G_m^-(s)]^{-1}$. This approach leads to a realizable controller, but there is no guarantee that it is the "best" in any sense. Alternative controller algorithm design methods are based on other goals. For example, the controller could be designed to minimize the integral of the error squared, ISE, of the controlled variable during the disturbance response. This approach is presented by Newton et al. (1957) and applied to IMC control by Morari and Zafiriou (1989). Naturally, a tunable filter remains in the design to achieve the desired robustness and manipulated-variable behavior.

Filter Design

The filter described in this chapter improves the robustness of the predictive system at the expense of increased deviation of the controlled variable from its set point. Alternative filter designs can be selected to improve the response of the system to a specific disturbance. For example, the response of predictive systems to disturbances, as described in this chapter, can be somewhat slow, but the disturbance response can be improved by designing a filter that infers the disturbance variable $D(s)$ from the feedback signal $E_m(s)$. When there is no model error, this calculation requires that the filter be related to the inverse of the disturbance transfer function. While this concept is theoretically sound, it can lead to aggressive controllers that are tailored to a specific disturbance and may not respond well to other disturbance types. Again, some of these ideas are in Morari and Zafiriou (1989).

Robustness and Tuning

A vast literature is developing in methods for designing controllers using knowledge of the likely model errors, or mismatch. The key aspect of the design methods is to provide not only stability, but also the best performance possible, over the likely model errors. If the mismatch characterization is simple, like the gain margin used in Ziegler-Nichols, the methods are easily applied, but they can yield conservative feedback performance. This approach was promoted by the discussions and methods in Doyle and Stein (1981).

PID Tuning

The IMC controller can be expressed as an equivalent classical controller design, and this equivalence can be used to express PID tuning as a function of only one parameter: the IMC filter τ_f. Results have been developed by Rivera et al. (1986) and are summarized here for the "improved PI" tuning:

$$K_p K_c = \frac{2\tau + \theta}{2\tau_f} \qquad T_I = \tau + \frac{\theta}{2} \tag{19.36}$$

The recommendation is that $\tau_f \geq \max\,(1.7\theta, 0.2\tau)$ (Morari and Zafiriou, 1989).

19.8 ▫ CONCLUSIONS

In this chapter, an alternative feedback control structure and algorithm were introduced for processes that are open-loop stable. This predictive structure employs an explicit model of the process in the control calculations. In addition, the controller $G_{cp}(s)$ is designed to be an approximation of the process model inverse. Since the feedback signal is the disturbance [when $G_m(s)$ is a perfect model], the predictive controller functions somewhat like a feedforward controller and potentially can implement more aggressive adjustments to the manipulated variables.

Analysis of the predictive control structure provides a unified viewpoint for evaluating the effects of dynamic elements in the feedback process on control performance. In particular, the importance of dead time, inverse response (right-half-plane zeros), constraints, and model error are clearly identified.

> **Recall that the controller cannot eliminate dead time or inverse response from the feedback process. Substantial improvement in single-loop control performance requires process changes to reduce these unfavorable dynamic characteristics.**

The predictive structure has the feature of removing process elements that are difficult to control from the *calculation of the feedback* adjustment, when the model is perfect. In this situation, the feedback signal E_m does not depend on the controller output MV, because it is affected only by disturbances. In single-loop control, predictive control offers potential for improved performance in feedback processes with large fraction dead times, inverse responses, or both. Also, predictive control can be employed in cascade control with similar secondary and primary dynamics and in feedforward-feedback control with a disturbance dead time less than the feedback dead time.

In addition to providing useful single-loop control algorithms, the material in this chapter provides a general manner for analyzing feedback systems. Specifically, the predictive structure is the basis for the powerful multivariable control algorithm presented in Chapter 23.

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ADDITIONAL RESOURCES

The results in this chapter are limited to stable processes, which eliminates unstable processes such as the chemical reactors in Appendix C and non-self-regulating levels. The approach can be extended, as described in Morari and Zafiriou (1989).

QUESTIONS

19.1. For the following processes, design IMC and Smith predictor model predictive controllers. Specify all parameters and give all equations for digital implementation. Simulate each for a set point change.

(*a*) The process in question 6.1, controlling temperature by adjusting the valve.

(*b*) The process in question 6.2, controlling temperature by adjusting the valve.

(*c*) The nonisothermal chemical reactor in Example 13.12, controlling the reactor concentration, C_A, by manipulating *(i)* the pure A feed valve (v_A) or *(ii)* the coolant flow valve (v_C).

(*d*) The chemical reactors in Example 3.3, controlling outlet composition C_{A2} by adjusting the inlet composition C_{A0}. Use an approximate first-order-with-dead-time model for $G_m(s)$ and in designing $G_{cp}(s)$.

19.2. The three-tank mixing process with IMC control was investigated in Example 19.8 for various flow rates. Using the deterministic-calculation approach introduced in Section 16.3, determine a method for maintaining good IMC control performance as the measured flow rate changes over the range in Table 16.1.

19.3. The following process models have been identified for processes that conform to the block diagram in Figure 19.11. For each process, determine whether cascade control or single-loop control is appropriate, assuming that $D_2(s)$ is a significant disturbance. For cascades, decide whether the performance might be improved by using an IMC controller as the primary.

Assume that $G_{d1}(s) = G_{p1}(s)$ and $G_{d2}(s) = G_{p2}(s)$.

	Secondary	**Primary**
(*a*)	$G_{p2}(s) = \dfrac{0.5}{\tau s + 1}$	$G_{p1}(s) = \dfrac{2.3}{(\tau s + 1)^5}$
(*b*)	$G_{p2}(s) = \dfrac{0.5}{(\tau s + 1)^3}$	$G_{p1}(s) = \dfrac{2.3}{(\tau s + 1)^3}$
(*c*)	$G_{p2}(s) = \dfrac{0.5}{(\tau s + 1)^5}$	$G_{p1}(s) = \dfrac{2.3}{(\tau s + 1)}$

19.4. (*a*) Verify the equalities given in equations (19.31) to (19.33), relating the IMC and Smith predictor approximate inverses for $G_m^-(s) = K_m/(1 + \tau s)$.
(*b*) Verify the relationship in equation (19.10) relating the model predictive and classical controllers.
(*c*) In Figures 19.10 to 19.12, which of the transfer functions represent control calculations and which represent process behavior?
(*d*) Determine the criteria for zero steady-state offset with model predictive control of a stable process with an impulse input change.

19.5. Describe a proper method for providing anti-reset windup for the Smith predictor. Include a block diagram and apply the final value theorem to prove that your design is adequate.

19.6. Perform the following analysis for the stirred-tank heat exchanger in Example 8.5 under IMC feedback control. To simplify the analysis, assume a perfect model when determining the analytical solution and that $G_d(s) = G_p(s)$.
(*a*) Analyze the degrees of freedom.
(*b*) Derive the linearized model for the process and controller.
(*c*) Determine the analytical solution for the controlled variable for a step set point change. Assume that $\tau_f = \tau$.
(*d*) Determine the analytical solution for the manipulated variable for a step set point change. Assume that $\tau_f = \tau$.
(*e*) Recalculate the results in (*c*) and (*d*) for $\tau_f = \beta\tau$, with $\beta = 0.5$ and 0.1. Sketch the shape of the dynamic responses of the controlled and manipulated variables for these three values of τ_f.
(*f*) Select the best value of τ_f for the heat exchanger, not necessarily one of the values considered in previous parts of this question.

19.7. The selection of the manipulated and controlled variables is discussed in Chapter 7. Discuss how these criteria should be modified for feedback control using model predictive control.

19.8. A mixing process with the structure in Figure 13.4 and with the following feedback and disturbance transfer functions is to be controlled with an IMC controller. The controlled variable is to be maintained within ±0.37 of the set point for a unit step disturbance. What value of the IMC controller filter is required to achieve this performance?

$$G_p(s) = \frac{1.0e^{-0.5s}}{1 + 1.5s} \qquad G_d(s) = \frac{1.0}{1 + 1.0s}$$

19.9. The chemical reactor process described in Examples I.2 and 13.8 has a feedback system with the outlet concentration controlled by adjusting the solvent flow rate. Design IMC and Smith predictor model predictive controllers for feedback control. Program one of them and compare the control performance with that achieved with PI feedback in Example 13.8. Discuss the relative performances and steps required to substantially improve the control performance.

19.10. Draw a block diagram for the Smith predictor control system. In each block that involves controller calculations, show the equations solved in the digital implementation. Assume that an adequate process model is first-order with dead time.

19.11. The results of the tuning study given in Figure 19.6 show that the IMC filter factor decreases as the fraction dead time increases. Since the filter was introduced to make the feedback adjustments less aggressive and more robust, one might expect larger filter values to be necessary at large fraction dead times. Discuss the effects of the filter and reconcile the tuning correlations with the robustness expectations just stated. (Hint: Consider all control performance criteria and process conditions involved in determining the tuning correlations. They are described in Table 9.1.)

19.12. Analyze the control performance for IMC (or Smith predictor) feedback control of the three-tank mixing process using *closed-loop frequency response*. The process is modelled in Example 7.2.

(*a*) Derive the expression used for this calculation assuming that the controller uses the first-order-with-dead-time approximation in Example 19.5 and $\tau_f = 6.1$. (Do not solve for the real and imaginary parts analytically.)

(*b*) Use a computer program to evaluate the magnitude of the transfer function over a range of frequencies; that is, determine $|CV(j\omega)|/|D(j\omega)|$.

(*c*) Compare the results in (*b*) with the equivalent results in Figure 13.16 (curve *a*) for PI control, discussing similarities and differences.

19.13. A method for analyzing the stability of the model predictive control systems should be available. Perform the following analysis for the three-tank mixing process under IMC control for two cases: $\tau_f = 0$ and $\tau_f = 6.1$. Use the continuous transfer functions from Example 19.6.

(*a*) Determine the expression for $G_{OL}(s)$ that could be used to analyze stability.

(*b*) Determine the magnitude and phase angle of $G_{OL}(j\omega)$ for various values of frequency, including several decades around the critical frequency. Present the results in Bode plots.

(*c*) Evaluate the Bode plot for the assumptions required for use in Bode stability analysis.

(*d*) Based on the results of (*b*) and (*c*), discuss the use of the Bode method for stability analysis of model predictive control systems. Suggest a more general method that is applicable.

19.14. Consider a process with first-order-with-dead-time feedback dynamics and first-order disturbance dynamics (e.g., the process in Figure 13.4). The system is to be controlled with an IMC system and is subject to an unmeasured step disturbance. Assume the model predictive control calculations can be designed with perfect models. Answer the following questions for two cases: (1) zero feedback dead time and (2) nonzero feedback dead time.

(*a*) Define the best, physically possible feedback control performance. For this question alone, control performance is determined completely by the ISE of the controlled variable.

(*b*) Determine the transient response of the manipulated variable that would result in the behavior of the controlled variable determined in (*a*).

(*c*) Derive the IMC controller and filter $G_f(s)G_{cp}(s)$ that would give the performance defined in (*a*) and (*b*) using the feedback measurement only.

19.15. Using the IMC tuning rules for a PI controller in equation (19.36) and Figure 19.6, develop graphs of K_pK_c and $T_I/(\theta+\tau)$ versus the fraction feedback dead time, $\theta/(\theta+\tau)$. Compare these graphs with Figure 19.9.