# Feedforward Control

CHAPTER 15

## 15.1 □ INTRODUCTION

*Feedforward* uses the measurement of an input disturbance to the plant as additional information for enhancing single-loop PID control performance. This measurement provides an "early warning" that the controlled variable will be upset some time in the future. With this warning the feedforward controller has the opportunity to adjust the manipulated variable before the controlled variable deviates from its set point. Note that the feedforward controller does not use an output of the process! This is the first example of a controller that does not use feedback control; hence the new name *feedforward.* As we will see, feedforward is usually combined with feedback so that the important features of feedback are retained in the overall strategy.

Feedforward control is effective in reducing the influences of disturbances, although not usually as effective as cascade control with a fast secondary loop. Since feedforward control also uses an additional measurement and has design criteria similar to cascade control, engineers often confuse the two approaches. Therefore, the reader is urged to master the design criteria for feedforward control introduced in this chapter and be able to distinguish between opportunities for cascade and feedforward designs.

## 15.2 □ AN EXAMPLE AND CONTROLLER DERIVATION

The process example used in this introduction is the same stirred-tank heat exchanger considered in Chapter 14 for cascade control. The control objective is still the maintenance of the outlet temperature very close to its set point, and the manipulated variable is still the heating medium valve position. The only difference is that the heating oil pressure is not varying significantly; thus, the cascade controller is not required, as shown in Figure 15.1. In this case, the inlet temperature varies with sufficient amplitude to disturb the outlet temperature significantly. The challenge is to design a feedforward controller that reduces or, in the ideal case, eliminates the effect of the inlet temperature on the outlet temperature by adjusting the heating oil valve.

The approach to designing a feedforward controller is based on completely cancelling the effect of the disturbance. This concept is sketched in Figure 15.2. The disturbance is shown as a step change to simplify the discussion, but the analysis and resulting feedforward controller are applicable to any disturbance of arbitrary time dependence. The change in the outlet temperature in response to the inlet temperature change, shown as curve A, is the response that would occur without control. For perfect control, the outlet temperature would not change; this is shown as curve B. To achieve perfect control the manipulated variable must be adjusted to compensate for the disturbance—that is, to cause the mirror image of the disturbance so that the sum of the two effects is zero. Thus, curve C shows the effect of the manipulated variable on the outlet temperature; the sum of curves A and C is a zero disturbance to the controlled variable, which gives perfect feedforward control. The feedforward control algorithm uses the measurement of the disturbance to calculate the manipulated variable with the goal of perfect feedforward compensation as shown in Figure 15.2.

The control calculation that achieves this goal can be derived by analyzing the block diagram of the feedforward control system in Figure 15.3. The individual blocks account for the process $G_p(s)$, the disturbance $G_d(s)$, and the feedforward controller $G_{ff}(s)$. The equation that relates the measured disturbance to the outlet variable is

$$CV(s) = [G_d(s) + G_{ff}(s)G_p(s)]D_m(s) \tag{15.1}$$

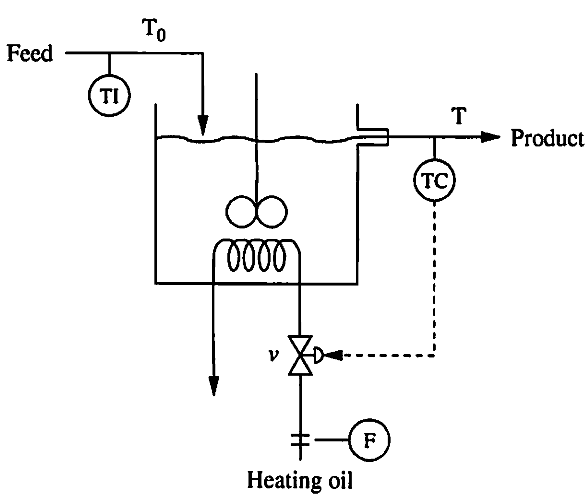


**FIGURE 15.1**

**Stirred-tank heat exchanger with single-loop feedback temperature control.**

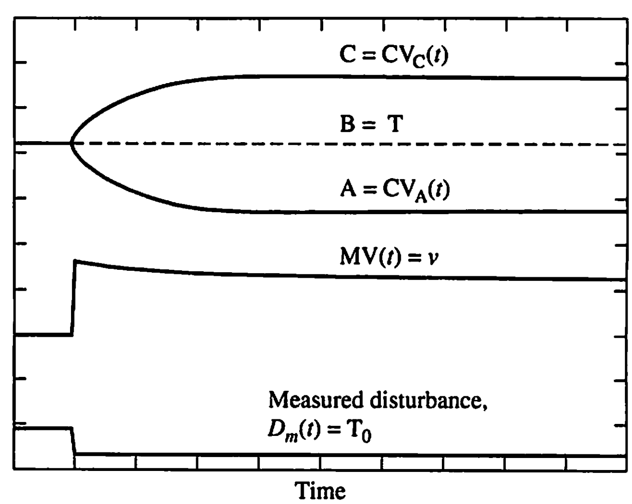


**FIGURE 15.2**

**Time domain plot showing perfect feedforward compensation for a measured disturbance.**

Since equation (15.1) involves deviation variables, the goal is to maintain the outlet temperature at zero, $CV(s) = 0$ $[T'_{out}(s) = 0]$. The only unknown, which is the controller, $G_{ff}(s)$, can be determined by rearranging equation (15.1). The result is

Feedforward controller design: $$G_{ff}(s) = -\frac{G_d(s)}{G_p(s)} \tag{15.2}$$

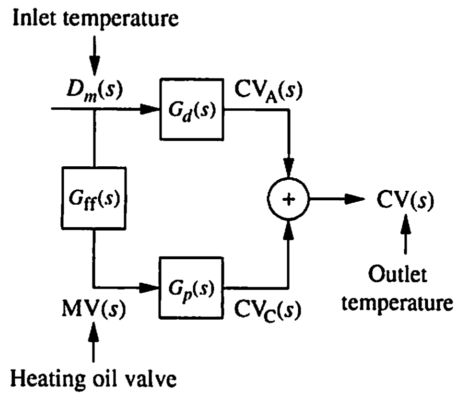


**FIGURE 15.3**

**Simplified block diagram of feedforward compensation.**

It is important to note that the feedforward controller depends on the models for the disturbance and the process. The feedforward controller is never a PID algorithm—a result that should not be surprising, because we have a new control design goal that does not apply feedback principles.

Equation (15.2) provides the general feedforward control equation. Typical transfer functions for the disturbance and the process are now substituted to derive the most common form of the controller. Assume that the transfer functions have the following first-order-with-dead-time forms:

$$\frac{CV(s)}{MV(s)} = G_p(s) = \frac{K_p e^{-\theta s}}{\tau s + 1} \qquad \frac{CV(s)}{D_m(s)} = G_d(s) = \frac{K_d e^{-\theta_d s}}{\tau_d s + 1} \tag{15.3}$$

By substituting equations (15.3) into equation (15.2), the feedforward controller would have the form

$$G_{ff} = \frac{MV(s)}{D_m(s)} = -\frac{G_d(s)}{G_p(s)} = K_{ff}\left(\frac{T_{ld}s + 1}{T_{lg}s + 1}\right)e^{-\theta_{ff}s} \tag{15.4}$$

where
lead/lag algorithm $= (T_{ld}s + 1)/(T_{lg}s + 1)$
feedforward controller gain $= K_{ff} = -K_d/K_p$
feedforward controller dead time $= \theta_{ff} = \theta_d - \theta$
feedforward controller lead time $= T_{ld} = \tau$
feedforward controller lag time $= T_{lg} = \tau_d$

In most (but not all) cases, this form of the feedforward controller provides sufficient accuracy; usually, second- or higher-order terms in the controller do not improve the control performance, especially because the models are not known exactly.

The special form of the feedforward controller in equation (15.4) consists of a gain, dead time, and a factor called a *lead/lag.* The dynamic behaviors of gains and dead times are well known by this point in the book, but lead/lag is new, so a few typical dynamic responses are presented in Figure 15.4. Each result uses the same lead/lag algorithm with different parameters as indicated. Again, for simplicity, the input is a step change, but the feedforward controller with a lead/lag performs well for any input function. The analytical expression for the output of a lead/lag, here represented as $y(t)$, for a *unit step input* can be determined from entry 5 in Table 4.1 to be

$$y(t) = 1 + \frac{T_{ld} - T_{lg}}{T_{lg}} e^{-t/T_{lg}} \tag{15.5}$$

As seen in the figure of dynamic responses, when the lead time is less than the lag time, the manipulated variable rises to the steady-state value as a first-

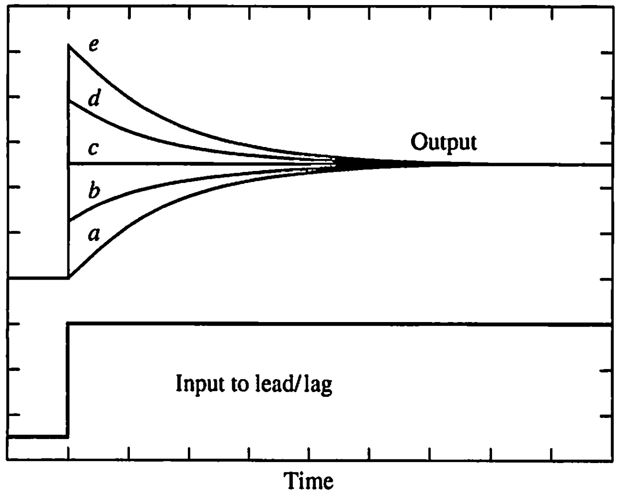


**FIGURE 15.4**

**Example of dynamic responses for a lead/lag algorithm. Ratios of lead to lag times are: (*a*) 0; (*b*) 0.5; (*c*) 1.0; (*d*) 1.5; (*e*) 2.0.**

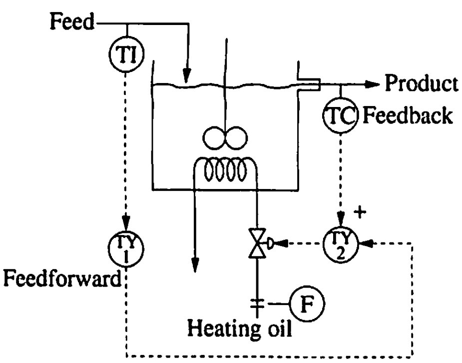


**FIGURE 15.5**

**Stirred-tank heat exchanger with feedforward-feedback control strategy.**

order response from an initial step that does not reach the final steady state. This is consistent with a process whose disturbance time constant is greater than its process time constant, requiring the control action to be slowed so that the effects of the disturbance and the feedforward controller cancel. When the lead and lag times are equal, the manipulated variable immediately attains its steady-state value. This is consistent with a process that has equal disturbance and process time constants. Finally, when the lead time is greater than the lag time, the manipulated variable initially exceeds, then slowly returns to, its steady-state value. This is consistent with a process whose disturbance time constant is smaller than its process time constant.

The derivation of the feedforward controller ensures perfect control if (1) the models used are perfect, (2) the measured disturbance is the only disturbance experienced by the process, and (3) the control calculation is realizable, that is, capable of being implemented as discussed in Section 15.7. Neither of conditions 1 or 2 is generally satisfied. Therefore, feedforward control is always combined with feedback control, when possible, to ensure zero steady-state offset! Since the process and control calculations are considered to be linear, the adjustments to the manipulated variable from the feedforward and feedback controllers can be added. A typical feedforward-feedback control system is given in Figure 15.5 for the stirred-tank heat exchanger.

## 15.3 FEEDFORWARD CONTROL DESIGN CRITERIA

The principles of feedforward control have been introduced with respect to the stirred-tank heater. In Table 15.1 the design criteria are summarized in a concise form so that they can be applied in general. Adherence to these criteria ensures that feedforward control is used when appropriate.

The first two items in the table address the application of feedforward control. Naturally, only when feedback control does not provide acceptable control

**TABLE 15.1**

**Feedforward control design criteria**

| **Feedforward control is desired when** |
|---|
| 1. Feedback control does not provide satisfactory control performance. |
| 2. A measured feedforward variable is available. |
| **A feedforward variable must satisfy the following criteria:** |
| 3. The variable must indicate the occurrence of an important disturbance. |
| 4. There must *not* be a causal relationship between the manipulated and feedforward variables. |
| 5. The disturbance dynamics must not be significantly faster than the manipulated–output variable dynamics (when feedback control is also present). |

performance is an enhancement like feedforward control employed. The second criterion requires that an acceptable measured feedforward variable be available or that it can be added at reasonable cost.

A potential feedforward variable must satisfy three criteria. First, it must indicate the occurrence of an important disturbance; that is, there must be a direct, reproducible correlation between the process disturbance and the measured feedforward variable, and the measured variable should be relatively insensitive to other changes in operation. Naturally, the disturbance must be important (i.e., change frequently and have a significant effect on the controlled variable), or there would be no reason to attenuate its effect. Second, the feedforward variable must *not* be influenced by the manipulated variable, because the feedback principle is not used. Note that this requirement provides a clear distinction between variables used for cascade and feedforward. Finally, the disturbance dynamics should not be faster than the dynamics from the manipulated to the controlled variable.

This final requirement is related to combined feedforward-feedback control systems. Should the effect of the disturbance on the controlled variable be very fast, feedforward could not affect the output variable in time to prevent a significant deviation from the set point. As a result, the feedback controller would sense the deviation and adjust the manipulated variable. Unfortunately, the feedback adjustment would be in addition to the feedforward adjustment; thus, a double correction would be made to the manipulated variable; remember, the feedforward and feedback controllers are independent algorithms. The double correction would cause an overshoot in the controlled variable and poor control performance. In conclusion, feedforward control should not be used when the disturbance dynamics are very fast and PID feedback control is present. Naturally, if feedback is not present (perhaps due to the lack of a real-time sensor), feedforward can be applied regardless of the disturbance dynamics.

## Feedforward and Feedback Are Complementary

Feedforward and feedback control each has important advantages that compensate for deficiencies of the other, as summarized in Table 15.2. The major advantage of

**TABLE 15.2**

**Comparison of feedforward and feedback principles**

| | **Feedforward** | **Feedback** |
|---|---|---|
| Advantages | Compensates for a disturbance before the process output is affected | Provides zero steady-state offset |
| | Does not affect the stability of the control system | Effective for all disturbances |
| Disadvantages | Cannot eliminate steady-state offset | Does not take control action until the process output variable has deviated from its set point |
| | Requires a sensor and model for each disturbance | Affects the stability of the closed-loop control system |

feedback control is that it reduces steady-state offset to zero for all disturbances. As we have seen, it can provide good control performance in many cases but requires a deviation from the set point before it takes corrective action. However, feedback does not provide good control performance when the feedback dynamics are unfavorable. In addition, feedback control can cause instability if not correctly tuned.

Feedforward control acts before the output is disturbed and is capable of very good control performance with an accurate model. Another advantage is that a stable feedforward controller cannot induce instability in a system that is stable without feedforward control. This fact can be demonstrated by analyzing the transfer function of a feedforward-feedback system shown in Figure 15.6, which accounts for sensors and the final element explicitly:

$$\frac{\mathrm{CV}(s)}{D_m(s)} = \frac{G_v(s)G_p(s)G_{\mathrm{ffs}}(s)G_{\mathrm{ff}}(s) + G_d(s)}{1 + G_v(s)G_p(s)G_{\mathrm{fbs}}(s)G_c(s)} \tag{15.6}$$

As long as the numerator is stable, which is normally the case, stability is influenced by the terms in the characteristic equation, which contain terms for the feedback process, instrumentation, and controller. The disturbance process, feedforward instrumentation, and feedforward controller appear only in the numerator. Therefore, a (stable) feedforward controller cannot cause instability, although it can lead to poor performance if improperly designed and tuned. The major limitation to feedforward control is its inability to reduce steady-state offset to zero. As explained, this limitation is easily overcome by combining feedforward with feedback.

**Feedforward control** uses a measured input disturbance to determine an adjustment to an input manipulated variable. All feedforward control strategies should conform to the design criteria in Table 15.1.

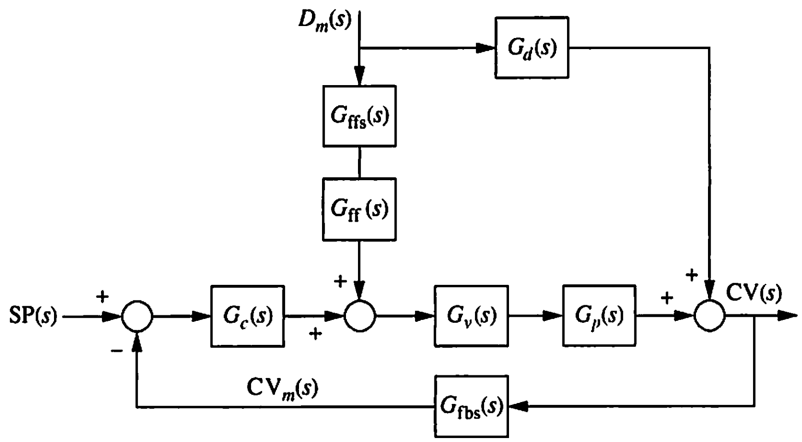


**FIGURE 15.6**

**Block diagram of feedforward-feedback control system with sensors and final element.**

## 15.4 FEEDFORWARD PERFORMANCE

In the introduction to this chapter, feedforward control was described as simple and effective. The foregoing material has demonstrated how simple a feedforward-feedback strategy is to evaluate and design. In this section, the performance is calculated for a few sample systems and compared to the single-loop feedback-only performance to demonstrate the effectiveness of feedforward. Due to the large number of process and controller parameters, no general correlations concerning feedforward control performance are available. The general trends in these examples should be applicable to most realistic processes.

Based on the feedforward design method, perfect control performance is theoretically possible; however, it is never achieved because of model errors. Therefore, a key factor in feedforward control performance is model accuracy. A typical feedforward-feedback system consistent with the block diagram in Figure 15.6 was simulated for various cases; this system can be thought of as the heat exchanger system in Figure 15.5 with the following process and controller models:

$$G_p(s) = \frac{e^{-15s}}{20s+1} \qquad G_d(s) = \frac{e^{-30s}}{20s+1}$$

$$G_{ff}(s) = -1.0e^{-15s}\frac{20s+1}{20s+1} = -1.0e^{-15s} \tag{15.7}$$

$$G_{ffs}(s) = 1 \qquad G_{fbs}(s) = 1 \qquad G_v(s) = 1 \qquad G_c(s) = 0.9\left(1 + \frac{1}{20s}\right)$$

The upset was a single step change and no noise was added to the measurements, so that the effect of the control alone could be determined. The actual process and disturbance responses remain unchanged for all cases; the feedforward controller tuning parameters are changed to determine the effect of controller model errors on performance. The feedback PI controller was tuned by conventional means for good regulation of the controlled variable without excessive variation in the manipulated variable. The control performance measure was the integral of the absolute value of the error (IAE).

The resulting control performances are shown in Figure 15.7 as a function of the feedforward model error. Note that the results are reported relative to the feedback-only performance, so any value less than 100 percent represents an improvement through feedforward control; recall that the system returns to the set

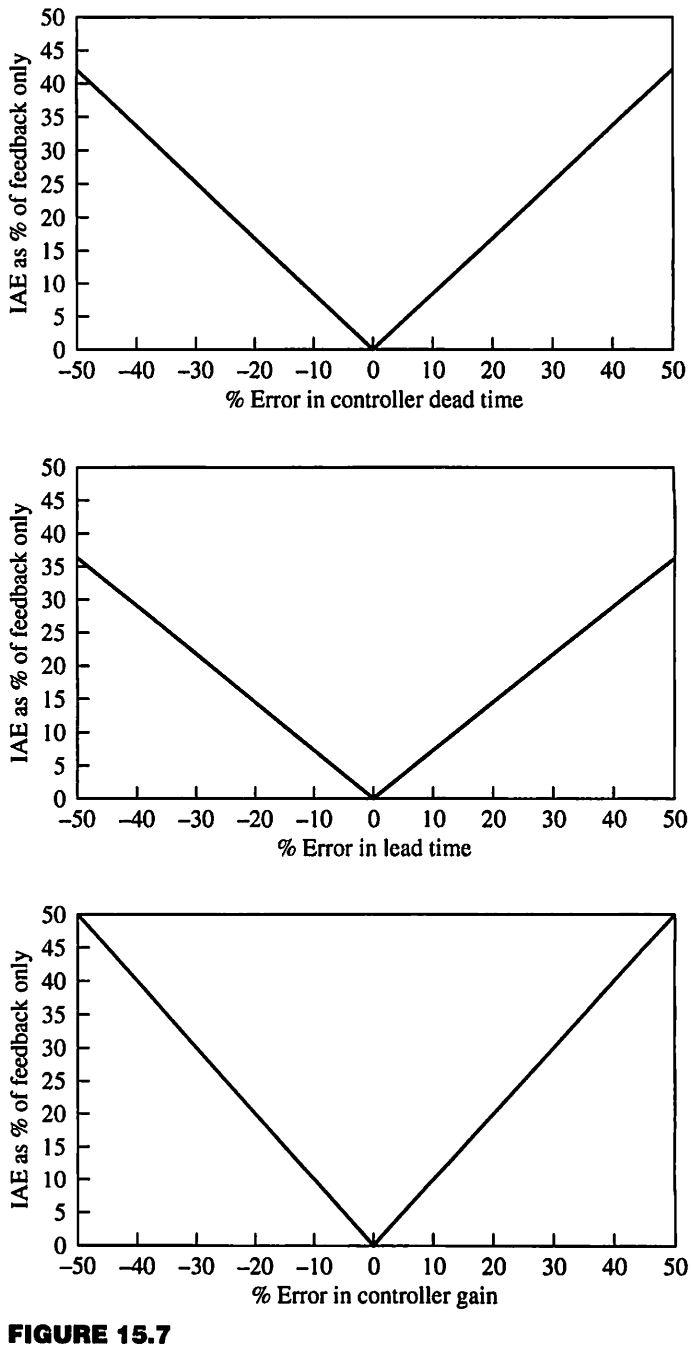


**FIGURE 15.7**

**Example of the effect of errors in the feedforward controller on the performance, reported as a percent of feedback-only IAE.**

point, even with feedforward errors, because of the feedback controller's integral mode. This provides a simple comparison of performances in a manner useful for answering the key question of whether or not to use feedforward to enhance feedback. Separate plots provide the control performance with errors in the gain, dead time, and ratio of lead to lag times. For each of these plots the other model parameters matched the process exactly.

The results demonstrate that feedforward control can substantially improve control performance, even with significant errors in the model used. For this process studied, feedforward would provide substantial improvement, maintaining the IAE much lower than that achieved by feedback-only for the large range of model errors considered. This insensitivity of performance to model error leads to robust control over a large range of process dynamics without updating feedforward controller parameters.

Typical transient responses with feedback/feedforward control are given in Figure 15.8*a* through *e* for the example system subject to a unit step disturbance. Figure 15.8*a* shows the performance of feedback-only, and the next three parts show the performance of the feedforward-feedback control system with model

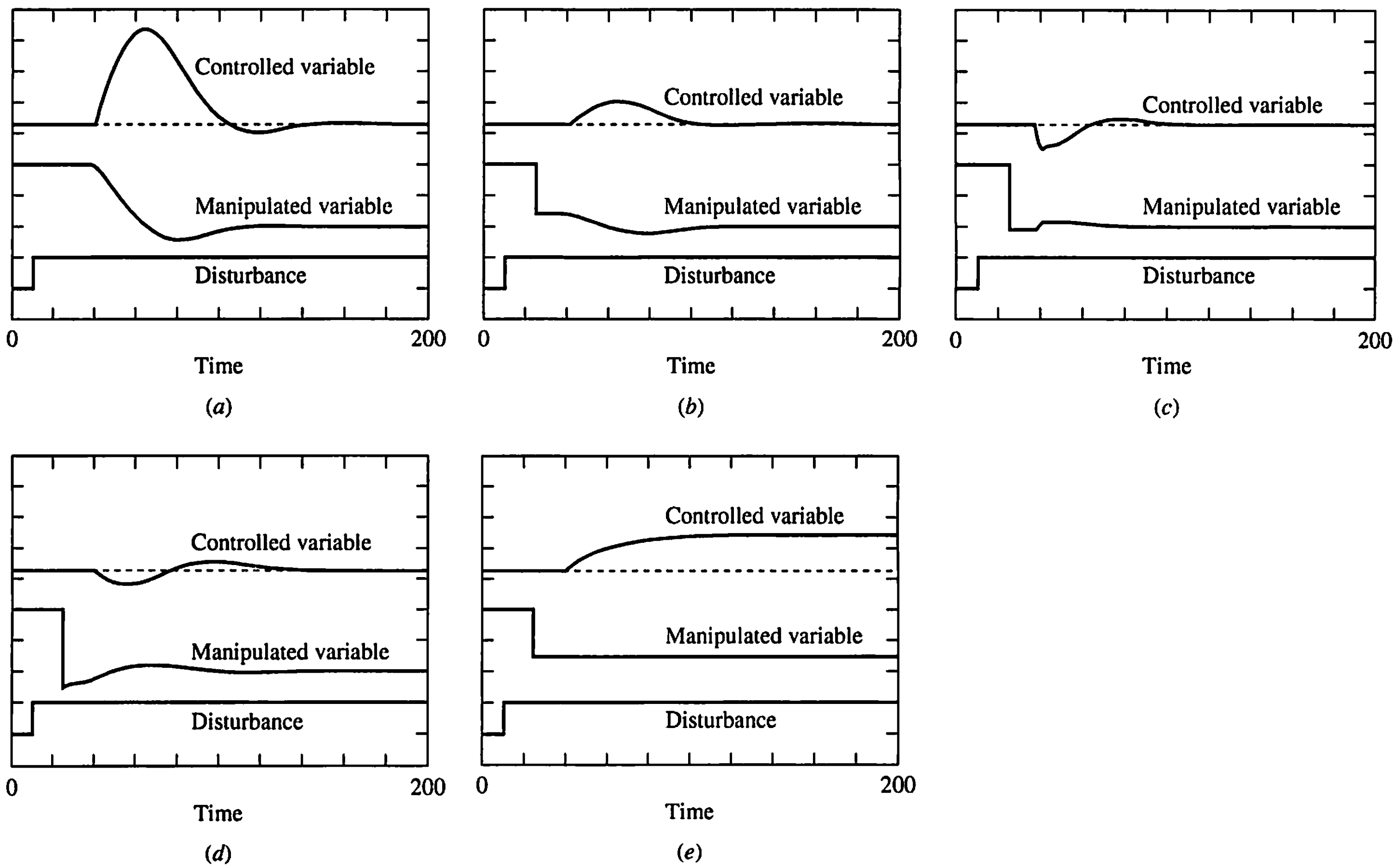


**FIGURE 15.8**

**Transient responses: (*a*) feedback-only; (*b*) feedforward-feedback with −25% error in $K_{ff}$; (*c*) feedforward-feedback with −20% error in $\theta_{ff}$; (*d*) feedforward-feedback control with +25% error in the $\tau_{ld}$; (*e*) feedforward-only control with −25% error in $K_{ff}$. One tick (10% of scale) is 0.2 for the controlled variable, 0.50 for the manipulated variable, and 1.0 for the disturbance.**

errors as indicated in the caption. These sample results, along with Figure 15.7, demonstrate the general insensitivity of feedforward control to model errors, which is an important property contributing to its successful application. The final sample result, Figure 15.8*e*, shows the performance with feedforward-only control, which gives steady-state offset unless the feedforward gain is perfect—a highly unlikely situation. The steady-state offset could be determined by applying the final value theorem to equation (15.1).

The results in Figures 15.7 and 15.8 support a frequently used simplification to feedforward control. Often the lead/lag and dead time elements are eliminated from the feedforward controller; the resulting controller is usually called *steady-state feedforward.* This simplification does not substantially degrade control performance when the feedforward controller dead time is small and the lead and lag times are nearly equal. In conclusion:

> **Feedforward control can substantially improve control performance of processes for which feedback alone does not provide acceptable control, and its performance does not degrade rapidly with model errors.**

## 15.5 ◻ CONTROLLER ALGORITHM AND TUNING

The approach to deriving the feedforward controller algorithm was described along with the first example in Section 15.2. The controller is expressed as a transfer function in that section. Analog implementation would require an electrical circuit that closely approximates the transfer function. Such a circuit would be costly and is seldom made for a range of model structures, but it is available for the lead/lag with gain. To clarify the application of feedforward, the *digital* implementation of a typical feedforward controller is developed here. The programming of the controller is shown schematically in Figure 15.9. The gain is simply a multiplication. The dead time can be implemented by using a table of data whose length times the sample period equals the dead time. The data location (or pointer) is shifted one step every time that the controller is executed. The lead/lag element must be transformed into a digital algorithm. One way to do this is to convert the lead/lag into a differential equation by remembering that multiplication of the Laplace transform

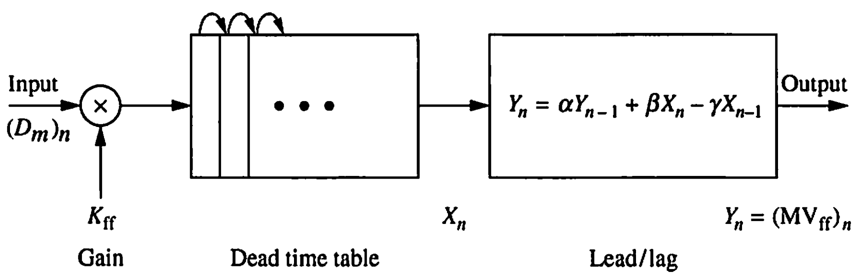


**FIGURE 15.9**

**Schematic diagram of a digital feedforward controller.**

by the variable $s$ is equivalent to differentiation. The resulting equation is

$$T_{\text{lg}} \frac{dY(t)}{dt} + Y(t) = T_{\text{ld}} \frac{dX(t)}{dt} + X(t) \tag{15.8}$$

with $X$ the input to the lead/lag and $Y$ the output from the lead/lag algorithm. The differential equation can be transformed into a difference equation by approximating the derivative by a backward difference, as follows:

$$\frac{dY(t)}{dt} \approx \frac{Y_n - Y_{n-1}}{\Delta t} \qquad \frac{dX(t)}{dt} \approx \frac{X_n - X_{n-1}}{\Delta t} \tag{15.9}$$

The resulting equation can be rearranged to yield the following equation, which can be used in a digital computer to implement the digital lead/lag.

$$Y_n = \left( \frac{\frac{T_{\text{lg}}}{\Delta t}}{\frac{T_{\text{lg}}}{\Delta t} + 1} \right) Y_{n-1} + \left( \frac{\frac{T_{\text{ld}}}{\Delta t} + 1}{\frac{T_{\text{lg}}}{\Delta t} + 1} \right) X_n - \left( \frac{\frac{T_{\text{ld}}}{\Delta t}}{\frac{T_{\text{lg}}}{\Delta t} + 1} \right) X_{n-1} \tag{15.10a}$$

where $Y_n =$ output signal from the lead/lag
$X_n =$ the input signal to the lead/lag

which can be combined with the gain and dead time for the digital form of a feedforward controller with lead/lag:

$$\begin{aligned} \text{MV}_n = & \left( \frac{\frac{T_{\text{lg}}}{\Delta t}}{\frac{T_{\text{lg}}}{\Delta t} + 1} \right) \text{MV}_{n-1} + K_{\text{ff}} \left( \frac{\frac{T_{\text{ld}}}{\Delta t} + 1}{\frac{T_{\text{lg}}}{\Delta t} + 1} \right) (D_m)_{n-\Gamma} \\ & - K_{\text{ff}} \left( \frac{\frac{T_{\text{ld}}}{\Delta t}}{\frac{T_{\text{lg}}}{\Delta t} + 1} \right) (D_m)_{n-\Gamma-1} \end{aligned} \tag{15.10b}$$

with $\Gamma = \theta/\Delta t$. The reader should note that the method in equation (15.10) is not the best, most general method for converting the algorithm to digital form. Limitations are presented by the delay table, which requires the dead time divided by the execution period to be an integer. In addition, the difference approximation is accurate only for execution times that are small compared to the lead and lag times. More general methods (which require the use of $z$-transforms) for deriving digital algorithms are available (see Appendix L or Smith, 1972).

Tuning the feedforward-feedback control system follows a simple, stepwise procedure. Either controller may be tuned first; assume that the feedback is tuned first, which requires the identification of the feedback process model $G_p(s)$. Because the tuning parameters for the feedforward controller are derived from both the disturbance and process models, the disturbance model must also be identified through plant experiments, as described in Chapter 6. The disturbance variable cannot normally be changed in a perfect step; thus, the statistically based methods are usually required for identifying $G_d(s)$. The feedforward control performance can be tested through application of feedforward-only control (i.e., with the feedback controller temporarily in manual mode). A typical transient result is given in Figure 15.8$e$. The steady-state offset gives an indication of the error in the feedforward gain $K_{ff}$, which can be further adjusted until the desired accuracy is achieved. Some information on the dynamic tuning parameters can be deduced from feedforward-only control. Should the controlled variable initially respond in the direction indicating too rapid a change in the manipulated variable, either the feedforward controller dead time is too short or the lead/lag time constant ratio is too high. Trial and error are required to establish the improved values. A method for adjusting the lead and lag times is available (Shinskey, 1988), but it requires a perfect step change in the disturbance variable. The disturbance is not usually controlled independently (if it were controlled, it would not be a disturbance), so the method is of limited applicability.

Finally, some common sense is required when tuning the lead/lag times. First, the effect of high-frequency noise in the feedforward measurement should be considered. The lead/lag calculation can amplify noise when the lead time is much greater than the lag time. This effect can be understood by noting that the lead/lag calculation approaches a proportional-derivative calculation as the lead time increases (i.e., $T_{lg} \approx 0$):

$$\frac{T_{ld}s + 1}{T_{lg}s + 1} \approx T_{ld}s + 1 \tag{15.11}$$

Even without high-frequency noise, the lead/lag could make large changes in the manipulated variable when the lead time is much larger than the lag time, as shown in Figure 15.4. To reduce the effect of noise and limit the overshoot in the manipulated variable, the ratio of lead to lag times should not exceed about 2:1, unless plant experience indicates otherwise.

> **Tuning a feedforward-feedback control system requires that each controller be tuned independently, following individual initial and fine-tuning methods.**

## 15.6 ▫ IMPLEMENTATION ISSUES

Feedforward control involves a new algorithm for which there is no accepted standard display used in commercial equipment. Since the feedforward controller responds to disturbances, it has no set point—a factor that changes the display significantly. One feature that should be provided in the display is the ability for the operator to turn the feedforward and the feedback on and off separately. Also, the operator should have a display of the result after the feedforward and feedback signal have been combined, because the operator always wants to know the signal sent to the final control element.

The calculations for feedforward, equations (15.4) and (15.10*b*), are simple and can be performed with standard algorithms available in most commercial control equipment. The engineer normally connects or "configures" the preprogrammed algorithms and enters the tuning constants. An important feature that must be included is smooth (i.e., "bumpless") transfer when feedforward or feedback controllers are turned on and off. One approach to bumpless transfer is to use incremental or velocity forms of the feedforward and feedback control equations. Whenever one or both of the controllers is turned off (i.e., put in manual), the change in its output becomes zero. When it is turned on, or put in automatic, its output calculation resumes. This is an example of an approach to bumpless transfer; other approaches are possible (for example, see Gallun et al., 1985).

The feedforward-feedback control system uses more control equipment—two sensors and controllers—than the equivalent single-loop system. Since the system performance requires all of this equipment to function properly, its reliability can be expected to be lower than that of the equivalent single-loop system. However, it is important to note that feedback control is not dependent on the feedforward; should any component in the feedforward controller fail, the feedforward part can be turned off, and the feedback controller will function properly. Usually, the lower reliability does not prevent the use of feedforward.

Since the feedforward-feedback design involves more equipment, it costs slightly more than the single-loop system. The increased costs include a field sensor and transmission to the control house (if the variable is not already available for monitoring purposes), a controller (whose cost may be essentially zero if a digital system with spare capacity is used), and costs for installation and documentation. These costs are not usually significant compared to the benefits achieved through a properly designed feedforward control strategy, except that expensive analyzers for feedforward are often not economically justified.

> Feedforward control, where applicable, provides a simple method for substantial improvement in control performance. The additional costs and slightly lower reliability are not normally deterrents to implementing feedforward control.

## 15.7 FURTHER FEEDFORWARD EXAMPLES

In this section the concept of feedforward control is consolidated, and a few new features are presented through further examples.

**EXAMPLE 15.1. Packed-bed chemical reactor**

For the first example the packed-bed chemical reactor analyzed in Chapter 14 is considered again. The process with its feedback control strategy is shown in Figure 15.10. The control objective is to maintain the outlet concentration close to its set point by adjusting the preheat. Suppose that the feed composition is a significant disturbance. The goal is to design a feedforward control strategy for this process using the sensors and manipulated variables given. (The reader is encouraged to design a control system before reading further.)

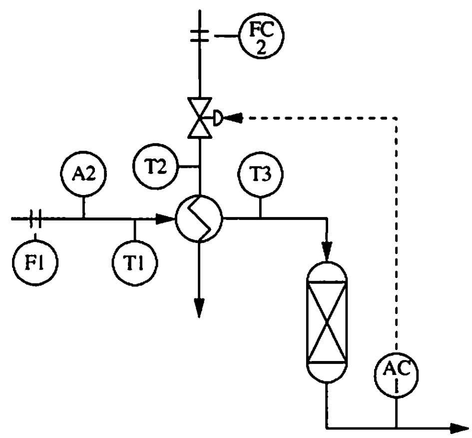


**FIGURE 15.10**

**Packed-bed chemical reactor with feedback control.**

Since we are dealing with a feedforward control strategy, the key decision is the selection of the feedforward variable. Therefore, the first step is to evaluate the potential measured variables using the design criteria in Table 15.1. The results of this evaluation are summarized in Table 15.3. Since all of the criteria must be satisfied for a variable to be used for feedforward, only the reactor inlet concentration, A2, is a satisfactory variable. The resulting control strategy is shown in Figure 15.11.

***Signal combination.*** The feedforward controller adjustment must be implemented in a manner that does not interfere with feedback control. First, we assume that the process behaves in (approximately) a linear manner, so that the feedforward and feedback adjustments can be calculated independently and added. Second, the correct location for combining the signals can be determined by

**TABLE 15.3**

**Evaluation of potential feedforward variables**

| Criterion | A2 | F1 | F2 | T1 | T2 | T3 |
|---|---|---|---|---|---|---|
| 1. Single-loop control not satisfactory | Y | Y | Y | Y | Y | Y |
| 2. Variable measured | Y | Y | Y | Y | Y | Y |
| 3. Indicates key disturbance | Y | N | N | N | N | N |
| 4. Not influenced by MV | Y | Y | N | Y | Y | N |
| 5. Suitable disturbance dynamics | Y | N/A | N/A | N/A | N/A | N/A |

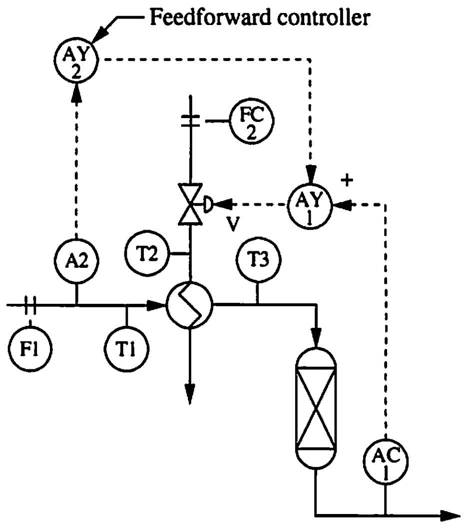


**FIGURE 15.11**

**Packed-bed chemical reactor with feedforward-feedback control.**

referring to the feedforward and feedback equations:

$$MV_{ff}(s) = G_{ff}(s)D_{A2}(s) \qquad MV_{fb}(s) = G_c(SP_{A1}(s) - CV_{A1}(s)) \tag{15.12}$$

The outputs of the two controllers can be combined when they both manipulate the same variable, that is, if $MV_{ff}(s)$ and $MV_{fb}(s)$ represent the same manipulated variable. This demonstrates that the feedforward controller output can be added to the output of the feedback controller which is regulating the same controlled variable. In this example, the feedforward controller output is added to the output of the outlet analyzer controller, AC-1, as shown in Figure 15.11. The combined signal is sent to the valve.

***Solution.*** To complete this example, the feedforward controller tuning constants are calculated from the following empirically determined disturbance and process models:

$$\begin{aligned} G_d(s) &= \frac{A1(s)}{A2(s)} = \frac{0.30e^{-42s}}{(35s+1)}\left(\frac{\text{outlet mole/m}^3}{\text{inlet mole/m}^3}\right) \\ G_p(s) &= \frac{A1(s)}{v(s)} = \frac{-0.108e^{-44s}}{(54s+1)}\left(\frac{\text{outlet mole/m}^3}{\%\text{ open}}\right) \end{aligned} \tag{15.13}$$

The resulting controller parameters are determined by applying equation (15.4):

Feedforward gain $= -[0.3/(-0.108)] = 2.78$ (% open/input g-mole/m$^3$)

Feedforward lead time $= \tau_p = 54$ min

Feedforward lag time $= \tau_d = 35$ min

Feedforward dead time $= \theta_d - \theta_p = 42 - 44 = -2$ min $< 0$ (not possible)

Note that the disturbance dead time is smaller than the process dead time. As a result, the feedforward controller requires a *negative* dead time for perfect compensation.

> A negative dead time is not possible since it requires a prediction of future disturbances; this situation is termed *not physically realizable.*

However, since the negative dead time is small compared to the process dynamics, we can set it equal to the smallest feasible number, which is zero. Based on the example sensitivity of feedforward control performance to errors in this chapter (Figure 15.7), a small error in the feedforward dead time should not significantly degrade the performance. Note that a better way to resolve this problem would be to relocate the inlet analyzer farther upstream; this preferred solution, if possible, would provide an earlier warning and give a longer disturbance dead time.

***Retaining cascade feedback.*** We started this example assuming that single-loop feedback would be applied. We learned in the previous chapter (Example 14.1) that cascade control could provide excellent control performance for many disturbances, but not for feed composition. Cascade and feedforward can be applied simultaneously to a process to achieve the advantages of both as shown in Figure 15.12. The general cascade and feedforward design rules apply to this combination; a new design decision involves the proper choice of how to combine the feedforward and feedback control signals, which is specified in the following.

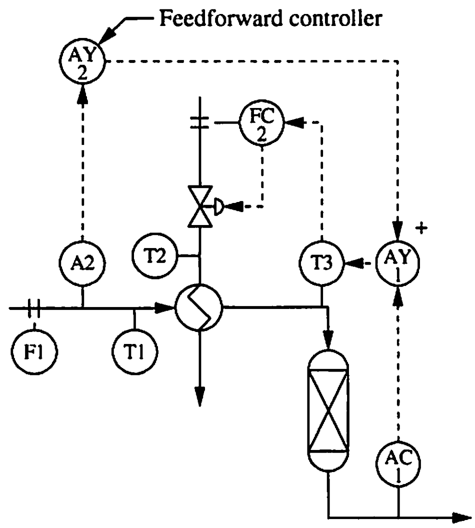


**FIGURE 15.12**

**Packed-bed chemical reactor with combined feedforward and cascade control.**

> For a feedforward controller designed to maintain a process output variable $X$ constant, the feedforward controller output signal is combined with the output from the feedback controller that is controlling the same variable $X$.

For the reactor example, the feedforward controller is designed to maintain A2 unchanged; therefore, the feedforward signal is added to the output of the A2 feedback controller. The feedforward controller design obeys the general design rule, equation (15.2), which gives the following result.

$$G_{ff}(s) = -\frac{A1(s)/A2(s)}{A1(s)/T3_{sp}(s)} = \frac{T3_{sp}(s)}{A2(s)} = \frac{MV_{ff}(s)}{D_m(s)} \tag{15.14}$$

**EXAMPLE 15.2. Multiple feedforward measurements**

In Chapter 14 we learned that a single cascade controller could attenuate the effects of several disturbances. Since feedforward must sense the disturbance to be effective, a separate feedforward controller is required for *each* disturbance. Assuming linearity, the resulting calculations from all feedforward controllers can be added. An example of two feedforward controllers is shown in Figure 15.13*a* for the stirred-tank heat exchanger. In this case, both the inlet temperature and the inlet flow change significantly and independently. Two separate feedforward controllers calculate individual adjustments for the heating oil flow. They are both added to the feedback signal in the completed strategy.

Sometimes the effects of several measured disturbance variables can be combined into a single feedforward controller. The combination relies on insight into the underlying process models. In the case of the stirred-tank heat exchanger, the following linearized model can be written:

$$\rho C_v \frac{dT}{dt} = \rho C_p F(T_{in} - T) + K F_{oil} \tag{15.15}$$

It is clear that the steady-state effects of the disturbances appear in the first term on the right-hand side of the equation. This can be rearranged to yield

$$\Delta F_{oil} = \frac{\rho C_p}{K} \Delta[F(T^* - T_{in})] \tag{15.16}$$

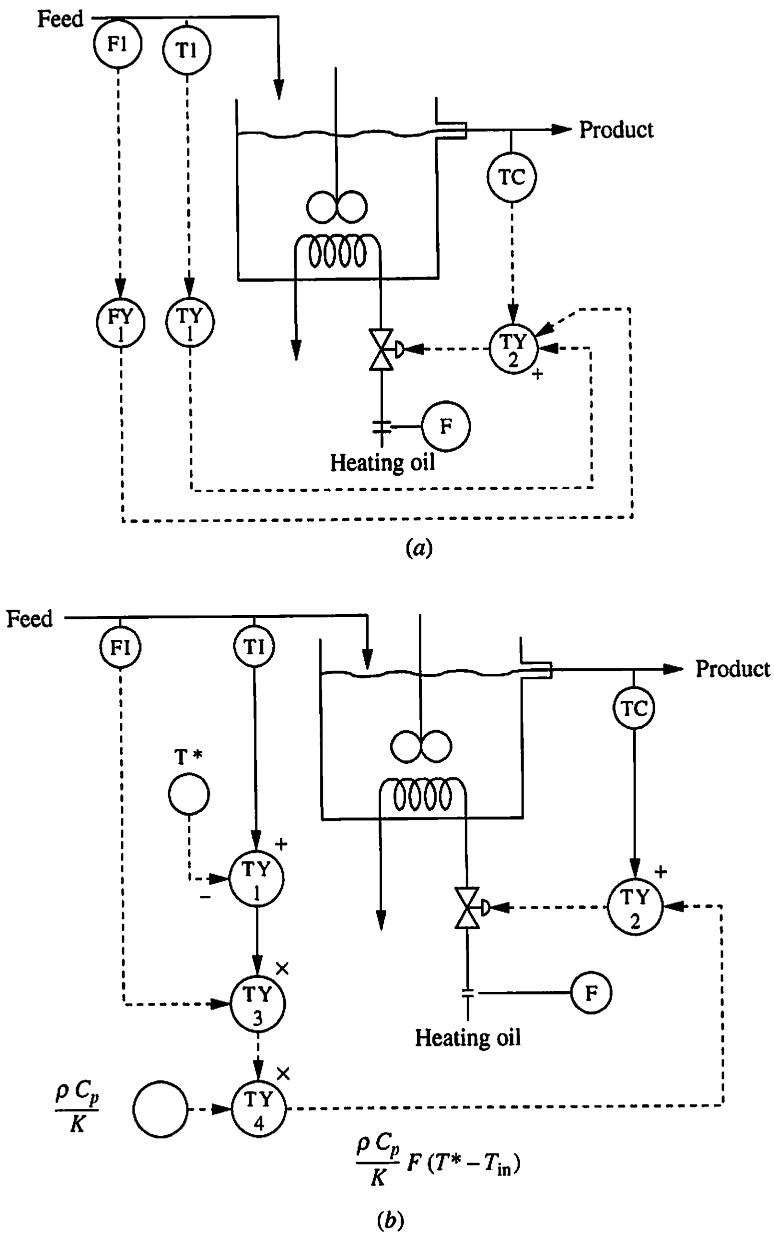


**FIGURE 15.13**

**(*a*) Stirred-tank heat exchanger with two feedforward controllers; (*b*) stirred-tank heat exchanger with two disturbance variables and one feedforward controller.**

where $T^*$ = the outlet temperature filtered or averaged so that this calculation does not give an improper response to an unmeasured disturbance

Therefore, the steady-state flow change required to compensate for a disturbance can be calculated directly and, assuming it is proportional to the heating valve position, output as the feedforward signal. The controller is shown schematically in Figure 15.13*b*.

**EXAMPLE 15.3. Feedforward-only control**

The derivation for the stirred-tank heat exchanger might lead one to propose a feedforward-only controller derived by setting equation (15.15) to zero and solving for $F_{oil}$. As mentioned several times already and demonstrated in Figure 15.8*e*, feedforward-only control cannot eliminate steady-state offset. Thus, it should be used only when feedback is not possible.

**EXAMPLE 15.4. Ratio control**

One particularly simple form of feedforward control is widely used to maintain flows at desired proportions. The process situation is shown in Figure 15.14*a*, where one of the flows is controlled by another strategy; as far as this process is concerned, it is uncontrolled or *wild.* The other stream can be manipulated with a valve to achieve the desired composition of the blended stream. The feedforward/feedback strategy measures the flow rate of the uncontrolled stream and adjusts the flow of the manipulated stream to maintain the desired ratio. The feedforward controller uses the measurement of the uncontrolled flow, multiplied by a gain, and outputs to the set point of the feedback flow controller. Because of the fast dynamics, no dead time or lead/lag is required. Note that the ratio control provides feedforward-only compensation; if strict composition control is required, a composition sensor can be placed in the mixed stream and used with a PID controller to achieve zero steady-state offset by adjusting the ratio $R$.

An alternative approach is also used in practice. This approach achieves the same goal, but it does not satisfy the criteria for a feedforward controller. The ratio controller shown in Figure 15.14*b* uses the two flow measurements to calculate the actual ratio and adjusts the valve to achieve the desired value. The control calculation in this design could be a feedback PI controller with a calculated controlled variable rather than a single measured variable. Again, this ratio design does not guarantee zero steady-state offset of the composition.

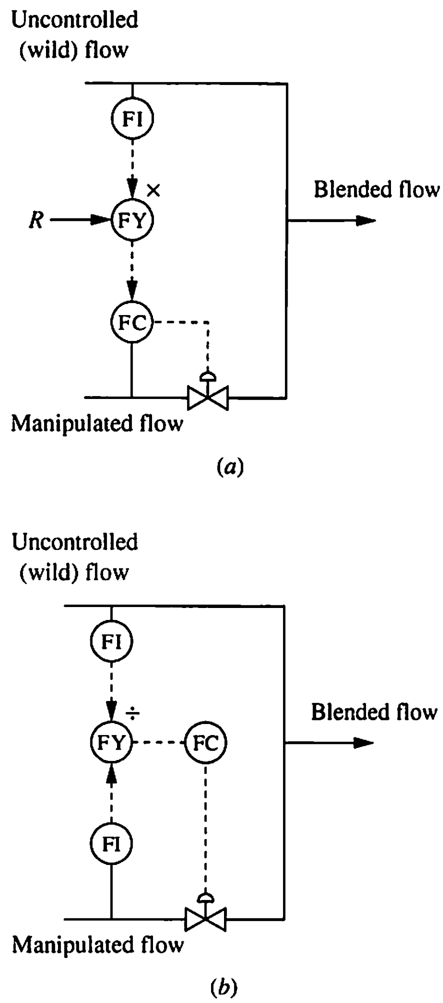


**FIGURE 15.14**

**Flow ratio control: (*a*) steady-state feedforward; (*b*) feedback.**

**EXAMPLE 15.5. Flow disturbances**

As the material passes through the plant, the flow rate is varied to control inventories. As a result, the flow may not be as constant throughout the plant as it is at the inlet. This situation is further explained in Chapter 18 on level control. Feedforward control is very effective in attenuating disturbances resulting from flow rate disturbances. An example of fired-heater control is given in Figure 15.15. The temperature of the fluid in the coil at the outlet of the heater is to be controlled. The flow rate sensor is a reliable, inexpensive feedforward measurement, and the combined feedforward-feedback strategy is very effective. Similar flow rate feedforward can be applied to other processes such as distillation and chemical reactors.

**EXAMPLE 15.6. Fired heaters.**

Several types of feedforward control to improve the control performance of a fired heater are possible. One approach, shown in Figure 15.16, measures the inlet temperature. If this temperature varies significantly and tight outlet temperature control is important, the feedforward strategy shown can be used to compensate for the disturbance.

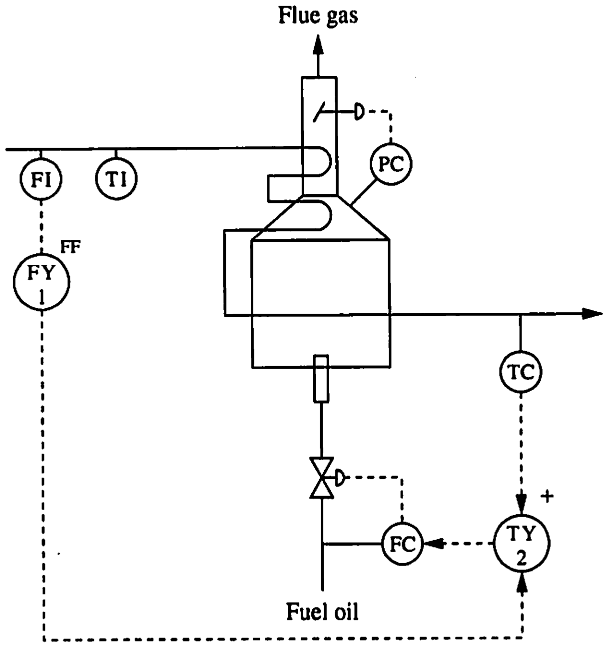


**FIGURE 15.15**

**Example of feed rate feedforward control applied to a fired heater.**

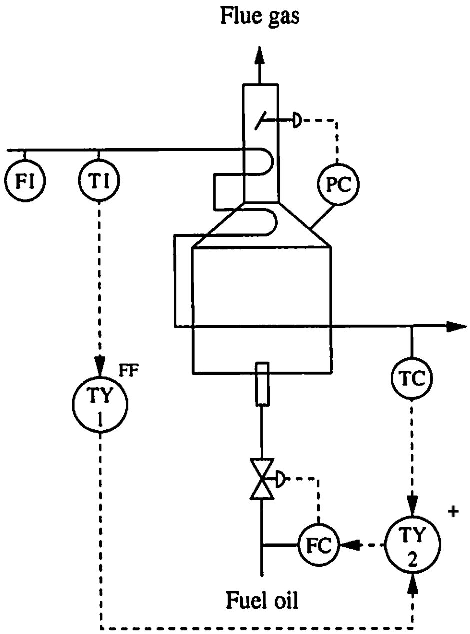


**FIGURE 15.16**

**Example of inlet temperature feedforward control applied to a fired heater.**

Another example of feedforward control is given in Figure 15.17 for a heater with two fuels. In this case, one fuel is not controlled by the heater system; this can occur when the fuel is a byproduct in another section of the plant and large economic incentives exist for consuming the fuel (designated as A in the figure). To prevent the variations in the byproduct fuel from upsetting the outlet temperature, a feedforward controller adjusts the manipulated fuel flow (B) to maintain the *heat fired* (i.e., the sum of the fuel rates times their heats of combustion) at the desired value. The feedforward controller usually needs no dynamic elements but must consider the differences in heats of combustion in its calculation. Control designs like the one in Figure 15.17 are widely used in petrochemical plants, which have large fuel byproduct streams.

Also, the general principle demonstrated in the two-fuel furnace can be applied to any process that has two potential manipulated variables of which one is adjusted by another control strategy (i.e., a wild stream). Other examples include (1) the use of two reboilers in distillation, with one (wild) reboiler duty varied to maximize heat integration and the other manipulated to control product purity, and (2) balancing electrical demand with varying (wild) in-plant generation and manipulated purchases.

**EXAMPLE 15.7. Distillation**

Distillation columns can have slow dynamics with long dead times and analyzer delays. Therefore, distillation is a good candidate for feedforward control when product composition control is important. In addition, a distillation column has two products, so a disturbance can affect two different controlled variables. The feedforward controller in Figure 15.18*a* provides compensation for changes in the feed flow rate by adjusting the reflux and reboiler flows. The feedforward controller shown in Figure 15.18*b* provides compensation for feed composition. (Note that

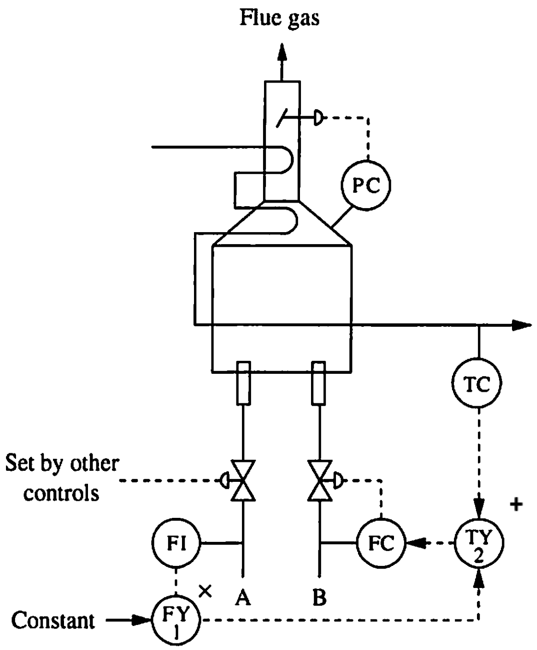


**FIGURE 15.17**

**Example of feedforward compensation for a wild fuel being consumed in a fired heater.**

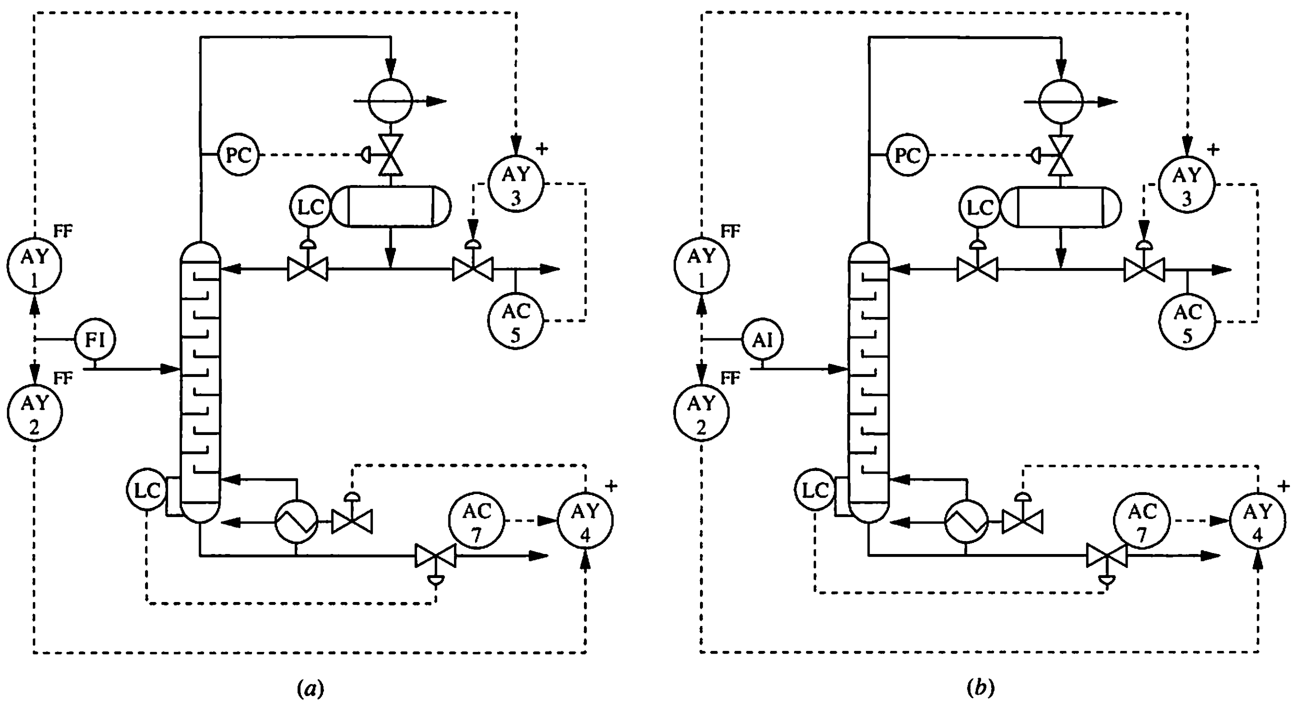


**FIGURE 15.18**

**Feedforward control in a two-product distillation tower: (*a*) from feed rate; (*b*) from feed composition.**

the feedforward controllers for multivariable systems cannot be designed using equation (15.2) for each controller; interaction must be considered. See question 21.17.) The disturbance models for this controller must be identified empirically.

**EXAMPLE 15.8. Controller Design Issues**

The direct application of the feedforward controller design equation (15.2) can lead to poor performance in some situations, which are discussed in this example.

***a. Unstable controller.*** The design equation is repeated below and applied to a process with an inverse response in the feedback dynamics. (Reasons for the numerator dynamics were explained in Section 5.4, and related process examples are presented in Appendix I.)

Disturbance: $$G_d(s) = \frac{K_d}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

Feedback: $$G_p(s) = \frac{K_p(\tau_3 s - 1)}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

Unstable controller: $$G_{ff}(s) = -\frac{G_d(s)}{G_p(s)} = \frac{K_d}{(\tau_1 s + 1)(\tau_2 s + 1)} \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{K_p(\tau_3 s - 1)}$$

$$= -\frac{K_d/K_p}{(\tau_3 s - 1)}$$

In these equations, all $\tau_i > 0$. The key result is instability of the feedforward controller, which is indicated by the positive pole (root of the denominator of the transfer function). This would lead to unacceptable performance. One corrective step, which is applied below to the example, is to simply remove the unstable pole from the controller.

Stable controller: $$G_{ff}(s) = -K_d/K_p$$

This approach will yield a stable feedforward controller but might not give good performance. A potentially better method is presented in Chapter 23 on model predictive control.

***b. Pure derivative controller.*** The design equation is repeated below and applied to a process with feedback dynamics of higher order than disturbance dynamics.

Disturbance: $G_d(s) = \dfrac{K_d}{\tau_1 s + 1}$ Feedback: $G_p(s) = \dfrac{K_p}{(\tau_1 s + 1)(\tau_2 s + 1)}$

Controller: $$G_{ff}(s) = -\frac{G_d(s)}{G_p(s)} = -\frac{K_d}{\tau_1 s + 1} \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{K_p} = -\frac{K_d(\tau_2 s + 1)}{K_p}$$

In these equations, all $\tau_i > 0$. The controller has a pure derivative, and if the feedback process were of even higher order the controller would have second or higher derivatives. It is good practice to have the order of the feedforward controller denominator at least the same or higher than the numerator order. One corrective step, which is applied below, is to add a filter to the controller.

Improved controller: $$G_{ff}(s) = -\frac{K_d(\tau_2 s + 1)}{K_p(\tau_{ff} s + 1)}$$

While the controller above does not satisfy the original design rule, *it is expected* to provide better performance for noisy disturbance measurements.

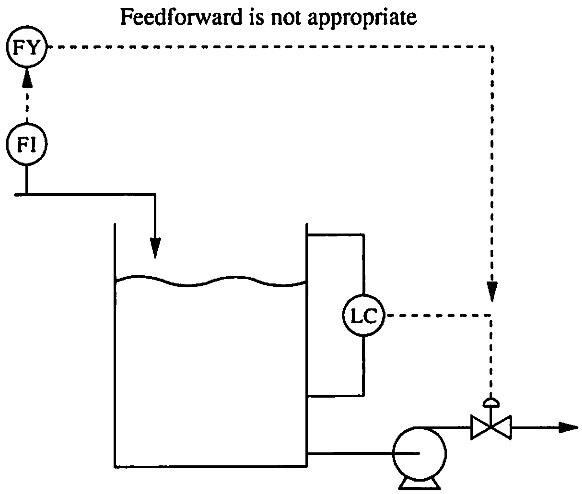


**FIGURE 15.19**
**Feedforward control is not generally required for level control, where the outlet flow manipulations should be smooth.**

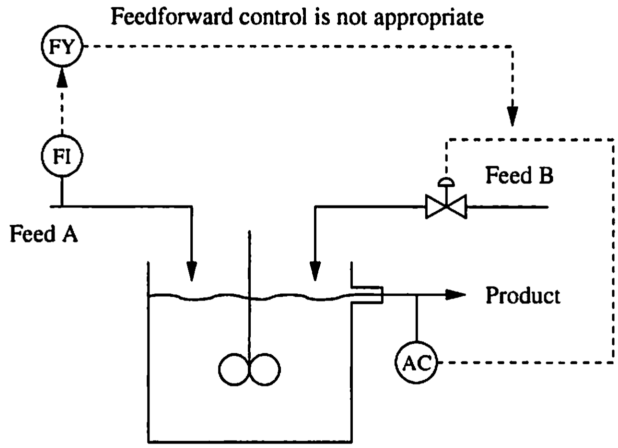


**FIGURE 15.20**
**Feedforward control is not generally required for nearly linear processes with little dead time.**

### Feedforward Is Not Used Everywhere

Sometimes engineers have the impression that because feedforward is generally a good idea, it should be applied in all process control strategies. This is not the case. As strongly emphasized in the first design criterion, feedforward is applied *when feedback control does not provide satisfactory control performance.* Thus, feedforward is not used if tight control is not needed or if feedback control provides good performance. An example of the first situation is given in Figure 15.19, where the level can vary within limits without influencing the plant economics or safety; thus, feedforward is not applied. An example of the second situation is given in Figure 15.20, where tight control of the mixing process is possible with feedback-only control because the process has almost no dead time.

## 15.8 ▪ FEEDFORWARD CONTROL IS GENERAL

Feedforward control is a way to take corrective action as soon as information on a disturbance is available. In the packed-bed reactor (Figure 15.12), the inlet analyzer AC-2 provides an early warning of a disturbance. The feedforward controller adjusts the inlet temperature without interfering with the feedback controller.

Feedforward control concepts are not limited to engineering control systems. Social organizations also benefit from early response to events. In business, feedforward may be termed "positive preactions"; whatever the name, the improved performance can be dramatic. A hypothetical example of university decision making is given in Figure 15.21. The goal is to have needed faculty, staff, and buildings available for all of the students attending the university. A major variable is the number of students. Therefore, the total number of young people (e.g., 14 years

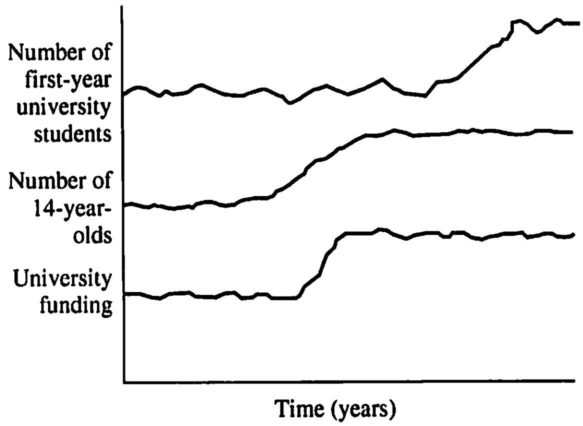


**FIGURE 15.21**

**Example of feedforward control applied to a planning decision in a university.**

old) in the population can be measured or estimated. Should this number increase significantly, the facilities can be increased over several years so that the university is able to accommodate the demand when it occurs.

## 15.9 ▩ CONCLUSIONS

Feedforward control does not employ the feedback principle; it manipulates a system input based on the measured value of a different system input. This approach to control requires new algorithms, with the proper algorithm depending on models of the disturbance and feedback dynamics. As shown in Figure 15.8*a* through *e*, improved performance is achieved without increased variation in the manipulated variable and without the requirement of highly accurate models. Based on this performance improvement and simplicity of implementation, the engineer is well advised to evaluate potential feedforward controls for important controlled variables.

The first few times engineers evaluate feedforward, they must perform careful studies like the one in Table 15.3, but after gaining some experience they will be able to design feedforward control strategies quickly without explicitly writing the criteria and table.

Feedforward control is not universally applicable; the design criteria in Table 15.1 can be used to determine whether feedforward is appropriate and, if so, to select the best feedforward variable. If it is not immediately possible and improved performance is required, the engineer should investigate the possibility of adding the necessary sensor. However, feedforward control is effective only for the measured disturbance(s); thus, additional enhancements, such as cascade and feedback from the final controlled variable, should be used in conjunction with feedforward.

## REFERENCES

Gallun, S., C. Matthews, C. Senyard, and B. Slater, "Windup Protection and Initialization for Advanced Digital Control," *Hydro. Proc.*, 63–68 (June 1985).

Shinskey, F. G., *Process Control Systems,* McGraw-Hill, New York, 1988.
Smith, C., *Digital Computer Process Control,* Intext, Scranton, PA, 1972.



## QUESTIONS

**15.1.** (*a*) In your own words, discuss the feedforward control design criteria. Give process examples in which feedforward control is appropriate and not appropriate.

(*b*) One of the design criteria for feedforward control requires that the feedforward variable not be affected by the manipulated variable. Why is this required? If the variable were influenced by the manipulated variable, what control design would be appropriate?

(*c*) In a feedforward-feedback control strategy, which controller should be tuned first? What would be the effect of reversing the order of tuning? Clearly state any assumptions you have used.

(*d*) Describe how the addition of feedforward control to an original feedback-only system affects the resonant frequency and the amplitude ratio (controlled to measured disturbance) at the resonant frequency.

(*e*) Discuss why the last design rule in Table 15.1 is valid when feedforward is applied in conjunction with feedback. Is it also valid for feedforward-only?

(*f*) Review the factors in Table 13.3 and determine which factors are influenced by feedforward control.

**15.2.** In this question, you will design control strategies for the system of stirred tanks in Figure Q15.2. The measurements and manipulated variable are shown in the figure; you may not alter them and need not use them all. The following information will help you design the strategy.

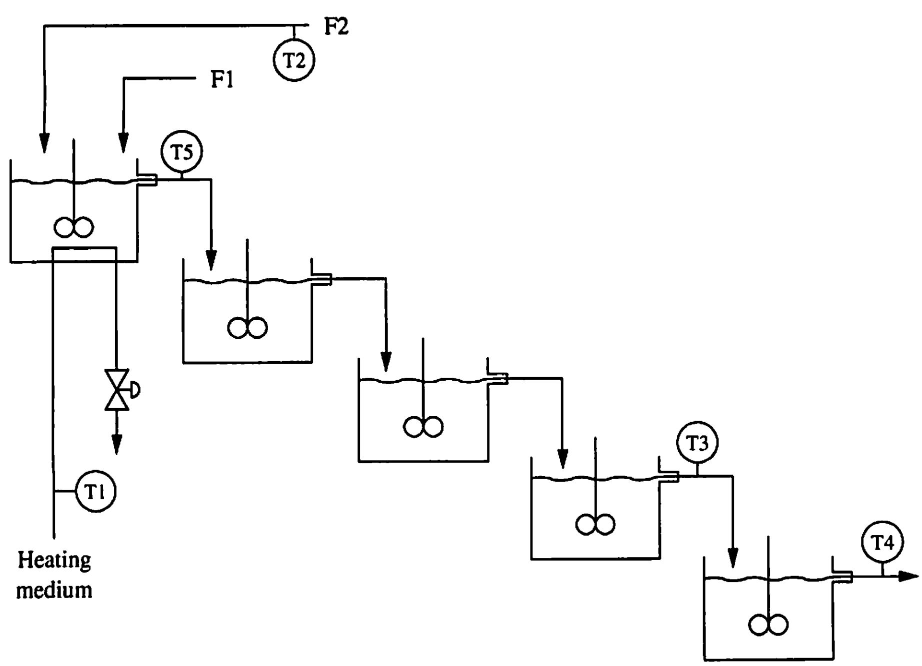


**FIGURE Q15.2**

(1) The goal is to control the temperature $T_4$ at the outlet of the last tank tightly. The major disturbance is the temperature of one of the feed streams, $T_2$.
(2) The flow rates to the stirred tanks cannot be changed by your control strategy and are essentially constant; $F_1 = 5\ m^3/min$ and $F_2 = 5\ m^3/min$.
(3) The volume of each tank is $10\ m^3$.
(4) At the flow rates given, the steady-state gain for the heating coil is 1°C/% (change in first tank temperature per change in % valve opening). The sensor, valve, and heating coil dynamics are negligible, and the heat losses are small but not negligible.

(*a*) Decide whether a single-loop feedback control strategy is possible. Explain your answer. If yes, draw the single-loop control system on the process figure and define each control algorithm.
(*b*) Decide whether a cascade control strategy is possible, yes or no. Explain your answer. If yes, draw the best cascade control system on the process figure and define each controller algorithm.
(*c*) Decide whether a feedforward/feedback control strategy is possible, yes or no. Explain your answer. If yes, draw the best feedforward/feedback control system on the process figure and define each controller algorithm.
(*d*) Rank the strategies in (*a*) through (*c*) that are possible according to their control performance; that is, the ability to control the outlet temperature $T_4$. Explain the ranking.
(*e*) For the *best* strategy, calculate all parameters for the control algorithms: gains, integral times, leads, lags, dead times, and so forth. (Hint: You must develop analytical models and transfer functions for the relevant input-output relationships.)

**15.3.** The feedforward-feedback strategy has an additional sensor and controller. How is it possible to add these and not violate the degrees of freedom of the system? For the heat exchanger example in Section 15.2,

(*a*) Derive all equations describing the process and the feedforward-feedback controllers.
(*b*) Analyze the degrees of freedom to verify that the system is exactly specified.
(*c*) Discuss how you would solve the equations in (*a*) numerically for a dynamic response (simulate the process with a digital control system).

**15.4.** Propose feedforward/feedback control designs for the following systems, where possible. Draw the design on a sketch of the process and verify the design using the feedforward design criteria. The processes, with [controlled/disturbance] variables, are

(*a*) Example 14.1 [A1/T2]
(*b*) Figure 14.17 (T2/fuel flow)
(*c*) Figure 14.20 [tank temperature/fresh coolant temperature]
(*d*) Figure Q13.2 [outlet concentration/$C_A$]
(*e*) Example 7.2 [$x_{A3}/(x_A)_B$]
(*f*) Figure Q8.12 [outlet concentration (AC)/ flow of stream C]

**15.5.** Derive the transfer function in equation (15.6) based on the block diagram in Figure 15.6.

**15.6.** Verify that a feedforward-feedback control system has zero steady-state offset for a measured disturbance. What restrictions must you place on the disturbance, feedback process, and control algorithms in your derivation?

**15.7.** The following transfer functions have been evaluated for the process in Figure 15.15, with time in min:

$$\frac{T_{out}(s)}{T_{in}(s)} = \frac{0.40e^{-1.8s}}{3.5s + 1} \qquad \frac{T_{out}(s)}{F_{fuel}(s)} = \frac{0.1e^{-1.1s}}{4.2s + 1}$$

(*a*) Determine the continuous feedforward and feedback algorithms and the values of all adjustable parameters.
(*b*) Determine the digital feedforward and feedback algorithms and the values of all adjustable parameters, including the execution period.

**15.8.** (*a*) Describe how to program a digital feedforward-feedback controller so that the automatic/manual status of each controller can be changed independently.
(*b*) Describe how to initialize the feedforward controller.
(*c*) Derive the algorithm for an incremental (or velocity) form of the feedforward algorithm that calculates the change in the manipulated variable at each execution.
(*d*) Discuss the possibility of integral windup caused by feedforward control without feedback.

**15.9.** In Example 15.2, the tank temperature was replaced with a "filtered" value, $T^*$. Explain why this was done. Can this analysis be generalized to an additional criterion for feedforward control with calculated variables?

**15.10.** In the description of the control design for a packed-bed reactor in Example 15.1, the correct location for combining the feedforward and feedback controllers is explained. Discuss the behavior of the control system for the two improper locations, adding the feedforward to (*a*) the outlet of the $T_3$ controller and (*b*) the outlet of the $F_2$ controller. How would the control system respond to a disturbance for each of the improper connections?

**15.11.** The feedforward control of a second-order process is analyzed in this question. The structures for the open-loop process models for two inputs to the controlled variable are given in the following equations:

Inverse response: $$G_1(s) = \frac{CV(s)}{X1(s)} = \frac{K_1(-\tau_1 s + 1)}{(\tau_2 s + 1)(\tau_3 s + 1)}$$

Overdamped: $$G_2(s) = \frac{CV(s)}{X2(s)} = \frac{K_2}{(\tau_4 s + 1)(\tau_5 s + 1)}$$

with all $\tau > 0$. Depending on other design factors, either $X1$ or $X2$ can serve as the manipulated variable, with the other being the measured disturbance. Answer the following questions about this system. Answer parts (*a*) to (*c*) with $X1$ the manipulated variable and $X2$ the measured disturbance.

(*a*) Is perfect feedforward control (no deviation in the controlled variable) possible for this system?

(*b*) Derive the feedforward controller transfer function for this system. Sketch the shape of the response of the manipulated and controlled variables to a step change in the measured disturbance with feedforward-only control.

(*c*) Based on the answers in (*a*) and (*b*), propose a modified feedforward controller which provides acceptable performance. Substitute numerical values for the reactor process.

(*d*) Answer the same questions in (*a*) through (*c*) for the modified process system in which $X2$ is the manipulated variable and $X1$ is the measured disturbance, opposite from the original situation.

(*e*) Can you use the results in parts (*a*) through (*d*) to develop a general conclusion on the effects of (positive) zeros on feedforward control performance?

(Note: The system in parts (*a*) through (*c*) can be thought of as the series chemical reactors in Example I.2, but the solution to this problem is general for processes with positive zeros.)

**15.12.** Given the processes in Figure Q15.12, place them in order of how much each would benefit from feedforward control for a disturbance measured by analyzer A. Explain your ranking.

**15.13.** The feedforward control from set point given in Figure Q15.13 has been suggested.

(*a*) Derive the transfer function for the set point feedforward controller, $G_{ffsp}(s)$.

(*b*) Discuss this controller. Is it possible to implement, and how would it affect the dynamic response of the controlled and manipulated variables?

(*c*) Discuss the need for a set point feedforward if the feedback controller uses a PID algorithm.

**15.14.** The feedforward controller was derived to provide perfect control. Using the block diagram in Figure 7.4, derive the *feedback* controller that gives perfect control. Are there any reasons why this controller is not practical?

**15.15.** (*a*) Verify that all designs in Section 15.7 satisfy the feedforward design criteria.

(*b*) In the description of flow ratio control, it was not specified whether the orifice $\Delta P$ measurements were used or their square roots were used. Which is correct and why?

(*c*) Derive the analytical relationship in equation (15.5) for the output of a lead/lag element when the input experiences a step change.

(*d*) Explain the feedforward calculation for Figure 15.17. Give the equations and the physical property data required.

**15.16.** Discuss one example of feedforward control in each of the following categories: university, government, and business organizations.

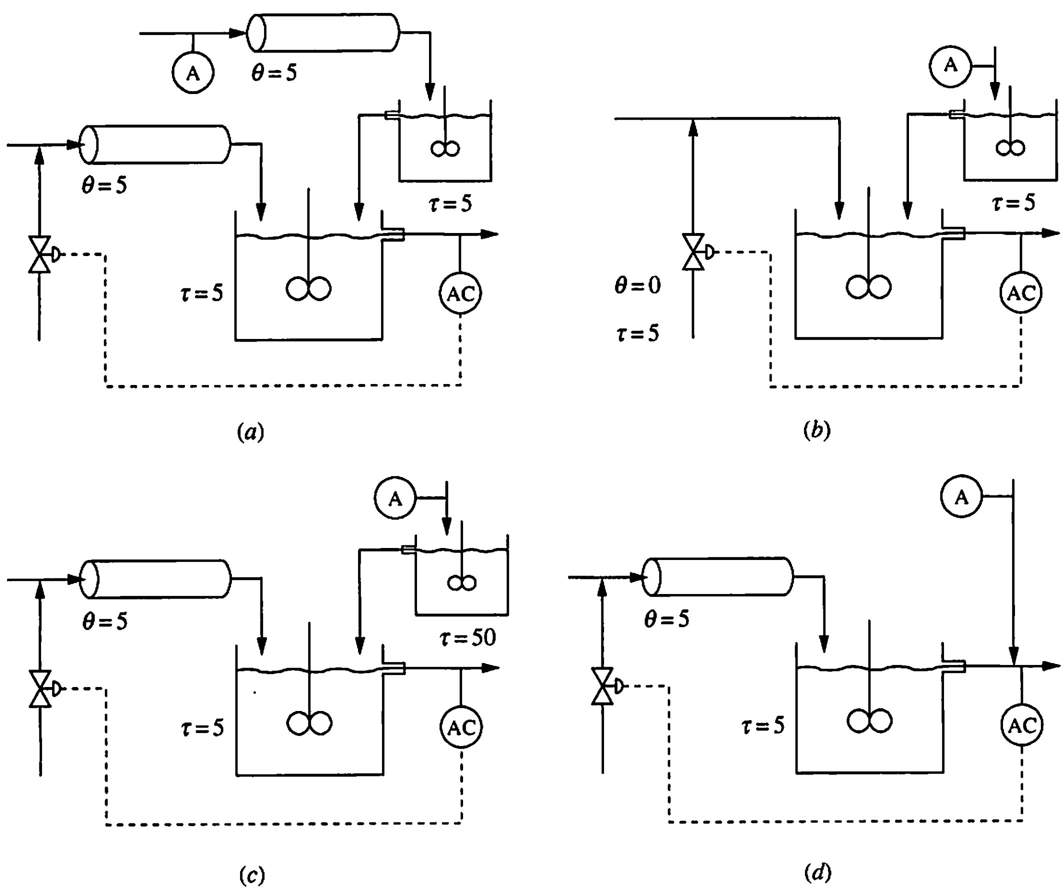


**FIGURE Q15.12**

**The solid lines contribute negligible dead time.**

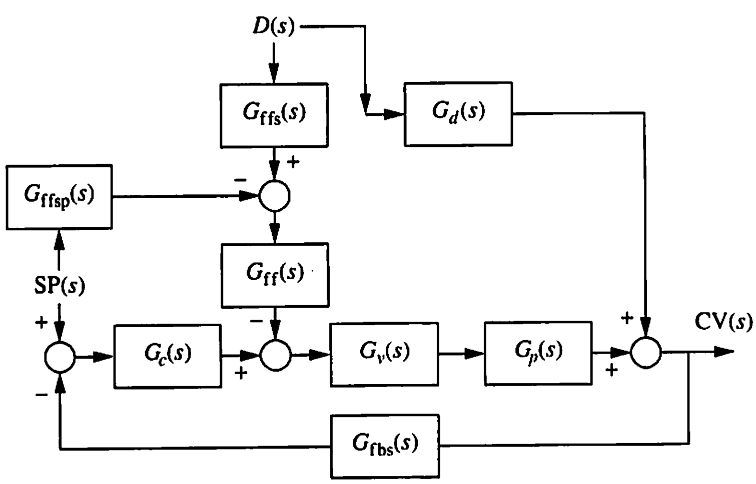


**FIGURE Q15.13**