The PID Algorithm

CHAPTER 8

8.1 ▫ INTRODUCTION

Continuous feedback control offers the potential for improved plant operation by maintaining selected variables close to their desired values. In this chapter we will emphasize the control algorithm, while remembering that all elements in the feedback loop affect control performance. Engineers should fully understand the algorithm for three reasons. First, the performance of the entire feedback system depends on the structure of the algorithm and the parameters used in the algorithm. Second, all other elements are process equipment and instrumentation, which are costly and time-consuming to alter, so a key area of flexibility in the loop is the control calculation. Third, while engineers use only a few algorithms, as will be explained, they are responsible for determining the values of adjustable parameters in the algorithms.

In this chapter, we will learn about the *proportional-integral-derivative* (PID) control algorithm. The PID algorithm has been successfully used in the process industries since the 1940s and remains the most often used algorithm today. It may seem surprising to the reader that one algorithm can be successful in many applications—petroleum processing, steam generation, polymer processing, and many more. This success is a result of the many good features of the algorithm, which are covered initially in this chapter and expanded on and evaluated in later chapters.

This algorithm is used for single-loop systems, also termed single input–single output (SISO), which have one controlled and one manipulated variable. Usually, many single-loop systems are implemented simultaneously on a process, and the performance of each control system can be affected by interaction with the other loops. However, the next few chapters will concentrate on ideal single-loop systems, in which interaction is negligible or nonexistent; extensions, including interaction, are covered in Parts V and VI.

As we cover the PID control algorithm here and in subsequent chapters, we will address important theoretical issues in feedback control including stability, frequency response, tuning, and control performance. Thus, by covering the PID controller in depth, we will acquire key analytical techniques applicable to all feedback control systems, including PID and alternative control algorithms, along with important knowledge about current practice.

8.2 DESIRED FEATURES OF A FEEDBACK CONTROL ALGORITHM

Many of the desired characteristics for feedback control were discussed in the previous chapter under quantitative measures of control performance. Here, a few of these characteristics are extended for use in this and upcoming chapters.

Key Performance Feature: Zero Offset

The performance measures discussed previously could be combined into two categories: dynamic (IAE, ISE, damping ratio, settling time, etc.) and steady-state. The steady-state goal—returning to set point—is further discussed here. This goal can be stated mathematically as follows by using the final value theorem,

$$\lim_{t\to\infty} E(t) = \lim_{s\to 0} sE(s) = 0 \tag{8.1}$$

with E denoting the *error:* the difference between the (desired value) set point and (measured) controlled variable. It would seem unreasonable to demand that the control system return to set point for all fluctuations in inputs. Therefore, we select the most important, most often occurring input (disturbance) variation from among the following cases:

1. The input variable varies but ultimately returns to its initial value; an example is a pulse. For this input type most (but not all) processes would require no feedback control to satisfy the condition in equation (8.1).
2. The input variable varies for some time and then attains a steady value different from its initial value; this type we shall term *steplike*, because the transition from initial to different final value does not have to be a perfect step. Feedback control is required to achieve zero steady-state offset.
3. The input variables never attain a steady state; for this discussion, a ramp input is often considered, $D(t) = at$, $D(s) = a/s^2$.

Case 2 is the most typical situation, while case 3 occurs occasionally, as in a batch system where the set point is changed as a ramp. For case 2, the expression in equation (8.1) becomes

$$\lim_{t\to\infty} E(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\left(\frac{\Delta X}{s}\right) G(s) = 0 \tag{8.2}$$

where $G(s) = E(s)/X(s)$, and $X(s)$ is the input disturbance $D(s)$ or set point change $SP(s)$. By satisfying equation (8.2), the control algorithm is guaranteed to return the controlled variable to its set point for that particular process and input function. Note that systems satisfying equation (8.2) are not guaranteed to achieve zero steady-state offset for other inputs, such as a ramp. To evaluate the control performance in this chapter, a step input, $X(s) = 1/s$, will be used, because it represents the most commonly occurring situation; other inputs will be considered in later chapters.

Insensitivity to Errors

As we learned in Part II, we can never model a process exactly. Because parameters in all control algorithms depend on process models, control algorithms will always be in error despite our best modelling efforts. Therefore, control algorithms should provide good performance when the adjustable parameters have "reasonable" errors. Naturally, all algorithms will give poor performance when the adjustable parameter errors are very large. The range of reasonable errors and their effects on control performance are studied in this and several subsequent chapters.

Wide Applicability

The PID control algorithm is a simple, single equation, but it can provide good control performance for many different processes. This flexibility is achieved through several adjustable parameters, whose values can be selected to modify the behavior of the feedback system. The procedure for selecting the values is termed *tuning*, and the adjustable parameters are termed *tuning constants*.

Timely Calculations

The control calculation is part of the feedback loop, and therefore it should be calculated rapidly and reliably. Excessive time for calculation would introduce an extra slow element in the control loop and, as we shall see, degrade the control performance. Iterative calculations, which might occasionally not converge, would result in a loss of control at unpredictable times. The PID algorithm is exceptionally simple—a feature that was crucial to its initial use but is not as important now due to the availability of inexpensive digital computers for control. Because of its wide use, the PID controller is available in nearly all commercial digital control systems, so that efficiently programmed and well-tested implementations are available.

Enhancements

No single algorithm can address all control requirements. A convenient feature of the PID algorithm is its compatibility with enhancements that provide capabilities not in the basic algorithm. Thus, we can enhance the basic PID without discarding it. Many of the common enhancements are presented in Part IV.

The main goal of this chapter is to explain the PID algorithm fully. Each element of the algorithm is termed a *mode* and uses the time-dependent behavior of the feedback information in a different manner, as indicated by the name *proportional-integral-derivative*. Each mode of the equation and the key capability it provides

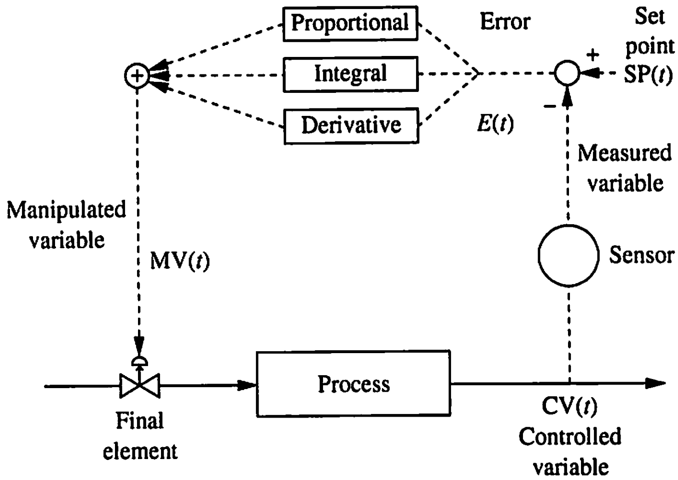


FIGURE 8.1

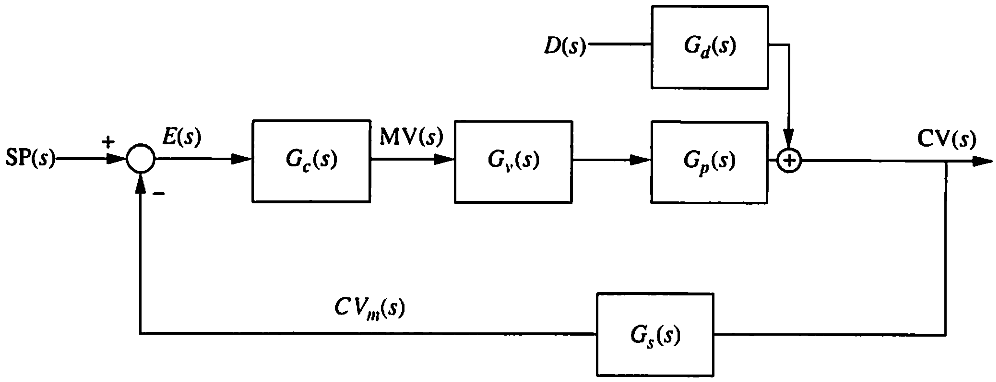
Overview schematic of a PID control loop.

are discussed thoroughly. The complete PID equation, which is the sum of the three modes as shown in Figure 8.1, is then reviewed, and a few example control responses are presented. The reader is cautioned that there is no consistency in commercial control equipment regarding the sign of the subtraction when forming the error; the convention used in this book is $E(t) = \text{SP}(t) - \text{CV}(t)$. Some preprogrammed equipment uses the opposite sign, a factor that does not affect the principles of this book but certainly affects the performance of actual control systems! (Since the error is multiplied by one of the adjustable tuning constants, the sign of the constant can be adapted to the sign of the error to give the desired direction of the control manipulation.)

8.3 ◻ BLOCK DIAGRAM OF THE FEEDBACK LOOP

In this chapter, key quantitative features of a dynamic process controlled by the proportional-integral-derivative (PID) controller will be presented. Since all elements in the loop affect the dynamic behavior, the modelling must combine the individual models of the process, instrumentation, and controller into one overall dynamic model of the loop. We learned in Chapter 4 how to combine individual models *using block diagrams*. Therefore, we begin the analysis of the control loop by deriving the transfer function models of the loop based on its constituent elements using block diagram algebra. By using general symbols of each of the loop elements, e.g., $G_p(s)$ for the process, we will derive overall transfer function models applicable to many specific systems. The model for any specific control loop can be developed by substituting the element models, e.g., $G_p(s) = K_p/(\tau s + 1)^2$ for a second-order process.

The block diagram is shown in Figure 8.2 with the terminology that will be used throughout the book. Notice that the equipment elements in the feedback loop are collected into three transfer functions: the valve or final element, $G_v(s)$; the process, $G_p(s)$; and the sensor, $G_s(s)$. The computing element is the controller $G_c(s)$. The process output variable selected to be controlled is termed the *controlled variable*, $\text{CV}(s)$, and the process input variable selected to be adjusted by the



Transfer Functions
$G_c(s)$ = Controller
$G_v(s)$ = Transmission, transducer, and valve
$G_p(s)$ = Process
$G_s(s)$ = Sensor, transducer, and transmission
$G_d(s)$ = Disturbance

Variables
$CV(s)$ = Controlled variable
$CV_m(s)$ = Measured value of controlled variable
$D(s)$ = Disturbance
$E(s)$ = Error
$MV(s)$ = Manipulated variable
$SP(s)$ = Set point

FIGURE 8.2

Block diagram of a feedback control system.

control system is termed the *manipulated variable*, MV(s). The desired value, which must be specified independently to the controller, is called the *set point*, SP(s); it is also called the *reference value* in some books on automatic control. The difference between the set point and the measured controlled variable is termed the *error*, $E(s)$. An input that changes due to external conditions and affects the controlled variable is termed a *disturbance*, $D(s)$, and the relationship between the disturbance and the controlled variable is the *disturbance transfer function*, $G_d(s)$. First, the transfer function of the controlled variable to the disturbance variable, CV$(s)/D(s)$, is derived, with the change in the set point, SP(s), taken to be zero.

The system involves a recycle, since the process output variable is used in determining the process input variable—our definition of feedback; therefore, special care must be taken in deriving the transfer function. The four-step procedure presented in Chapter 4 is used here. The first step is to begin with the variable in the numerator of the transfer function, which in this case is CV(s). In the second step, the expression for this variable as a function of input variables is derived in reverse direction to the information flow in the block diagram. The result is

$$\begin{aligned} \text{CV}(s) &= G_p(s)G_v(s)\text{MV}(s) + G_d(s)D(s) \\ &= G_p(s)G_v(s)G_c(s)G_s(s)[\text{CV}(s)] + G_d D(s) \end{aligned} \tag{8.3}$$

This procedure is followed until one of two situations is reached: the numerator variable can be expressed as a function of the denominator variable alone (which occurs for series systems), or the numerator variable can be expressed as a function of itself and the denominator variable (which occurs for a simple feedback system). The expression in equation (8.3) is clearly of the second type. The third step in the procedure is to rearrange the equation so that the variables are separated as follows:

$$[1 + G_p(s)G_v(s)G_c(s)G_s(s)]\text{CV}(s) = G_d(s)D(s) \tag{8.4}$$

Equation (8.4) can be rearranged to yield the closed-loop disturbance transfer function, and the same procedure can be used to derive the set point transfer function.

Closed-loop transfer functions for a feedback loop

Disturbance response: $$\frac{CV(s)}{D(s)} = \frac{G_d(s)}{1 + G_p(s)G_v(s)G_c(s)G_s(s)} \tag{8.5}$$

Set point response: $$\frac{CV(s)}{SP(s)} = \frac{G_p(s)G_v(s)G_c(s)}{1 + G_p(s)G_v(s)G_c(s)G_s(s)} \tag{8.6}$$

In summary, the block diagram procedure for deriving a transfer function involves four steps:

1. Select the numerator of the transfer function.
2. Solve in reverse direction to the causal relationships (arrows) in the block diagram to eliminate all variables except the numerator and denominator in the transfer function.
3. Separate variables in the equation.
4. Divide by the denominator variable to complete the transfer function.

For simple systems like the one in Figure 8.2, the foregoing procedure will yield the transfer function. In more complex systems, it will not be possible to eliminate all intermediate variables immediately in step 2. Therefore, steps 2 and 3 must be performed several times, as will be demonstrated in later chapters.

The use of block diagrams entails one potential difficulty, especially for the person just learning process control. Since the block diagram represents the model of the system, there is no distinction in the symbols used for various physical components in the system. For example, the block diagram in Figure 8.2 represents a system composed of elements from the process, $G_p(s)$ and $G_d(s)$; instrumentation, $G_v(s)$ and $G_s(s)$; and a control calculation performed by a computing device, $G_c(s)$.

Two generalizations can be made about the closed-loop transfer functions to assist in checking the derived transfer functions using block diagram manipulations. First, the numerator is simply the product of all transfer functions between the input (denominator variable) and the output (numerator variable). Second, the denominator of the right-hand side is of the form $1 + G''(s)$. The term $G''(s)$ is the product of all elements in the feedback loop. These guidelines can be checked by applying them to equations (8.5) and (8.6).

Finally, the transfer function notation is often simplified by lumping all instrumentation and process dynamics into one term, $G_p(s)$. This is equivalent to the following expression.

$$G_p(s) = G'_p(s)\, G_v(s) G_s(s) \tag{8.7}$$

with $G'_p(s)$ being the process alone. This is a natural simplification, since the dynamics of all elements from the controller output to the controller input contribute to the control system performance. Also, when the dynamics are determined empirically, the only model determined is the overall product of all instrumentation and process elements, and the individual elements are not known. The resulting simplified transfer function is

$$\frac{CV(s)}{D(s)} = \frac{G_d}{1 + G_p(s)G_c(s)} \quad \text{with} \quad G_p(s) = G'_p(s)G_v(s)G_s(s) \tag{8.8}$$

This simplification is not used when the effects of sensors and final elements are to be shown clearly; however, it is used often to simplify notation. If the process transfer function $G_p(s)$ is shown in a closed-loop block diagram or transfer function without the sensor and final element, the reader should assume that it includes the dynamics of the sensor and final element, since feedback control requires all elements in the loop.

> **The block diagram analysis yields several valuable results:**
>
> 1. **The block diagram provides a visual "picture of the equations" showing the feedback loop.**
> 2. **The general closed-loop transfer function model can be applied to any specific system by substituting the transfer function models for the loop elements.**
> 3. **Entries in the overall transfer function denominator demonstrate that only the elements *in the feedback loop* affect the system stability; neither the disturbance nor the set point change affects stability.**

The results of the block diagram analysis are not restricted to the proportional-integral-derivative (PID) controller. Any linear controller algorithm $[G_c(s)]$ would yield the conclusions in the boxed highlight above.

8.4 □ PROPORTIONAL MODE

It seems logical for the first mode to make the control action (i.e., the adjustment to the manipulated variable) proportional to the error signal, because as the error increases, the adjustment to the manipulated variable should increase. This concept is realized in the proportional mode of the PID controller:

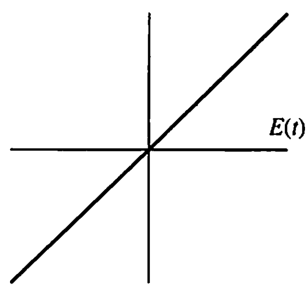
$$\text{Proportional mode:} \quad MV_p(t) = K_c E(t) + I_p$$

$$G_c(s) = \frac{MV_p(s)}{E(s)} = K_c \tag{8.9}$$

The *controller gain* K_c is the first of three adjustable parameters that enable the engineer to tailor the PID controller to various applications. The controller gain has units of [manipulated]/[controlled] variables, which is the inverse of the process gain K_p. Note that the equation includes a constant term or bias, which is used during initialization of the algorithm I_p. During initialization the value of the manipulated variable should remain unchanged; therefore, the initialization constant can be calculated at the time of initialization as

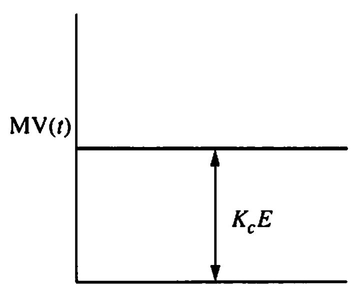
$$I_p = [MV(t) - K_c E(t)]\,|_{t=0} \tag{8.10}$$

The behavior of the proportional mode is summarized in Figure 8.3*a* and *b*. In deviation variables, a plot of manipulated variable versus error gives a straight line



Note: slope = K_c

(*a*)



Note: E(t) = constant

(*b*)

FIGURE 8.3

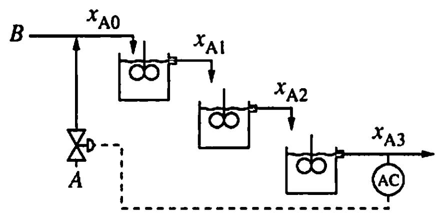
Summary of proportional mode.

with slope equal to the controller gain and zero intercept. A plot of the manipulated variable versus time for constant error gives a constant value.

Although the concept seems logical, we do not yet know whether the control performance of the proportional controller satisfies the desired control performance goals presented in the previous chapter and Section 8.2. To evaluate performance it is useful to have the closed-loop transfer function. The transfer function for the disturbance response of the system in Figure 8.2 is given in equation (8.5). Substituting the transfer function model for a proportional controller, $G_c(s) = K_c$, gives the following transfer function:

$$\frac{\mathrm{CV}(s)}{D(s)} = \frac{G_d(s)}{1 + G_p(s)G_v(s)K_cG_s(s)} \tag{8.11}$$

One of the most important goals in control performance is zero offset at the final steady state. For a disturbance response, the zero steady-state offset requires $E'(t)\,|_{t\to\infty} = -\mathrm{CV}'(t)\,|_{t\to\infty} = 0$.



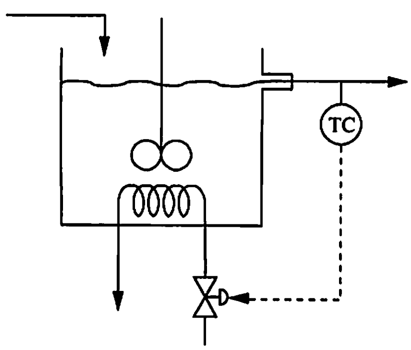
EXAMPLE 8.1.

The three-tank mixing process under control modelled in Example 7.2 is now analyzed. Recall that the feedback and disturbance processes are third-order. The steady-state value for error under proportional control can be determined by rearranging equation (8.11), substituting the models for $G_p(s)$ and $G_d(s)$, and applying the final value theorem to the system with a steplike disturbance, $D(s) = \Delta D/s$. Recall that the valve transfer function is included in $G_p(s)$, and the sensor transfer function is assumed to be unity, implying instantaneous, error-free measurement.

$$G_s(s) = 1 \qquad G_p(s)G_v(s) = \frac{K_p}{(\tau s + 1)^3} \qquad G_d(s) = \frac{K_d}{(\tau s + 1)^3} \qquad G_c(s) = K_c$$

$$\begin{aligned} \mathrm{CV}'(t)\,|_{t\to\infty} &= \lim_{s\to 0}\left[(s)(\Delta D/s)\frac{K_d\left(\frac{1}{\tau s+1}\right)\left(\frac{1}{\tau s+1}\right)\left(\frac{1}{\tau s+1}\right)}{1 + K_cK_p\left(\frac{1}{\tau s+1}\right)\left(\frac{1}{\tau s+1}\right)\left(\frac{1}{\tau s+1}\right)}\right] \\ &= \frac{K_d\Delta D}{1 + K_cK_p} \neq 0 \end{aligned} \tag{8.12}$$

Note that the feedback control system with proportional control does *not* achieve zero steady-state offset! This result can be understood by recognizing the proportional relationship between the error and the manipulated variable in the controller algorithm; the only way in which the control equation (8.9) can have the error return to zero is for the value of the manipulated variable to return to its initial condition. However, for the error to be zero in the process equation, the manipulated variable must be different from its initial value, because it must compensate for the disturbance. Thus, steady-state offset occurs with proportional-only control. This is a serious shortcoming, which must be corrected by one of the remaining two modes.



EXAMPLE 8.2.

Another important property of a control system is a fast response to a disturbance or set point change. The expression for a disturbance response is analyzed using equation (8.11) for a simple process with the disturbance and feedback processes being first-order with the same time constant. This system can be thought of as the heat exchanger in Example 3.7 and has been selected to simplify the analytical solution.

$$G_p(s) = \frac{K_p}{\tau s + 1} \qquad G_d(s) = \frac{K_d}{\tau s + 1} \qquad G_c(s) = K_c$$

$$\frac{\text{CV}(s)}{D(s)} = \frac{\dfrac{K_d}{\tau s + 1}}{1 + \dfrac{K_c K_p}{\tau s + 1}} = \frac{\dfrac{K_d}{1 + K_c K_p}}{\left(\dfrac{\tau}{1 + K_c K_p}\right)s + 1} \tag{8.13}$$

with $K_c K_p > 0$ for negative feedback control. The analytical solutions for the step disturbance response, $D(s) = \Delta D/s$, for the process with and without proportional control are

$$\text{CV}'(t) = \Delta D K_d (1 - e^{-t/\tau}) \qquad \text{(no control)} \tag{8.14}$$

$$\text{CV}'(t) = \frac{\Delta D K_d}{1 + K_c K_p}(1 - e^{-t/[\tau/(1+K_c K_p)]}) \qquad \text{(proportional control)} \tag{8.15}$$

Equation (8.15) demonstrates that the feedback controller alters both the time constant of the closed-loop system and the final deviation from set point by a factor of $1/(1 + K_c K_p)$ for a first-order process. This means that the feedback system responds faster than the open-loop system to a step disturbance and has a smaller deviation from set point. Both of these modifications to the system behavior are generally desired. The results in equation (8.15) indicate that as the controller gain is increased, the final value of the error decreases in magnitude and the system reaches steady state faster. We might be tempted to generalize this result (improperly) to all systems and apply high controller gains to all processes.

To test this idea on a more complex process, several dynamic responses for the linearized model of the three-tank mixing process under proportional control are shown in Figure 8.4*a* through *d*. Again, the input is a step disturbance in the feed concentration. The case without control ($K_c = 0$) shows the response of a third-order system to a step input; it is overdamped and reaches a final value of the disturbance magnitude. As the controller gain is increased to 10, the final value of the error decreases, as predicted by equation (8.12). Also, the time to reach the steady state decreases; that is, the dynamic response becomes faster, as predicted. As the controller gain is increased to 100, the nature of the dynamic response changes from overdamped to underdamped. As the controller gain is increased further to 220, the system becomes *unstable!*

These results demonstrate an important feature of feedback control systems: the closed-loop response can become underdamped and ultimately unstable as the controller parameters are adjusted to make the controller very aggressive (increasing the controller gain, K_c). This example suggests, and later theoretical analysis will confirm, that it is generally not possible to maintain the controlled variable close to the set point by setting the controller gain to a very large value (although this approach would work for the first-order process in Example 8.2). The reasons for the instability and methods for predicting the stability limits are presented in Chapter 10 after the control algorithm has been fully explained.

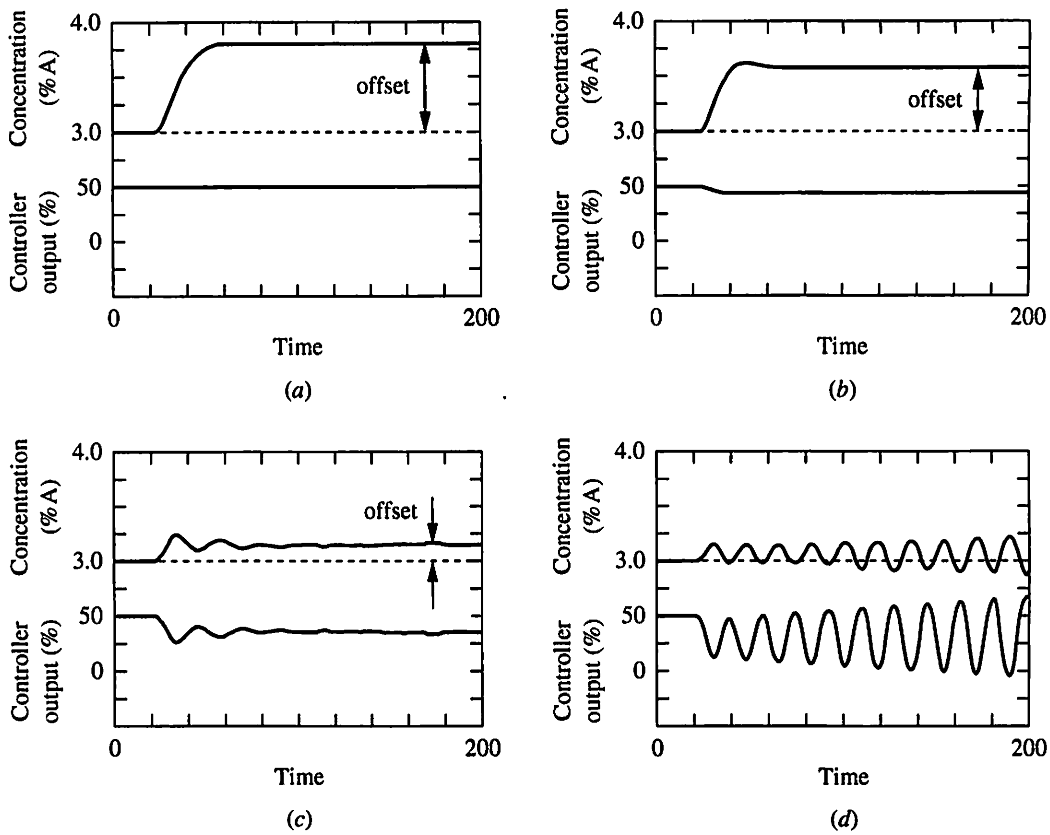
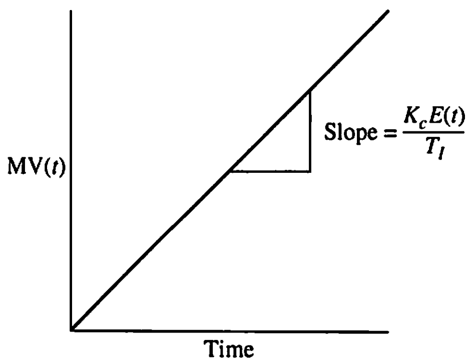


FIGURE 8.4

Disturbance responses for three-tank mixing process under proportional control subject to a disturbance in feed composition $(x_A)_B$ of 0.8%A and K_c = [%open/(%A)]: (*a*) without control; (*b*) with proportional control, $K_c = 10$; (*c*) with proportional control, $K_c = 100$; (*d*) with proportional control, $K_c = 220$.

> The proportional mode is simple, provides a rapid adjustment of the manipulated variable, does not provide zero offset although it reduces the error, speeds the dynamic response, and can cause instability if tuned improperly.



Note: $E(t)$ = constant

FIGURE 8.5

Summary of the behavior of the integral mode.

8.5 ◻ INTEGRAL MODE

Since the proportional mode does not completely eliminate the effects of disturbances, the next mode should be "persistent" in adjusting the manipulated variable until the magnitude of the error is reduced to zero for a steplike input. This result is achieved by the integral mode:

$$\text{Integral mode:} \quad MV_I(t) = \frac{K_c}{T_I}\int_0^t E(t')\,dt' + I_I$$
$$G_c(s) = \frac{MV_I(s)}{E(s)} = \frac{K_c}{T_I s} \tag{8.16}$$

The new adjustable parameter is termed the integral time, T_I, which has the units of time; it is combined with the controller gain in equation (8.16) because this is the conventional form of the integral mode used in the commercial PID controller. This form is used throughout the book for consistency and so that later correlations for parameter values can be used. Again, the integral mode equation has a constant of initialization.

The behavior of the integral mode is summarized in Figure 8.5. For a constant error, the manipulated variable increases linearly with a slope of $E(t)K_c/T_I$. This behavior is different from the proportional mode, in which the value is constant over time for a constant error.

EXAMPLE 8.3.

The effect of the integral mode can be determined by evaluating the offset of the three-tank mixing process under integral-only control for a step disturbance, $D(s) = \Delta D/s$.

$$G_v(s)G_p(s) = \frac{K_p}{(\tau s + 1)^3} \qquad G_d(s) = \frac{K_d}{(\tau s + 1)^3} \qquad G_c(s) = \frac{K_c}{T_I s} \qquad G_s(s) = 1$$

$$\mathrm{CV}'(t)\big|_{t=\infty} = \lim_{s\to 0}\left[\frac{(s)(\Delta D/s)K_d\left(\frac{1}{\tau s+1}\right)\left(\frac{1}{\tau s+1}\right)\left(\frac{1}{\tau s+1}\right)}{1 + K_c\left(\frac{1}{T_I s}\right)K_p\left(\frac{1}{\tau s+1}\right)\left(\frac{1}{\tau s+1}\right)\left(\frac{1}{\tau s+1}\right)}\right] \quad (8.17)$$

$$= 0$$

The integral control mode achieves zero steady-state offset, which is the primary reason for including this mode.

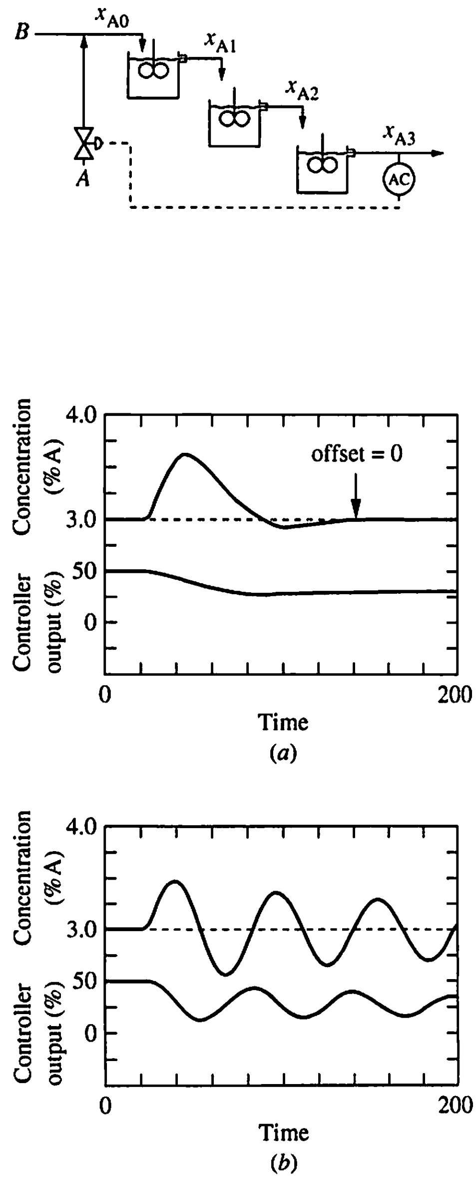


FIGURE 8.6

Three-tank mixing process under integral-only control subject to a disturbance in feed composition $(x_A)_B$ of 0.8%A and K_c = [%open/%A], T_I = [min]: (a) $K_c = 1, T_I = 1$; (b) $K_c = 1,\ T_I = 0.25$.

Again, some dynamic responses of the three-tank mixing process are plotted, this time with an integral controller, in Figure 8.6*a* and *b*. As can be seen, the manipulation of the controller output is slower for integral-only control than for proportional-only control. As a result, the controlled variable returns to the set point slowly and experiences a larger maximum deviation. If the integral time is reduced small enough, as in Figure 8.6*b*, the controller will be very aggressive, and the system will become highly oscillatory; further reduction in T_I can lead to an *unstable* system. Under integral-only control with properly selected tuning constants, the controlled variable returns to its set point, but the other aspects of control performance are usually not acceptable. In summary:

> The integral mode is simple; achieves zero offset; adjusts the manipulated variable in a slower manner than the proportional mode, thus giving poor dynamic performance; and can cause instability if tuned improperly.

8.6 □ DERIVATIVE MODE

If the error is zero, both the proportional and integral modes give zero adjustment to the manipulated variable. This is a proper result if the controlled variable is not changing; however, consider the situation in Figure 8.7 at time equal to t when the disturbance just begins to affect the controlled variable. There, the error and integral error are nearly zero, but a substantial change in the manipulated variable would seem to be appropriate because the rate of change of the controlled variable is large. This situation is addressed by the derivative mode:

$$\text{Derivative mode:} \qquad MV_d(t) = K_c T_d \frac{dE(t)}{dt} + I_d$$
$$G_c = \frac{MV_d(s)}{E(s)} = K_c T_d s \tag{8.18}$$

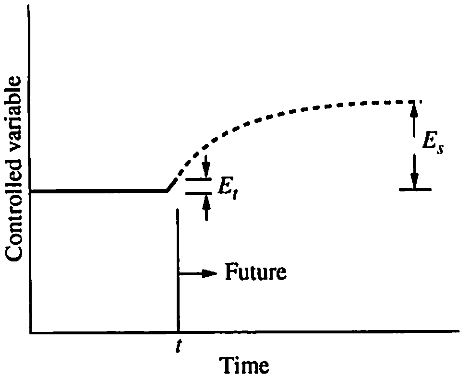


FIGURE 8.7

Assumed effect of disturbance on controlled variable.

The final adjustable parameter is the derivative time T_d, which has units of time, and the mode again has an initialization constant. Note that the proportional gain and derivative time are multiplied together to be consistent with the conventional PID algorithm.

Some further insight can be gained by examining the following development of a proportional-derivative controller (Rhinehart, 1991). Again consider the dynamic response in Figure 8.7, in which the data available at the current time t, which is at the beginning of the disturbance response, is shown by the solid line. The future response that would be obtained without feedback control is shown as the dotted line; note that this is simply the disturbance response. The value of the E_s, the total effect of the disturbance on the controlled variable as time approaches infinity, can be predicted using the assumption that the error is following a first-order response with a time constant equal to the disturbance process time constant:

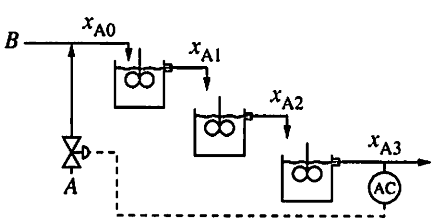
$$\tau_d \frac{dE}{dt} + E = E_s \tag{8.19}$$

Since the error will increase to E_s ultimately, the manipulated variable will have to be adjusted by a value proportional to E_s, or $MV' = E_s/K_c$. Rather than wait until the error becomes large, when the proportional and integral modes would adjust the manipulated variable, the controller could anticipate the future error using the foregoing equation to give

$$MV = K_c \left(E + \tau_d \frac{dE}{dt} \right) + I_d \tag{8.20}$$

Thus, the proportional-derivative modes are a natural result of the assumption that the error will respond as given in Figure 8.7. If the assumption is good, the derivative mode may improve the control performance.

The behavior of the calculation for the derivative-only mode is shown in Figure 8.8. When the controlled variable is constant, the derivative mode makes no change to the manipulated variable. When the controlled variable changes, the derivative mode adjusts the manipulated variable in a manner proportional to the rate of change.



EXAMPLE 8.4.

The offset of a derivative controller can be determined by applying the final value theorem to the three-tank mixing process for a step disturbance, $D(s) = \Delta D/s$.

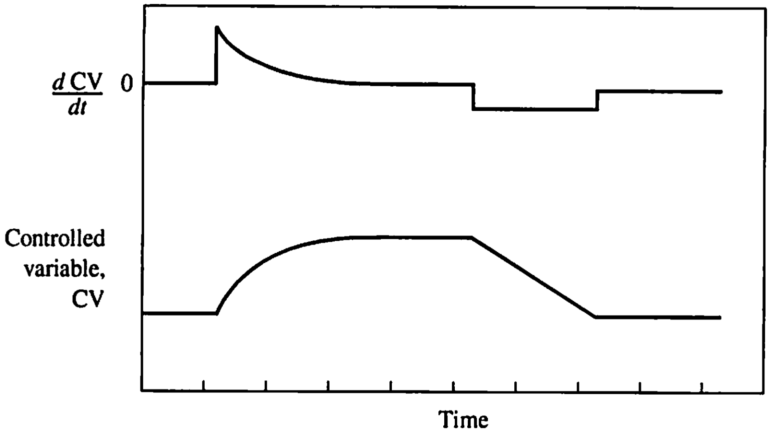


FIGURE 8.8

Example of the calculation of the derivative mode with constant set point.

$$G_v(s)G_p(s) = \frac{K_p}{(\tau s+1)^3} \qquad G_d(s) = \frac{K_d}{(\tau s+1)^3} \qquad G_c(s) = K_c T_d s$$

$$\text{CV}'(t)|_{t=\infty} = \lim_{s\to 0}\left[\frac{(s)(\Delta D/s)K_d\left(\frac{1}{\tau s+1}\right)\left(\frac{1}{\tau s+1}\right)\left(\frac{1}{\tau s+1}\right)}{1+K_c T_d s\left(\frac{1}{\tau s+1}\right)\left(\frac{1}{\tau s+1}\right)\left(\frac{1}{\tau s+1}\right)}\right] \tag{8.21}$$

$$= K_d \Delta D \neq 0$$

As is apparent, the derivative mode does not give zero offset. In fact, it does not reduce the final deviation below that for a system without control for any disturbance whose derivative tends toward zero as time increases; thus, its only benefit can be in improving the transient response. Since the derivative is never used as the only controller mode, dynamic responses are not included in this section, but dynamic responses for the PID controller will be given.

The derivative mode amplifies sudden changes in the controller input signal, causing potentially large variation in the controller output that can be unwanted for two reasons. First, step changes to the set point lead to step changes in the error. The derivative of a step change goes to infinity or, in practical cases, to a completely open or closed control valve. This control action could lead to severe process upsets and even to unsafe conditions. One approach to prevent this situation is to alter the algorithm so that the derivative is taken on the controlled variable, not the error. The modified derivative mode, remembering that $E(t) = \text{SP}(t) - \text{CV}(t)$, is

$$\text{MV}_d(t) = -K_c T_d \frac{d\text{CV}(t)}{dt} + I_d \tag{8.22}$$

While equation (8.22) reduces the extreme variation in the manipulated variable resulting from set point changes, it does not solve the problem of high-frequency noise on the controlled-variable measurement, which will also cause excessive variation in the manipulated variable. An obvious step to reduce the effects of noise is to reduce the derivative time, perhaps to zero. Other steps to reduce the effects of noise are presented in Chapter 12. In summary:

> **The derivative mode is simple; does not influence the final steady-state value of error; provides rapid correction based on the rate of change of the controlled variable; and can cause undesirable high-frequency variation in the manipulated variable.**

8.7 ◻ THE PID CONTROLLER

Naturally, it is desired to retain the good features of each mode in the final control algorithm. This goal can be achieved by adding the three modes to give the final expression of the PID controller. Where the derivative mode appears, two forms are given: (*a*) the standard and (*b*) the form recommended in this book because it prevents set point changes from causing excessive response, as described in the preceding section.

Time-Domain Controller Algorithms

PROPORTIONAL-INTEGRAL-DERIVATIVE.

$$\mathrm{MV}(t) = K_c\left(E(t) + \frac{1}{T_I}\int_0^t E(t')\,dt' + T_d\frac{dE(t)}{dt}\right) + I \qquad (8.23a)$$

$$\mathrm{MV}(t) = K_c\left(E(t) + \frac{1}{T_I}\int_0^t E(t')\,dt' - T_d\frac{d\,\mathrm{CV}(t)}{dt}\right) + I \qquad \text{(Recommended)} \qquad (8.23b)$$

Again, the controller has an initialization constant. Depending on the desired performance, various forms of the controller are used. The proportional mode is normally retained for all forms, with the options being in the derivative and integral modes. The most common alternative forms are as follows:

PROPORTIONAL-ONLY CONTROLLER.

$$\mathrm{MV}(t) = K_c[E(t)] + I \qquad (8.24)$$

PROPORTIONAL-INTEGRAL CONTROLLER.

$$\mathrm{MV}(t) = K_c\left(E(t) + \frac{1}{T_I}\int_0^t E(t')\,dt'\right) + I \qquad (8.25)$$

PROPORTIONAL-DERIVATIVE CONTROLLER.

$$\mathrm{MV}(t) = K_c\left(E(t) + T_d\frac{dE(t)}{dt}\right) + I \qquad (8.26a)$$

$$\mathrm{MV}(t) = K_c\left(E(t) - T_d\frac{d\,\mathrm{CV}(t)}{dt}\right) + I \qquad \text{(Recommended)} \qquad (8.26b)$$

Selection from among the four forms will be discussed after many features of the controllers have been introduced.

Laplace-Domain Transfer Functions

The control algorithms are used often in block diagrams and in closed-loop transfer functions. In these analyses the main purposes are to determine limiting behavior for control systems (stability and frequency response), usually for disturbance response; thus, the PID form with derivative on the error is used for simplicity. The transfer functions for the common forms are as follows. Note that each transfer function is the output over the input, with the input and output taken with respect to the *controller*, which is the opposite of the process. Also, since transfer functions are always in deviation variables, the initialization constant does not appear.

PROPORTIONAL-INTEGRAL-DERIVATIVE.

$$G_c(s) = \frac{\mathrm{MV}(s)}{E(s)} = K_c\left(1 + \frac{1}{T_I s} + T_d s\right) \tag{8.27}$$

PROPORTIONAL-ONLY.

$$G_c(s) = \frac{\mathrm{MV}(s)}{E(s)} = K_c \tag{8.28}$$

PROPORTIONAL-INTEGRAL.

$$G_c(s) = \frac{\mathrm{MV}(s)}{E(s)} = K_c\left(1 + \frac{1}{T_I s}\right) \tag{8.29}$$

PROPORTIONAL-DERIVATIVE.

$$G_c(s) = \frac{\mathrm{MV}(s)}{E(s)} = K_c\,(1 + T_d s) \tag{8.30}$$

The reader is strongly encouraged to learn the various forms of the algorithms in the time and Laplace domains, because they will be used in all subsequent topics.

8.8 ▫ ANALYTICAL EXPRESSION FOR A CLOSED-LOOP RESPONSE

It is clear that the algorithm structure and adjustable parameters affect the closed-loop dynamic response. A straightforward method of determining how the parameters affect the response is to determine the analytical solution for the linear process with PID feedback. This is generally *not done in practice,* because of the complexity of the analytical solution for realistic processes, especially when the process has dead time. However, the analytical solution is derived here for a simple process, to aid in understanding the interplay between the process and the controller.

EXAMPLE 8.5.

To facilitate the solution, a simple process—the stirred-tank heater in Example 3.7—is selected, with the controlled variable being the tank temperature and the manipulated variable being the coolant flow valve, as shown in Figure 8.9. Since proportional control was considered in Example 8.2, a proportional-integral controller is selected, because this will ensure zero steady-state offset. The response to a step set point change will be determined.

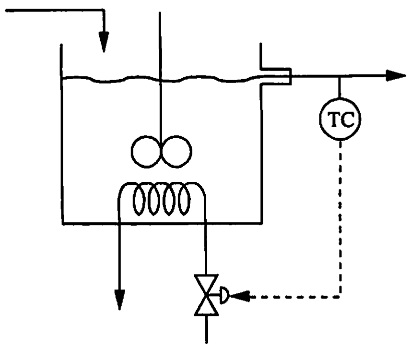


FIGURE 8.9

Heat exchanger control system in Example 8.5.

Formulation. The model for this process was derived in Example 3.7. It is repeated here with the models for the other elements in the control loop: the valve and the controller (the sensor is assumed to be instantaneous).

$$V\rho C_p \frac{dT}{dt} = C_p \rho F(T_0 - T) - \frac{aF_c^{b+1}}{F_c + \dfrac{aF_c^b}{2\rho_c C_{pc}}}(T - T_{cin}) \tag{8.31}$$

$$F_c = \left(K_v \sqrt{\frac{\Delta P}{\rho_c}}\right) v \tag{8.32}$$

$$v = K_c \left((T_{sp} - T) + \frac{1}{T_I}\int_0^t (T_{sp} - T)\,dt'\right) + I \tag{8.33}$$

First, the degrees of freedom of the closed-loop control system will be evaluated.

Dependent variables: $T,\ F_c,\ v$

External variables: $T_0,\ F,\ T_{cin},\ T_{sp}$ $\qquad$ $\text{DOF} = 3 - 3 = 0$

Constants: $\rho,\ C_p,\ C_{pc},\ a,\ b,\ K_v,\ \Delta P,\ \rho_c,\ K_c,\ T_I,\ I,\ V$

Thus, when the controller set point T_{sp} has been defined, the system is exactly specified. Note that the system without control requires the valve position to be defined, but that the controller now determines the valve opening based on its algorithm in equation (8.33). The three equations can be linearized and the Laplace transforms taken to obtain the following transfer functions:

$$G_p(s) = \frac{K_p}{\tau s + 1} \tag{8.34}$$

$$G_v(s) = K_v \sqrt{\frac{\Delta P}{\rho}} \approx 1.0 \tag{8.35}$$

$$G_c(s) = \frac{v(s)}{T_{sp}(s) - T(s)} = K_c\left(1 + \frac{1}{T_I s}\right) \tag{8.36}$$

The process gain and time constant are functions of the equipment design and operating conditions and are given in Example 3.7. We assume that the valve opening is expressed in fraction open and that $G_v(s) = 1$. The block diagram of the single-loop control system is given in Figure 8.2, and the closed-loop transfer function is rearranged to give

$$\text{CV}(s) = \frac{G_p(s)G_v(s)G_c(s)}{1 + G_p(s)G_v(s)G_c(s)G_s(s)}\text{SP}(s) \tag{8.37}$$

The general symbols are used for the controlled and set point variables, $\text{CV}(s) = T(s)$ and $\text{SP}(s) = T_{sp}(s)$. The transfer functions for the process, the PI controller, and the instrumentation ($G_s(s) = G_v(s) = 1$) can be substituted into equation (8.37) to give

$$\begin{aligned}
CV(s) &= \frac{G_p(s)G_c(s)}{1+G_p(s)G_c(s)}SP(s) \\
&= \frac{\frac{K_p}{\tau s+1}K_c\left(1+\frac{1}{T_I s}\right)}{1+\frac{K_p}{\tau s+1}K_c\left(1+\frac{1}{T_I s}\right)}SP(s) \\
&= \frac{T_I s+1}{\frac{\tau T_I}{K_c K_p}s^2+\frac{T_I(1+K_c K_p)}{K_c K_p}s+1}SP(s)
\end{aligned} \tag{8.38}$$

This can be rearranged to give the transfer function for the closed-loop system:

$$\frac{CV(s)}{SP(s)} = \frac{T_I s+1}{(\tau')^2 s^2 + 2\xi\tau' s + 1} \tag{8.39}$$

This is presented in the standard form with the time constant (τ') and damping coefficient expressed as

$$\xi = \frac{1}{2}\sqrt{\frac{T_I}{K_c K_p}}\left(\frac{1+K_c K_p}{\sqrt{\tau}}\right) \qquad \tau' = \sqrt{\frac{\tau T_I}{K_c K_p}} \tag{8.40}$$

Equation (8.39) can be rearranged to solve for CV(s) with SP(s) = $\Delta SP/s$ (step change). This expression can be inverted using entries 15 and 17 in Table 4.1 to give, for $\xi < 1$,

$$\begin{aligned}
T'(t) &= \Delta SP\left[\frac{T_I}{\tau'\sqrt{1-\xi^2}}e^{-\xi t/\tau'}\sin\left(\frac{\sqrt{1-\xi^2}}{\tau'}t\right)\right] \\
&+\Delta SP\left[1-\frac{1}{\sqrt{1-\xi^2}}e^{-\xi t/\tau'}\sin\left(\frac{\sqrt{1-\xi^2}}{\tau'}t+\phi\right)\right]
\end{aligned} \tag{8.41}$$

with $\quad \phi = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$

or using entry 10 in Table 4.1 to give, for $\xi > 1$,

$$T'(t) = \Delta SP\left[T_I\frac{(e^{-t/\tau'_1}-e^{-t/\tau'_2})}{\tau'_1-\tau'_2}+1+\frac{\tau'_1 e^{-t/\tau'_1}-\tau'_2 e^{-t/\tau'_2}}{\tau'_2-\tau'_1}\right] \tag{8.42}$$

with τ'_1 and τ'_2 the real, distinct roots of the characteristic polynomial when $\xi > 1.0$.

Solution. Before an example response is evaluated, some important observations are made:

1. The feedback system is second-order, although the process is first-order. Thus, we see that the integral controller increases the order of the system by 1.
2. The integral mode ensures zero steady-state offset, which can be verified by evaluating the foregoing expressions as time approaches infinity.
3. The response can be over- or underdamped, depending on the parameters in equation (8.40). Again, we see that feedback can change the qualitative characteristics of the dynamic response.

4. The response for this system is always stable (for negative feedback, $K_c K_p > 0$); in other words, the output cannot grow in an unbounded manner, because of the structure of the process and controller equations. This is not generally true for more complex and realistic process models (and essentially all control systems involving real processes), as will be explained in Chapter 10.

The final observation concerns the manipulated variable, which is also important in evaluating control performance. The transfer function for the manipulated variable can be derived from block diagram algebra to be

$$\frac{\text{MV}(s)}{\text{SP}(s)} = \frac{G_c(s)}{1 + G_p(s)G_c(s)G_v(s)G_s(s)} \tag{8.43}$$

The characteristic polynomials for the transfer functions in equations (8.37) and (8.43) are identical; thus, the periodic nature of the responses (over- or underdamped) of the controlled and manipulated variables are the same since they are affected by the same factors in the control loop. Thus, it would not be possible to obtain underdamped behavior for the controlled variable and overdamped behavior for the manipulated variable. The close relationship between these variables is natural, because the manipulated variable is calculated by the PI controller based on the controlled variable.

Results analysis. A sample dynamic response is given in Figure 8.10 for this system with $K_p = -33.9$°C/(m^3/min) and $\tau = 11.9$ min from Example 3.7 and tuning constant values of $K_c = -0.059$(m^3/min)/°C and $T_I = 0.95$ min, giving $\tau' = 2.38$ min and $\xi = 0.30$, and $\text{SP}'(s) = 2/s$. The response is clearly under-

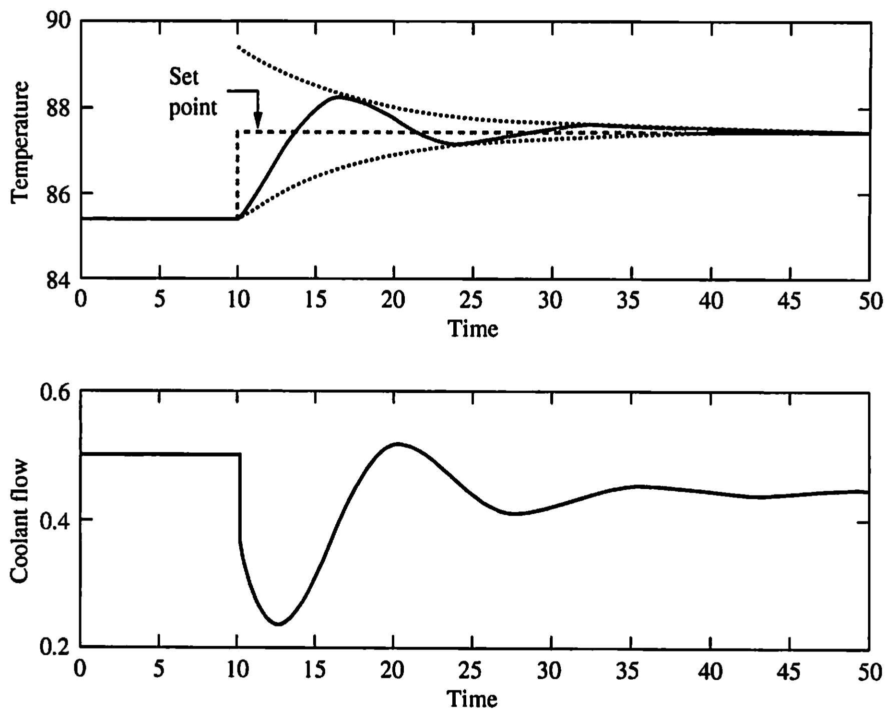


FIGURE 8.10

Dynamic response of feedback loop: set point (dotted), temperature (solid), and limits on magnitude (dashed).

damped, as indicated by the damping coefficient being less than 1.0. Also shown in the figure is the boundary defined by the exponential in the analytical solution, which determines the maximum amplitude of the oscillation at any time. Note that another set of controller tuning constants could yield overdamped behavior for the closed-loop system. The parameters used in this example were selected somewhat arbitrarily, and proper tuning methods are presented in the next two chapters.

Since both tuning constants, K_c and T_I, appear in τ' and ξ, it is not possible to attribute the damping or oscillations to a single tuning constant; they both affect the "speed" and damping of the response. It is apparent from the expression for ξ that the response becomes more oscillatory as K_c is increased and as T_I is decreased; the reason for the difference is that K_c is in the numerator of the controller, whereas T_I is in the denominator of the control algorithm. It is also apparent from equation (8.41) that the controlled-variable overshoot and decay ratio increase as the damping coefficient decreases.

This analysis could be extended to other simple systems, but it cannot be applied to most realistic systems, for which the inverse Laplace transform cannot be evaluated. Therefore, the derivation of complete analytical solutions will not be extended here. However, the general principles learned in this example are applicable to the methods of analysis introduced in the next few chapters. Also, one important class of processes—inventories (levels)—is simple enough to allow process equipment and controller design based on analytical solution of the linearized models, as covered in Chapter 18.

8.9 □ IMPORTANCE OF THE PID CONTROLLER

The process industries, which operate equipment at high pressures and temperatures with potentially hazardous materials, needed reliable process control many decades before digital computers became available. As a result, the control methods developed many decades ago were tailored to the limited computing equipment available at that time. The main method of automated computing during this period, and one which continues to be used today, is analog computation. The principle behind analog computing is the design of a physical system that follows the same equations as the equations desired to be solved (Korn and Korn, 1972). Naturally, the computing system must be simple and should have easy ways to alter parameters. An example of an analog control system is shown schematically in Figure 8.11. Here the level in a tank is controlled by adjusting the flow into the tank. The sensor is a float in the tank, and the final control element is the valve stem position. The controller is a proportional-only algorithm, so that the controller output is proportional to the error signal. This algorithm is implemented in the figure by a bar that pivots on a fulcrum. As the level increases, the float rises and the valve closes, reducing flow into the tank. The control parameters can be changed by (1) increasing the height of the fulcrum to increase the set point (with an appropriate adjustment of the connecting bars) or (2) altering the fulcrum position along the bar to change the controller proportional gain.

Although a few systems like the one in Figure 8.11 are in use (indeed, a form of that system is found in domestic toilet tanks), most of the analog controllers in the process industries use more sophisticated pneumatic or electronic principles

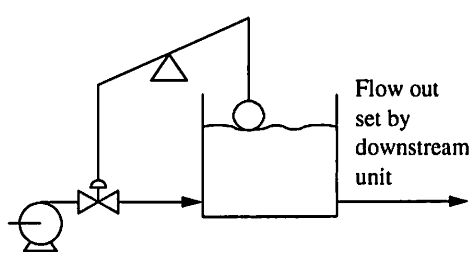


FIGURE 8.11

Example of an analog level controller.

to automate the PID algorithm. The typical industrial implementation yields the following transfer function for an electronic analog controller calculation (Hougen, 1972):

$$\frac{\text{MV}(s)}{\text{CV}(s)} = K_c \left[\frac{1 + T_I s}{T_I s}\right]\left[\frac{1 + T_d s}{1 + \alpha T_d s}\right] \tag{8.44}$$

Equation (8.44), often referred to as the *interactive PID algorithm*, is an approximation to the PID algorithm when α is small. The tuning constants are adjusted by changing values of resistors and capacitors used in the circuit. Note that since the equation structure is different from the forms already introduced, this equation would require different values of their tuning constants; the tuning rules in this book are for the forms in equation (8.23*b*). Analog controllers were used for many decades prior to the introduction of digital controllers and continue to be used today. Pneumatic analog controllers use air pressure as the source of power for the calculation to approximate the PID calculation (Ogata, 1990).

The techniques in this book are based on the analysis of continuous systems, because we will be using Laplace transforms and similar mathematical methods. Most processes are continuous (e.g., stirred tanks and heat exchangers), and the controller is also continuous when implemented with analog computation. However, the controller is discrete when implemented by digital computation; discrete systems perform their function only at specific times. For most of this book, the assumption is made that the control calculations are continuous, and this assumption is generally very good for digital controllers as long as the time for calculation is short compared with the process dynamic response. Since this situation is satisfied in most process control systems, the approach taken here is usually valid. Special features of digital control systems are introduced in Chapter 11 and covered thereafter as appropriate for subsequent topics, and numerous resources are dedicated entirely to the special aspects of digital control, for example, Appendix L, Franklin and Powell (1980) and Smith (1972).

8.10 ▪ CONCLUSIONS

In this chapter, the important proportional-integral-derivative control algorithm was introduced, and the key features of each mode were demonstrated. The proportional mode provides fast response but does not reduce the offset to zero. The integral mode reduces the offset to zero but provides relatively slow feedback compensation. The derivative mode takes action based on the derivative of the controlled variable but has no effect on the offset. The combination of the modes, or a subset of the modes, is required to provide good control in most cases.

A few examples have demonstrated that the PID controller can achieve good control performance with the proper choice of tuning constants. However, the control system can perform poorly, and even become unstable, if improper values of the controller tuning constants are used. An analytical method for determining good values for the tuning constants was introduced in this chapter for simple first-order processes with P-only and PI control. More general methods are presented for more complex systems in the next two chapters.

The dramatic influence of feedback on the dynamic behavior of a process was discussed in Chapter 7 and demonstrated mathematically in this chapter. Naturally, the ability to maintain the controlled variable near its set point is a desirable feature of feedback, but the potential change from an overdamped system to an underdamped or even unstable one is a facet of feedback that must be understood and monitored carefully to prevent unacceptable behavior. In Chapter 4, it was demonstrated that the key facets of periodicity and stability are determined by the roots of the characteristic equation, that is, by the poles of the transfer function. For the three-tank mixing process without control, the characteristic equation is

$$(\tau s + 1)^3 = 0 \tag{8.45}$$

giving the repeated poles $s = -1/\tau$. Since they are real and negative, the dynamic response is overdamped and stable. When proportional feedback is added, the transfer function is given in equation (8.12), and the characteristic equation is

$$(\tau s + 1)^3 + K_c K_p = 0 \tag{8.46}$$

Thus, the controller gain influences the poles and the exponents in the time-domain solution for the concentration. The influence of feedback control on stability is the major topic of Chapter 10.

Finally, it is important to note that the PID controller is emphasized in this book because of its widespread use and its generally good performance. The dominant position of this algorithm is not surprising, because it evolved over years of industrial practice. However, in nearly no case is it an "optimal" controller in any sense (i.e., minimizing IAE or maximum deviation). Thus, other algorithms can provide better performance in particular situations. Some alternative algorithms will be introduced in this book after the basic concepts of feedback control have been thoroughly covered.

REFERENCES

Franklin, G., and J. Powell, *Digital Control of Dynamic Systems*, Addison-Wesley, Reading, MA, 1980.

Hougen, J., *Measurements and Control Applications for Practicing Engineers*, Cahners Books, Boston, MA, 1972.

Korn, G., and T. Korn, *Electronic Analog and Hybrid Computers*, McGraw-Hill, New York, 1972.

Ogata, K., *Modern Control Engineering*, Prentice-Hall, Englewood Cliffs, NJ, 1990.

Rhinehart, R., personal communication, 1991.

Smith, C., *Digital Computer Process Control*, Intext, Scranton, PA, 1972.

ADDITIONAL RESOURCES

A brief history of operator interfaces for process control, showing the key graphical and pattern recognition features, is given in

Lieber, R., "Process Control Graphics for Petrochemical Plants," *Chem. Eng. Progr.* 45–52 (Dec. 1982).

Additional analytical solutions to low-order closed-loop systems can be found in

Weber, T., *An Introduction to Process Dynamics and Control*, Wiley, New York, 1973.

For a more complete discussion of system types than presented in Section 8.2, see

Distephano, S., A. Stubbard, and I. Williams, *Feedback Control Systems*, McGraw-Hill, New York, 1976.

> **With models for the process and controller now available, the dynamic behavior of a closed-loop system can be analyzed quantitatively. These questions provide some learning examples while using the mathematical tools available; additional analytical methods are introduced in the next chapters. The key concept is the manner in which the process and controller both influence the feedback system.**

QUESTIONS

8.1. Determine the analytical expression for a step set point change in the following processes under P-only and PI feedback control. You should select values for the tuning constant that give acceptable performance.

(*a*) Example 3.1 with C_A as the controlled variable, C_{A0} as the manipulated variable, and $\Delta SP = 0.1$ mole/m^3.

(*b*) Example 3.7 with T as the controlled variable, F as the manipulated variable, and $\Delta SP = 3°C$. (F_c is constant.)

(*c*) Example 3.3 with C_{A2} as the controlled variable, C_{A0} as the manipulated variable, and $\Delta SP = 0.05$ mole/m^3.

8.2. Program a dynamic simulation for the three-tank mixing system based on the equations derived in Example 7.2.

(*a*) Determine the open-loop responses in the third tank outlet concentration to a step change in

(1) The inlet concentration of component A in stream *B* (1 to 1.5% A)
(2) The valve position in the *A* stream (50 to 60% open)

(*b*) Determine the closed-loop (PID) responses of the third tank outlet concentration to

(1) A step set point change (3 to 3.5% A)
(2) A disturbance step change in the concentration of component A in stream *B* (1 to 1.5% A)

8.3. Using the appropriate transfer functions and applying the final value theorem, determine the final values of the error for a step set point change for the heater in Example 8.5 under P-only, PI, and PID control.

8.4. The control system given in Figure Q8.4 controls the level by adjusting the valve position of the flow out of the tank. Because of the pump, the flow out can be assumed to be a function of only the valve percent open and not of the level. Assume that the valve-flow relationship is linear (i.e., $F_{\text{out}} = K_v v$).

(*a*) Derive the differential equation and transfer function relating the level to the flows in and out.

(*b*) For the process with feedback control, determine the final value of the error for a step change in the inlet flow for P-only and PI controllers. Are the criteria for zero steady-state offset the same as for the three-tank example? Explain why/why not.

(*c*) Discuss the differences between this and question 8.13.

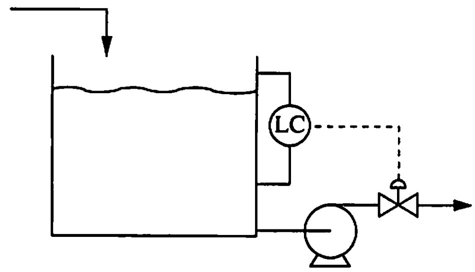


FIGURE Q8.4

8.5. The application to the final value theorem in equation (8.17) showed that the three-tank mixing system under I-only control has zero steady-state offset for a step disturbance. Is this a general conclusion for PID control for all (*a*) processes, (*b*) disturbance types, and (*c*) values of the tuning constants? Discuss the implications of your answers on the success of feedback control.

8.6. (*a*) The final value theorem seems to demonstrate that the offset tends to zero as the controller gain approaches infinity. Discuss this result, especially with regard to the definition of the Laplace transform and the dynamic responses shown in Figure 8.4*a* through *d*.

(*b*) The final value theorem provides one method for calculating the final value of a variable in a control system. Describe another way to determine the final value of variables without using the final value theorem. Use both methods to determine the final value of the manipulated variable in the three-tank mixing process for a step disturbance in the concentration of stream *B*, (*a*) without control and (*b*) with P-only feedback control.

8.7. (*a*) Calculate the roots of the characteristic equations and relate them to the dynamic behaviors of the closed-loop systems in Figure 8.4*a* through *d*.

(*b*) Select different tuning constant values that yield substantially different dynamic behavior for the closed-loop system in Example 8.5. Describe the different time-domain behavior.

8.8. Answer the following questions.

(*a*) The transfer function of the PID controller in equation (8.27) has no initialization constant. Why?

(*b*) Describe how to calculate the initialization constant I in equation (8.23*a* and *b*) for a PID controller.

(*c*) The transfer functions $G_c(s) = \text{MV}(s)/\text{CV}(s)$ and $G_p(s) = \text{CV}(s)/\text{MV}(s)$. Why isn't $G_c(s) = G_p^{-1}(s)$? Why do they have units that are the inverse of one another?

(*d*) Verify the Laplace transform of the controller, equation (8.27), from equation (8.23*a*).

(*e*) Determine the final value for the three-tank mixing process under PI control for an impulse disturbance in the feed composition. Can you determine a conclusion generally applicable to all processes?

(*f*) Repeat part (*e*) for a ramp disturbance.

8.9. When designing the feedback control algorithm, why were the following modes not included, or when would they be applicable?

(*a*) $\mathrm{MV}(t) = K_c\left(E(t) + \frac{1}{T_I}\int_0^t\left[\int_0^{t'} E(t'')\,dt''\right]dt'\right) + I$

(*b*) $\mathrm{MV}(t) = K_c(E(t))^2\left(E(t) + \frac{1}{T_I}\int_0^t E(t')\,dt'\right) + I$

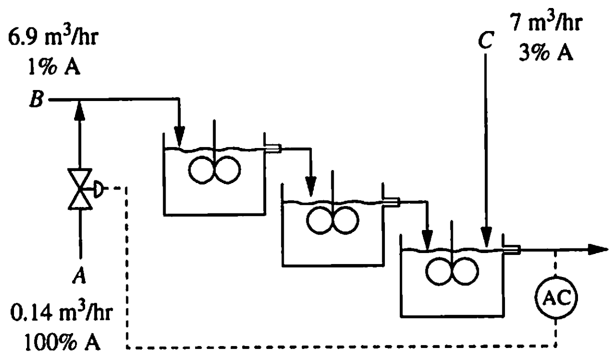
(*c*) $\mathrm{MV}(t) = K_c\left((E(t))^2 + \frac{1}{T_I}\int_0^t (E(t'))^2\,dt'\right) + I$

8.10. The controller display for the plant personnel does not present all possible variables associated with the PID algorithm. For each variable, state whether or not it is displayed and why: (*a*) controlled variable, (*b*) error, (*c*) set point, (*d*) manipulated variable, (*e*) integral of the error, (*f*) derivative of the error, and (*g*) initialization constant.

8.11. Describe how you would calculate the PID algorithm in a digital computer. Prepare a flow chart of the calculations.

8.12. Consider the modified stirred-tank mixing system in Figure Q8.12. The original concentration of the third tank remains 3 percent.

(*a*) Derive the equations describing the system.
(*b*) Draw a block diagram of the system.
(*c*) Derive the transfer functions for each element in the block diagram.
(*d*) Derive the closed-loop transfer function, CV(*s*)/SP(*s*).



Disturbance is change in the concentration of stream *C* with the flow rate constant.

FIGURE Q8.12

8.13. The level control system with a proportional-only algorithm in Figure Q8.13 is to be analyzed; the inlet flow is a function of only the valve opening. The process is not typical; usually, the flow out would be pumped, but here it drains by gravity. However, this is a simple system to begin analyzing control systems; more realistic processes will be considered in subsequent chapters.

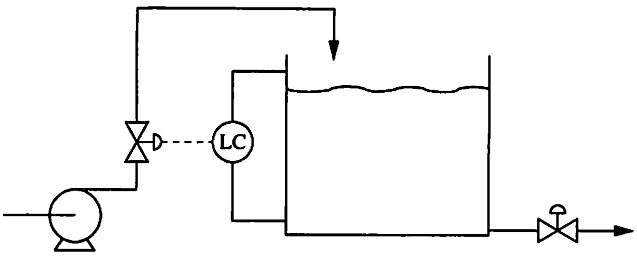


FIGURE Q8.13

(*a*) Derive a linearized model and transfer functions for the process and for the proportional-only controller.
(*b*) Draw a block diagram, and derive the closed-loop transfer function.
(*c*) Calculate the steady-state offset.
(*d*) Select an appropriate sign for the gain and calculate the time to reach 63 percent of the final steady-state error after a step disturbance in the outlet valve position.
(*e*) Discuss the differences between this and question 8.4.

8.14. Consider the PID algorithm in equation (8.23*a*). For each of the individual modes—proportional, integral, and derivative—describe with a sketch the result of its calculation when the error is each of the following idealized functions: (*a*) a constant, (*b*) an impulse, and (*c*) a sine (consider one cycle). (This question provides a thought exercise to help understand the three PID modes; this type of analysis is not performed when monitoring a control system.)

8.15. For the series reactors in Figure Q8.15, the outlet concentration is controlled at 0.414 mole/m^3 by adjusting the inlet concentration with a *proportional-only* feedback controller. At the initial base case operation, the valve is 50 percent open, giving $C_{A0} = 0.925$ mole/m^3. One first-order reaction A $\rightarrow$ B occurs; the data are $V = 1.05$ m^3, $F = 0.085$ m^3/min, and $k = 0.040$ min^{-1}. The process transfer function is derived in Example 4.2 as $C_{A2}(s)/C_{A0}(s) = 0.447/(8.25s + 1)^2$; the additional model relates the valve to inlet concentration, which for a linear valve and small flow of A ($F >> F_A$) gives $C_{A0}(s)/v(s) = 0.925/50 = 0.0185$ (mole/m^3)/%open; you may assume for this question that the sensor dynamics are negligible.
(*a*) Determine whether the reactors are stable *without* feedback control.
(*b*) Determine the closed-loop transfer function for a set point response.
(*c*) By analyzing the denominator of the transfer function (the characteristic polynomial), determine the stability of the feedback system for controller gain, K_c, values of (i) 0.0, (ii) 121, (iii) 605, and (iv) 2420 (in %valve opening/mole/m^3).
(*d*) By analyzing the total closed-loop transfer function, determine the steady-state offset for a set point change with controller gain, K_c, values of (i) 0.0, (ii) 121, (iii) 605, and (iv) 2420 (in %valve opening/mole/m^3).
(*e*) Without simulating, sketch the *general shape* of the dynamic response for a set point step change for each of the cases in (*c*) and (*d*) above.

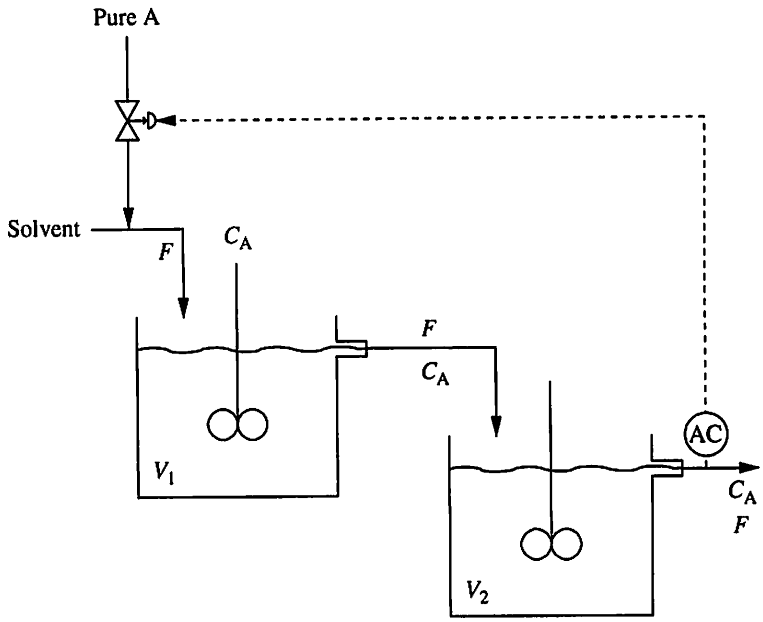


FIGURE Q8.15

8.16. Analyze the following systems for the feasibility of feedback control.

(*a*) Example I.1 with temperature T_3 as the controlled variable, F_{exch} as the manipulated variable, and $\Delta SP = 1°C$.

(*b*) Example I.2 with C_{A2} as the controlled variable, F_s as the manipulated variable, and $\Delta SP = 0.01$ mole/m^3.

8.17. The continuous control system in Figure Q8.17 is to be tuned for an *underdamped* open-loop process, $\xi < 1.0$. As a physical example, you may think of the CSTR with underdamped temperature dynamics in response to a change in the coolant flow described in Section 3.6. However, the question should be answered for the general system in Figure Q8.17.

(*a*) Determine the range of a P-only feedback controller gain that results in an *overdamped* closed-loop system. Discuss the implications of your results for the quality of feedback control performance.

(*b*) Repeat the analysis for a proportional-derivative controller and discuss the effect of the derivative mode on the closed-loop dynamic behavior, especially the periodicity.

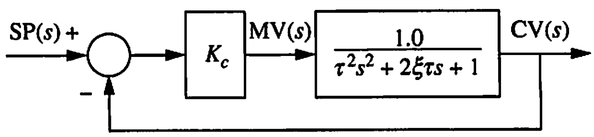


FIGURE Q8.17

8.18. (*a*) Determine the PID controller modes that are required for zero steady-state offset for an impulse disturbance for the following processes:

(1) The three-tank mixing process in Examples 7.2 and 7.3 with x_{AB} an impulse

(2) A non-self-regulating level system, like equation (5.15), with F_0 an impulse and F_1 adjusted by the controller

(*b*) Discuss the application of integral-only control to both processes.

8.19. The elements in several control systems are shown in Figure Q8.19. For each system, determine the transfer functions for $CV(s)/SP(s)$ and $CV(s)/D(s)$, where a disturbance is given.

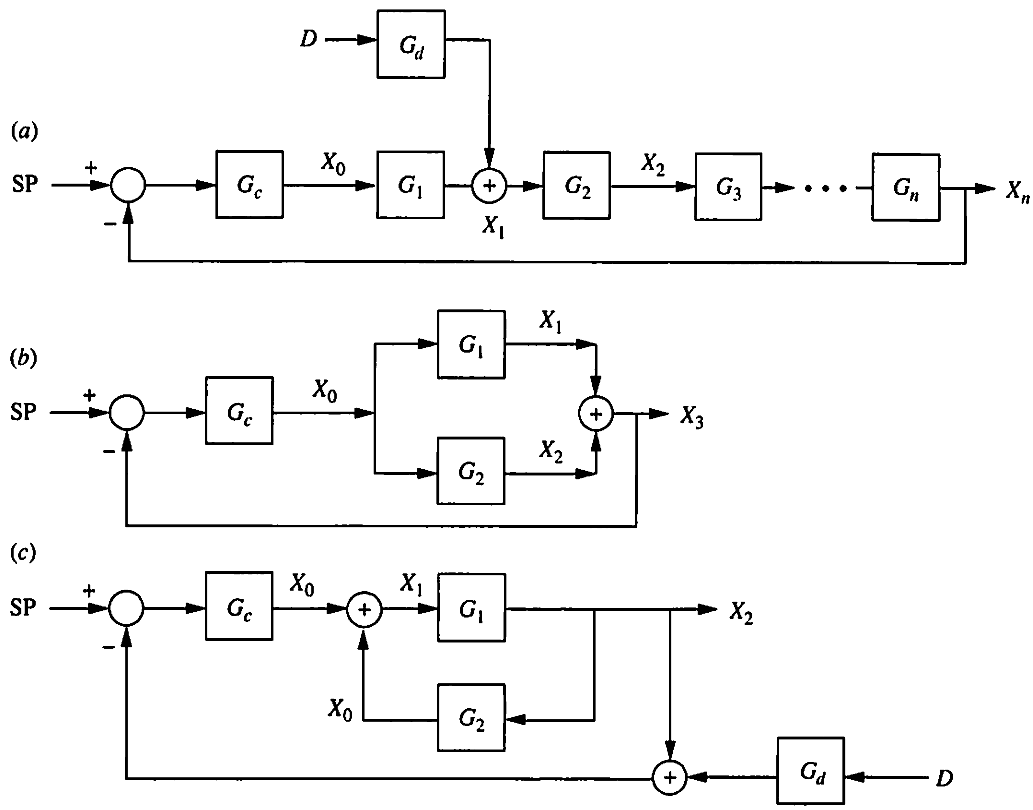


FIGURE Q8.19

Block diagrams for several control systems. All quantities are Laplace-transformed; the variable (s) is omitted for simplicity.