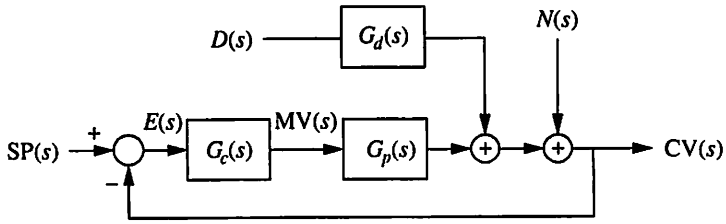
Determining Controller Constants to Satisfy Performance Specifications

APPENDIX E

This appendix presents a procedure for determining the tuning constants for feedback controllers that satisfy robust, time-domain performance specifications. The specifications involve the behavior of the controlled and manipulated variables and include measurement noise and variable process dynamics, as defined in Table 9.1. Because the goals are formulated to minimize the controlled-variable IAE subject to limitation on the manipulated-variable values, the tuning constants are determined using optimization principles. It is not possible to derive analytical expressions relating the tuning constants to the IAE and manipulated-variable transient response; therefore, the control system performance is determined by numerical solution of the model, and the best tuning values are determined using an optimization method.

E.1 □ SIMULATION OF THE CONTROLLED SYSTEM TRANSIENT RESPONSE

The single-loop control system considered in this appendix is shown in Figure E.1. The real system consists of elements that are continuous and cannot be solved analytically. As an approximation, the closed-loop transient is determined by numerical solution of the equations that define the system. As discussed in



CV(s) = controlled variable
D(s) = disturbance
E(s) = error
MV(s) = manipulated variable
N(s) = noise
SP(s) = set point

$G_p(s)$ = process, valve, and sensor
$G_d(s)$ = disturbance
$G_c(s)$ = controller

FIGURE E.1

Block diagram of the feedback control system.

Chapter 3, this approach can provide a set of points very close to the exact transient response—certainly accurate enough for use in the optimization approach.

The model for the feedback process is assumed to be first-order with dead time. As discussed in Appendix F and Section 6.4, this model can be approximated by the following algebraic equation at each time step:

$$(\mathrm{CV_{fb}})_n = K_p(1 - e^{-\Delta t/\tau})\mathrm{MV}_{n-\Gamma-1} + e^{-\Delta t/\tau}(\mathrm{CV_{fb}})_{n-1} \tag{E.1}$$

The dead time is simulated by a delay, $\Gamma = \theta/\Delta t$. In equation (E.1) the dead time must be an integer multiple of Δt, but advanced modelling methods using modified z-transforms enable modelling of systems with noninteger dead times (Ogata, 1987). The model for the effect of the disturbance on the controlled variable is first-order.

$$(\mathrm{CV}_d)_n = K_d(1 - e^{-\Delta t/\tau_D})D_{n-1} + e^{-\Delta t/\tau_D}(\mathrm{CV}_d)_{n-1} \tag{E.2}$$

The noise, $(\mathrm{CV}_N)_n$, is based on a random perturbation passed through a dynamic process (Ciancone, 1990). It has a standard deviation, σ_N. The measured value of the controlled variable is the sum of the three effects.

$$\mathrm{CV}_n = (\mathrm{CV_{fb}})_n + (\mathrm{CV}_d)_n + (\mathrm{CV}_N)_n \tag{E.3}$$

These equations determine the behavior of the controlled variable given the manipulated and disturbance variables. The disturbance D is a step for the disturbance response cases and zero for the set point cases, and the set point is constant for disturbance response cases and a step for set point response cases.

The manipulated variable is determined by the feedback controller. The digital form of the PID controller is explained in Section 11.4 and repeated here:

$$\mathrm{MV}_n = \mathrm{MV}_{n-1} + K_c\left[(\mathrm{SP}_n - \mathrm{CV}_n) - (\mathrm{SP}_{n-1} - \mathrm{CV}_{n-1}) + \frac{\Delta t(\mathrm{SP}_n - \mathrm{CV}_n)}{T_I} + \frac{T_d}{\Delta t}(-\mathrm{CV}_n + 2\mathrm{CV}_{n-1} - \mathrm{CV}_{n-2})\right] \tag{E.4}$$

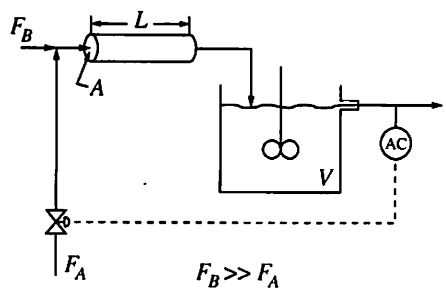
Equations (E.1) through (E.4) are solved at each time step from an initial steady state to a final time of about $6(\theta + \tau)$, which is sufficient to reach essentially the final steady state for a well-tuned system. The process equations and digital controller are executed at a frequency that gives $\Delta t/(\theta + \tau) = 0.1$, which is sufficient to approximate the continuous system closely although not exactly. By this method, the transient is evaluated for any set of tuning constants.

E.2 □ OPTIMIZATION OF THE TUNING CONSTANTS

The "best" values of the tuning constants are those that satisfy the performance goals. One goal requires that the integral of the controlled variable deviation, measured as IAE, be minimum. The IAE can be approximated using the discrete samples of the transient response as

$$\text{IAE} = \int_0^\infty |\text{SP} - \text{CV}|\, dt \approx \sum_{n=1}^{M} |\text{SP}_n - \text{CV}_n|\, \Delta t \qquad \text{(E.5)}$$

with M the number of points in the transient. The second goal requires that model error be considered to ensure a reasonable amount of robustness. The approach used here is to evaluate the entire transient responses for three feedback control systems with *different* process models, each with the *same* controller tuning constants. Thus, the measure of the controlled-variable performance is modified to be



$$\sum_{i=1}^{3} \text{IAE}_i = \sum_{i=1}^{3} \left(\sum_{n=1}^{M} |\text{SP}_n - \text{CV}_n|\, \Delta t \right)_i \qquad \text{(E.6)}$$

To include a range of process dynamics, the model parameters all change in a correlated manner as 75%, 100%, and +125% of their nominal values. This corresponds to changes in the feed flow rate in the example process in Figure 9.1.

The third performance goal places a limitation on the variation of the manipulated variable. Here, the manipulated variable is restricted in the extent to which it may exceed its final steady-state value; the final value (with no measurement noise) would be $-\Delta D/K_p$ or $\Delta \text{SP}/K_p$ for disturbance or set point response, respectively. The region of allowable values for the manipulated variable is large during the initial part of the transient and becomes smaller as the final steady state is reached to prevent excessive oscillations. The final variability is *nonzero,* because higher-frequency noise in the controlled variable is propagated to cause (undesirable but unavoidable) variation in the manipulated variable. Thus, this third goal also includes a bound on the variability of the manipulated variable because of the measurement noise N, which is apparent at the end of the transient response. The equations for the manipulated-variable bound select the least limiting,

$$\begin{aligned}
(\text{MV}_n)_1 &\le \frac{-\Delta D}{K_p} + \left[(\Delta \text{MV}_{\max}) \frac{-\Delta D}{K_p} - \frac{-\Delta D}{K_p} \right] \exp\left[\frac{-t_{\text{MV}}}{\beta_t(\theta + \tau)} \right] \\
(\text{MV}_n)_2 &\le \left(\frac{-\Delta D}{K_p} - \sigma_{\text{MV}} \right) \qquad \text{(E.7)} \\
(\text{MV}_n)_{\min} &= \min\left[(\text{MV}_n)_1,\ (\text{MV}_n)_2 \right] \\
(\text{MV}_n)_{\min} &\le \text{MV}_n
\end{aligned}$$

with time t_{MV} measured from the initiation of the input step. The term ΔD is replaced with $-\Delta\text{SP}$ for a set point change. Several parameters in this equation are related to the dynamic response of the process. Other parameters are fixed at reasonable values selected by the author to suit the widest range of industrial process applications, as given in Table E.1. Naturally, this definition will not be appropriate for all systems, but it should provide good starting values for the tuning of many feedback systems.

Some values of the tuning constants will result in manipulated-variable values that violate the constraints defined in equation (E.7). These values will be considered invalid because of the violation and will not be acceptable, even if they result in a low value for $\sum$IAE. Only tuning constant values that result in the constraints in equation (E.7) being satisfied for the entire transient response will be considered when minimizing $\sum$IAE. This mathematical problem is of the general class of nonlinear, constrained optimization. Determining the best tuning consistent with the goals is conceptually straightforward; the engineer could perform many simulations and, by trial and error, eventually find the best values of the tuning constants. However, the trial-and-error approach would be very time-consuming and require excessive calculations. The approach taken here was to formulate equations (E.1) through (E.7) for all time steps and solve them simultaneously using a method which employs intermediate results to direct the search efficiently toward the best values of the tuning constants (Ciancone, 1990).

The transient responses in Figure E.2*a* through *c* show the results of the optimization for one value of the fraction dead time, the nominal $\theta/(\theta + \tau) = 0.3$.

TABLE E.1

Parameters used in tuning optimization

Factor	Symbol	Value	Comment
Measurement noise	σ_N	0.55% of scale	$\pm 4\sigma_N = \pm 2.2\%$ of scale
Maximum change in MV	$\Delta \text{MV}_{\max}$	2.7	This allows 170% maximum overshoot at $t_{\text{MV}} = 0$ and decreases rapidly as time increases
Tune the time dependence for the allowable change in MV	β_t	1.5	This value reduces the allowable variation rapidly as time increases, damping the response
Allowable variation in MV at steady state, i.e., end of transient	σ_{MV}	2.5% of range	This is approximately $4\sigma_N$, the noise propagated via the proportional mode with $K_c K_p = 1.0$
Disturbance time constant	$\tau_D = \tau$	Depends on case	The disturbance time constant is the nominal feedback time constant
Input step magnitude	$-\Delta$SP or ΔD $(K_d = 1)$	10% of scale	Should be larger than measurement noise and must be of the sign shown for the sign conventions in this appendix
Model error		25% of each parameter	The errors are due to a change in operating conditions in a nonlinear process and thus are correlated (all increase or decrease concurrently)
Execution period	Δt	$0.1(\theta + \tau)$	Relatively small compared with feedback process

The "optimal" tuning for this case is $K_p K_c = 1.4$, $T_I/(\theta + \tau) = 0.7$, and $T_d/(\theta + \tau) = 0.02$, using the model parameters from the nominal case. The transient responses for the three cases, nominal (perfect) model and ±mismatch,

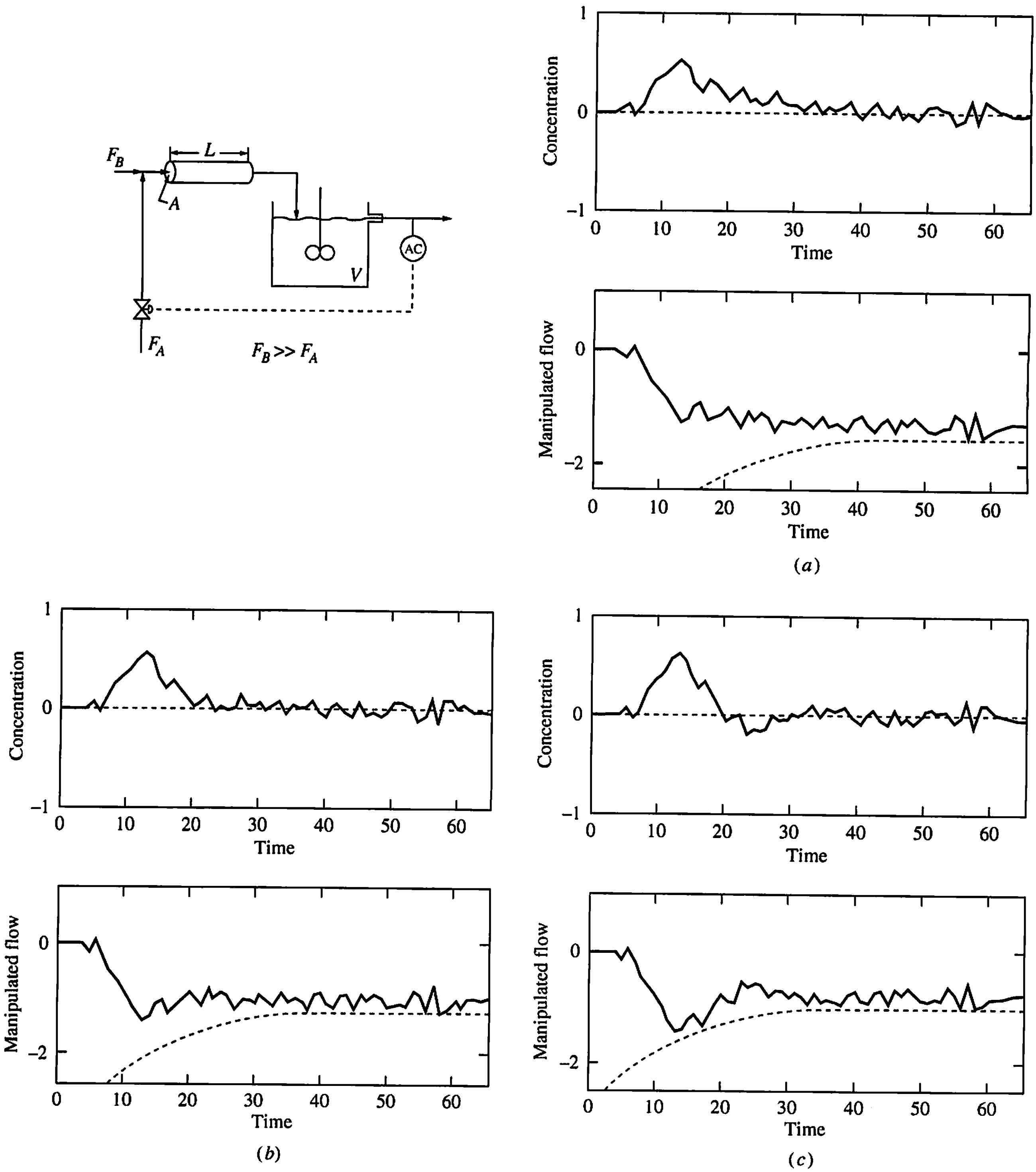


FIGURE E.2

Transient response for $\theta/(\theta + \tau) = 0.3$: (*a*) 75% of nominal feedback model parameters (high flow); (*b*) 100% of nominal feedback model parameters (nominal flow); (*c*) 125% of nominal feedback parameters (low flow).

demonstrate the importance of explicitly considering model error. Note that the feedback control is not too aggressive for the nominal case and is quite slow for the 75% case. However, the 125% case involving a slower process dynamics and a higher feedback process gain (i.e., smaller feed flow in the example process) is at the limit of the allowable manipulated-variable variation and exhibits oscillatory behavior. Thus, making the tuning more aggressive would result in unacceptable behavior for process dynamics for the 125% case, which is considered to occur often in this problem definition. Thus, all three goals are relevant in determining the best initial controller tuning. Finally, model errors larger than anticipated in the definition could cause closed-loop behavior deemed unacceptable; this situation would be rectified during fine tuning.

REFERENCES

Ciancone, R., "Selecting Appropriate Control Technology," undergraduate research project, McMaster University, 1990.

Ogata, K., *Discrete-Time Control Systems*, Prentice-Hall, Englewood Cliffs, NJ, 1987.